Tracking a Moving Vessel from Bearing Measurements

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(Invited Paper)

Abstract—**This paper presents a method for tracking a distant moving target using only bearing measurements obtained from a tracking platform. The method is an improvement of the Hinich–Bloom passive tracking approach presented in [1]. The target is assumed to be moving at constant speed on a fixed heading, whereas the platform maneuvers during the measurement period. The direction cosines of the bearings are computed with respect to a rotation of the coordinate system that places 0° at the mean estimated target bearing. This is done to minimize the approximation bias due to the linearization of sine bearing as a function of inverse range and time. The coordinate system is rotated back to estimate the target coordinates. When the noise is Gaussian, the estimates of target range and heading are approximately maximum likelihood when the target's relative range is slowly varying during the observation period. In this case the mean square errors of the target parameter estimates are the smallest achievable within the order of the approximation.**

I. INTRODUCTION

THIS PAPER considers the problem of estimating the track of a moving target using only bearing measurements obtained from a tracking ship. Suppose that the tracker detects a distant target and then maneuvers for a short period to obtain bearings on the target. These bearings are used to estimate the target's range, heading, and position at the end of the maneuver under the assumption that the target is moving at a constant speed on a fixed heading. A decision maker uses these estimates and their standard errors, along with intelligence information about the target and its environment, to decide on a course and speed for the tracking ship to get closer to the target. This estimation-decision procedure is repeated until the target is intercepted or lost. Success depends, to a great extent, on the use of estimators that are robust to perturbations of the model of the errors in bearing measurements. Simplicity is the main virtue of the least squares batch method presented in this paper.

This paper presents an extension and improvement of the Hinich–Bloom [1] method for estimating the range and heading of a distant constant speed target using noisy bearing measurements from a maneuvering tracking platform. When the target's range from the tracker is slowly varying, the bearing direction cosines are approximately linear functions of time and inverse range. One linear approximation is fitted by the method of least squares to obtain estimates of the linear equation's coefficients.

These coefficient estimates are combined to obtain estimates of the average range to the target and its coordinates at the end of the maneuver. If the target's velocity can be estimated from Doppler shift measurements of the signal spectrum, then the target's heading is determined.

The author's method is an open loop scheme that presents information at the command center aboard the tracking ship so as to facilitate conventional command decisions. Decisions can be made to alter the tracking strategy as a function of the estimates derived from all measurements made during the tracking maneuver. This open loop approach is fundamentally different from the "fire control" tracking approaches that use extended Kalman filters to constantly update estimates (Moose, Valandingham, and McCabe [2], and Hassab, Guimond, and Nardone [3]).

The author's method has several important advantages over the Kalman filter approach for such a decision problem. There exist relatively simple modifications of least squares that yield estimates that are robust to non-Gaussian bearing errors in the sample (an introduction to robust regression methods is given in Hinich and Talwar [4] and Martin [5]). Another advantage is that the computations used are very simple (the estimates can be obtained from a programmable calculator). Simple methods are usually robust to perturbations of the structural assumptions made about the statistical model.

II. SINGLE TARGET STATISTICAL MODEL

Assume that the tracker's sonar is receiving coherent acoustic signals from a distant target. Time delay measurements of wave curvature for acoustic radiation from a distant source have such large measurement errors in conventional sonar processors that the only practical model for the received signal is a plane wave. Let $B(t)$ denote the target's bearing with respect to true North, and let $\hat{B}(t)$ denote the estimated bearing at time t during a tracking maneuver of time duration T.

Bearings are usually obtained by delay-and-sum beamforming, but there is no need here to specify the type of processing used to obtain bearings. The accuracy of $\hat{B}(t)$ is obviously important for accurate tracking, so that accurate tracking requires a sonar system's signal processor to yield unbiased $\hat{B}(t)$ with small root mean square error (rmse).

Assume that the target's motion is sufficiently slowly varying so that the target velocity v_T and heading α_T (with respect to North) can be treated as constants during the tracking segment (see Fig. 1). As a consequence, the target's coordinates at time t are

$$x_T(t) = x_T(0) + v_T t \sin \alpha_T$$

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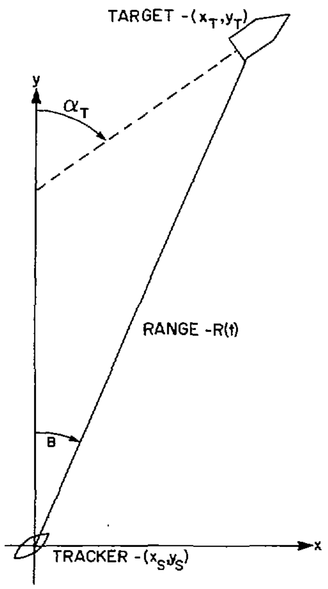

Fig. 1. Geometry of target bearing.

and

$$y_T(t) = y_T(0) + v_T t \cos \alpha_T \tag{1}$$

for the sampling period $0 < t < T$.

Let $R(t)$ denote the target's range from the tracking ship located at $\{x_S(t), y_S(t)\}$ at time t. Assume that the change of range during the segment is small relative to $R = R(T/2)$, the range at the middle of the period. More precisely, assume that

$$\delta = \max_{0<t<T} |R(t)/R - 1| \ll 1. \tag{2}$$

This assumption is reasonable when the range is large and the target and tracking vessels are moving at normal ship speeds.

To facilitate discussion of the direction cosines approximations that motivate the estimation method, suppose that the coordinate system is rotated so that the y-axis cuts through the middle of the target track. This rotation implies that the bearings $B(t)$ are small when R is large for the tracking segment. This rotation will be precisely defined in the next section. It then follows from (2) that the bearing direction cosines are approximated as follows:

$$\sin B(t) = [x_T(t) - x_S(t)]/R + 0(\delta^2) \tag{3}$$

where $x_S(t)$ is the x-coordinate of the tracker and

$$\cos B(t) = [1 - \sin^2 B(t)]^{1/2} = 1 - 0(\delta^2). \tag{4}$$

Note that $\sin B(t)$ is of order $0(\delta)$.

The method presented in this paper uses only $\sin \hat{B}(t)$ to estimate R in contrast to my previous approach. The $\cos \hat{B}(t)$ fit is not used since the dependent variable has a much smaller variance than $\sin \hat{B}(t)$. One equation is all that is needed to estimate the target's track since it will be assumed that v_T is determined from Doppler shift measurements.

Suppose that bearings are obtained at discrete times $t_n = n\tau$, where τ is the integration time for the sonar's signal processor. The data set is $\{\hat{B}(t_n), x_S(t_n): n = 1, \cdots, N = [T/\tau]\}$, and $x_S(t_n)$ is assumed to be known without error. Setting $\hat{s}_n = \sin \hat{B}(t_n)$, it follows from (1) and (3) that the data satisfy the following linear statistical model:

$$\hat{s}_n = R^{-1}x_T(0) + (R^{-1}v_T \sin \alpha_T)t_n - R^{-1}x_S(t_n) + \epsilon_n \tag{5}$$

where the approximation error $0(\delta^2)$ is absorbed into the error term ϵ_n. There is no need to assume here that ϵ_n has a zero mean since any bias in ϵ_n will be absorbed in the constant term, which is not used to estimate the range.

It is tempting to consider using a nonlinear least squares fit of the data to estimate the target parameters $x_T(0)$, $y_T(0)$, and α_T. Several related nonlinear least squares algorithms are discussed in the literature, but the estimates that are obtained by using them are often sensitive to the initial values used, and in some cases the algorithms fail to converge (Draper and Smith [6]). When δ is small, the numerical errors in the nonlinear estimates will be larger than the improvements in accuracy from the estimators derived from the linear approximation (5).

For example, suppose that the bearing errors are Gaussian with a common variance. Then the nonlinear least squares estimators are maximum likelihood.[1] The log likelihood of the sample of $\hat{s}_n$ is approximately a quadratic function of the parameters (and $1/R$) when δ is small. An ordinary least squares fit of the linearized model (5) will then yield approximately maximum likelihood estimators when the ϵ_n have common variance.

III. ESTIMATING RANGE BY LEAST SQUARES

The approximation (5) is best when the middle of the target track segment is on the y-axis. Although such a condition is unlikely for a given maneuver, it can be artificially achieved by rotating the coordinate system by the angle of the *mean bearing* $B = N^{-1} \sum_{n=1}^{N} \hat{B}(t_n)$. In other words, compute the ship's coordinates and the target bearings with respect to a rotated coordinate system whose $\tilde{y}$-axis has the angle B in the true coordinate system. In the new $(\tilde{x}, \tilde{y})$ coordinate system, the average bearing is zero, and the bearings are small.

Rather than cluttering up the notation with tildes over letters, $x_S(t_n)$, α_T, and $B(t_n)$ will now denote the ship and target parameters as measured with respect the *rotated coordinate system.* The reader will be reminded about the relative meaning of these terms wherever needed in the text, and the

[1] This standard result is given in many mathematical statistics texts. One good source is Draper and Smith [6, section 2.6.]

The method can easily be modified to handle unequal bearing error variances. The variance of ϵ_n is inversely proportional to the energy signal-to-noise ratio (SNR) for most processors, such as beamforming to find the beam whose average power is largest (Hinich [7]). The SNR is estimated by $\hat{\gamma}/(1 - \hat{\gamma})$, where $\hat{\gamma}$ is the average coherence between hydrophone channels. Thus a scalar multiple of the variance can be estimated by $(1 - \hat{\gamma}_n)/\hat{\gamma}_n$ computed at t_n. If $\hat{s}_n$, t_n, and $x_S(t_n)$ are multiplied by $[\hat{\gamma}_n/(1 - \hat{\gamma}_n)]^{1/2}$. For each n, then the least squares fit of these multiplied variates yields maximum likelihood estimates of β and b.

final estimates will be expressed in terms of the true coordinates.

To reparameterize (5), let $a = (x_T(0)/R) + \mu$ where μ is the mean of the errors ϵ_n in the rotated system, $\beta = (v_T \sin \alpha_T)/R$, and $b = -1/R$ where $-90° < \alpha_T \leqslant 90°$. Then (5) becomes

$$\hat{s}_n = a + \beta t_n + b x_S(t_n) + \epsilon_n \tag{6}$$

for $n = 1, \cdots, N$. Assume, at first, that the ϵ_n are uncorrelated random variables, a least squares regression of $\hat{s}_n$ on the variables t_n and $x_S(t_n)$ with an intercept yields unbiased and approximately Gaussian estimates of β and b.

To simplify exposition of the least squares estimators, shift the time index so that $n = -N/2 + 1, \cdots, N/2$ (assuming N even).

Define

$$c_{st} = \sum_{n=-N/2+1}^{N/2} (s_n - \bar{s}) t_n$$

$$c_{sx} = \sum_{n=-N/2+1}^{N/2} (\hat{s}_n - \bar{s})(x_S(t_n) - \bar{x}_S)$$

$$c_{tt} = \sum_{n=-N/2+1}^{N/2} t_n^2$$

$$c_{tx} = \sum_{n=-N/2+1}^{N/2} t_n (x_S(t_n) - \bar{x}_S)$$

and

$$c_{xx} = \sum_{n=-N/2+1}^{N/2} (x_S(t_n) - \bar{x}_S)^2 \tag{7}$$

where the overbars denote *sample means*. The least squares estimators of β and b are (from Hinich and Bloom)

$$\hat{\beta} = D_x^{-1}(c_{xx}c_{st} - c_{tx}c_{sx}) \tag{8}$$

$$\hat{b} = D_x^{-1}(c_{tt}c_{sx} - c_{tx}c_{st}) \tag{9}$$

where

$$D_x = c_{tt}c_{xx} - c_{tx}^2. \tag{10}$$

From the triangle inequality, $D_x = 0$ if and only if $x_S(t_n)$ is linear in t_n. Thus $D_x > 0$ if the tracking ship changes course, or changes speed if its heading is not $\boldsymbol{B}$ or $-\boldsymbol{B}$ with respect to true North. The variances of these estimators are

$$\sigma_\beta^2 = \sigma_\epsilon^2 c_{xx}/D_x \tag{11}$$

$$\sigma_b^2 = \sigma_\epsilon^2 c_{tt}/D_x \tag{12}$$

where σ_ϵ^2 is the variance of ϵ_n, and the correlation between them is

$$\rho_{\beta b} = -(c_{tt}c_{xx})^{-1/2} c_{tx}. \tag{13}$$

Note that $\rho_{\beta b} = 0$ if and only if $c_{tx} = 0$.

An example would be helpful at this point. Suppose that the tracking ship is moving in a constant heading with a speed $v_S(t) = 2v(1 - 2|t|/T)$ for $-T/2 < t < T/2$. The average tracker's velocity is v. The tracker accelerates for the first half of the segment and then decelerates to a stop at the end. This example is used to facilitate exposition of the statistical properties of the method, and is not optimal for fixed T, v, and σ_ϵ. More accurate estimates can be obtained by changing course during the maneuver.

Let ϕ denote the tracker's heading with respect to the *rotated* y-axis, i.e., the true heading is $\boldsymbol{B} + \phi$. Setting the origin so that $\boldsymbol{x_S} = 0$, the x-axis trajectory of the tracker is

$$x_S(t_n) = 2n(1 - 2|n|/N)\tau v \sin \phi. \tag{14}$$

The projected speed of the tracker on the rotated x-axis is $v_x = v \sin \phi$.

The following additional approximations are used to approximate σ_b^2 and σ_β^2 for large N:

$$\sum_{n=1}^{N/2} n^2 \cong N^3/24 \qquad \sum_{n=1}^{N/2} n^3 \cong N^4/64$$

and

$$\sum_{n=1}^{N/2} n^4 \cong N^5/160.$$

It then follows that $\boldsymbol{x_S} = 0$, $c_{tt} \cong (N\tau)^2 N/12$

$$c_{tx} \cong v_x (N\tau)^2 N/24 \tag{15}$$

and

$$c_{xx} \cong v_x^2 (N\tau)^2 N/30. \tag{16}$$

Applying (14) and (15) to (10), (11), and (12)

$$D_x \cong v_x^2 T^4 N^2/960 \tag{17}$$

$$\sigma_\beta^2 \cong 32\sigma_\epsilon^2/(T^2 N) \tag{18}$$

and

$$\sigma_b^2 \cong 80\sigma_\epsilon^2/(v_x^2 T^2 N). \tag{19}$$

From (13), the correlation between $\hat{\beta}$ and $\hat{b}$ is $\rho_{\beta b} \cong -0.79$. The maneuver in this example is far from optimal since an optimal maneuver would yield $\rho_{\beta b} = 0$.[2]

Suppose that the bearing errors are correlated over time. If so, assume that they are a sample from a stationary random process with a known covariance function. As long as the covariance is reasonably behaved for large lags, it is possible to design a prewhitening filter that will remove the serial cor-

[2] The circular course at constant speed that is used in the simulations presented in Hinich and Bloom gives estimates where $c_{tx} = 0$. A circular course is optimal when there is no *a priori* information about the target prior to detection.

relation from the dependent and independent variables in expression (6).

Consider the following simple example. Suppose that the ϵ_n satisfy the first order autogregression $\epsilon_n = \gamma\epsilon_{n-1} + u_n$ where $\{u_n\}$ is white noise. Then the linear model (6) can be transformed as follows: Let

$$z_n = \hat{s}_n - \gamma\hat{s}_{n-1} \qquad p_n = t_n - \gamma t_{n-1}$$

and

$$q_n = x_S(t_n) - \gamma x_S(t_{n-1}).$$

Then

$$z_n = (1-\gamma)a + \beta p_n + bq_n + u_n \tag{20}$$

yields unbiased and approximately Gaussian estimates of β and b. The serial correlation in the bearings has been removed by this simple filtering operation.

Assume that $R\sigma_b$ is small. It will now be shown that the precision of the range estimate depends upon $R\sigma_b$, which is proportional to $R\sigma_\epsilon/v_x TN^{1/2}$ for this example.

Since $\hat{b}$ estimates $-1/R$, a natural estimator of R is $\hat{R} = -1/\hat{b}$. This estimator is approximately maximum likelihood if the errors are Gaussian and δ is small. The bias in $\hat{R}/R$ due to the nonlinear transformation is given by

$$E(\hat{R}/R - 1) = (R\sigma_b)^2 + 0((R\sigma_b)^4). \tag{21}$$

The approximate proportional root mean square error (rmse) of $\hat{R}/R$ (or $\hat{R}/R - 1$) is

$$\text{rmse}\,(\hat{R}/R) = R\sigma_b + 0((R\sigma_b)^2). \tag{22}$$

The bias is an order of magnitude smaller than the rmse of $R\sigma_b \ll 1$.

To illuminate the meaning of this approximation condition, consider the example just presented. Applying (19) to (22), the proportional rmse is

$$\text{rmse}\,(\hat{R}/R) \cong \frac{(80)^{1/2}R}{v_x TN^{1/2}}\sigma_\epsilon. \tag{23}$$

Suppose that the target is detected at a range of 25 km. The tracker begins the aforementioned maneuver towards the bearing sequence for a period $T = 6$ min at an average speed of $v =$ 8 nmi/h (247 m/min). Assuming a closing speed of 4 nmi/h, $\delta = 0.03$ from (2) and thus the approximation error is $0(10^{-3})$. Suppose that in the rotated coordinate system, $\sigma_\epsilon =$ 120° and $\phi = 90°$ ($v_x = v$). If $\tau = 1$ s so that $N = 360$, and the standard deviation of the bearing error is $\sigma_\epsilon = 1°$ ($\sigma_\epsilon = 0.017$ rad), then it follows from (23) that the rmse $(\hat{R}/R) \cong 0.135$, and thus the rmse $(\hat{R}) \cong 3.4$ km.

This error is about as small as can be achieved from such a sampling procedure if $\sigma_\epsilon = 1°$. If $\sigma_\epsilon = 0.5°$ and $T = 10$ min with $\tau = 1$ s as before, the rmse $(\hat{R}) \cong 0.8$ km.

IV. ESTIMATING TARGET HEADING

The target's velocity can be accurately estimated from the Doppler shift of a characteristic peak of the signal spectrum in certain ocean environments. Suppose that the target's signal spectrum has at least one peak at a known frequency, and that the tracker's sonar signal processing has the capability to estimate v_T from the Doppler shift. It is then reasonable to assume that v_T is known for purposes of estimating α_T from $\hat{\beta}$ and $\hat{b}$. If σ_β and σ_b are small, then the errors $\epsilon_\beta = \hat{\beta} - \beta$ and $\epsilon_b = \hat{b} - b$ are small. Thus expanding the estimator $\hat{\alpha}_T = \sin^{-1}(\hat{R}\hat{\beta}/v_T)$ in terms of ϵ_β and ϵ_b, it follows that

$$\begin{aligned}\hat{\alpha}_T &= \sin^{-1}[(\beta+\epsilon_\beta)/v_T(-b-\epsilon_b)] \\ &= \alpha_T + (\cos\alpha_T)^{-1}R(v_T{}^{-1}\epsilon_\beta + \epsilon_b\sin\alpha_T) \\ &\quad + 0(\sigma_\beta{}^2) + 0(\sigma_b{}^2).\end{aligned} \tag{24}$$

If the absolute value of $\hat{R}\hat{\beta}/v_T$ is near one, then $\hat{\alpha}_T$ should be computed differently. If the statistic is less than one, then use

$$\hat{\alpha}_T = (+/-)\cos^{-1}[1-(\hat{R}\hat{\beta}/v_T)^2]$$

where the sign is determined by the sign of $\hat{\beta}$. If it is greater than one (but nearly one) then let $\hat{\alpha}_T = 90°$ or 270°.

In general, (24) estimates either α_T or $180° + \alpha_T$. This ambiguity is resolved by the sign of the average of $\hat{B}(t_{n+1}) - \hat{B}(t_n)$, or any other measure of the direction of bearing changes.

From (11)-(13) and (24), the mean square error of α_T is approximately

$$(\cos\alpha_T)^{-2}(R\sigma_\epsilon)^2 D_x{}^{-1}[v_T{}^{-2}c_{xx} - 2v_T{}^{-1}(\sin\alpha_T)c_{tx} + (\sin\alpha_T)^2 c_{tt}]. \tag{25}$$

Applying (18) and (19) to (24) and (25), the rmse of α_T for the example is

$$\text{rmse}\,(\hat{\alpha}_T) \cong \frac{4rR\sigma_\epsilon}{v_x TN^{1/2}\,|\cos\alpha_T|}\ \text{rad} \tag{26}$$

where

$$r^2 = 2(v_x/v_T)^2 - 5(v_x/v_T)\sin\alpha_T + 5(\sin\alpha_T)^2. \tag{27}$$

For example, suppose that $v_T = 10$ nmi/h and $\alpha_T = 150°$ in the rotated coordinate system. From (26) the rmse $(\hat{\alpha}_T) \cong$ 8.5°. For $\alpha_T = 30°$, the rmse $(\hat{\alpha}_T) \cong 2.9°$. The large difference between the accuracy of the estimates for these two directions implies that better *a priori* accuracy can be achieved by a maneuver that involves course as well as speed changes.

V. ESTIMATING TARGET COORDINATES

Set the origin at (x_S, y_S). Then a good and simple estimator of $x_T(0)$ with respect to this origin is $\hat{R}s$, where $s = N^{-1}\sum_{n=1}^N \hat{s}_n$, if the bearings are unbiased. If μ is not zero, then the estimated target coordinates will be biased even if $\hat{R}$ is not. Assume then that $\mu = 0$.

To justify this estimator, note that s is an unbiased estimator of $a = x_T(0)/R$ (it is the least squares estimator of a). Its variance is $\sigma_\epsilon{}^2/N$ and it is uncorrelated with $\hat{\beta}$ and $\hat{b}$ since $t_n =$

$x_S = 0$. From an error expansion similar to expression (24), it follows that the rms error of $\hat{R}s - x_T(0)$ is approximated as follows:

$$\text{rmse}\,(\hat{R}\hat{s}) \cong R(N^{-1}\sigma_\epsilon{}^2 + x_T{}^2(0)\sigma_b{}^2)^{1/2}. \tag{28}$$

A simple estimate of $y_T(0)$ in the rotated coordinate system is $\hat{R}$. Combining $\hat{R}s$, $\hat{R}$, and $\hat{R}\hat{\beta}$, the estimate of $v_T \sin \alpha_T$, the target coordinates at the *end of the segment* are estimated by

$$\tilde{x}_T = \hat{R}\hat{s} + \hat{R}\hat{\beta}T/2 \tag{29}$$

and

$$\tilde{y}_T = \hat{R} + v_T\,(\cos \hat{\alpha}_T)T/2. \tag{30}$$

Reversing the rotation, the coordinate estimates in *conventional coordinates* are

$$\hat{x}_T = \tilde{x}_T \cos B + \tilde{y}_T \sin B \tag{31}$$

and

$$\hat{y}_T = \tilde{y}_T \cos B - \tilde{x}_T \sin B. \tag{32}$$

These estimates can be checked by comparing them to $\hat{R}$ sin $B(T)$ and $\hat{R}$ cos $B(T)$, where $B(T)$ denotes the average of several of the last bearings measured in the segment.

VI. MULTIPLE TARGETS

The method is sufficiently simple to estimate ranges of several targets if they are moving on a constant heading with constant speed. To use the method for each target, an algorithm must be developed to classify bearings to targets when the target tracks cross. The calculations used in the method are sufficiently simple for a computer to try all logical possibilities. The bearings then can be sorted out given the constant heading assumption.

The errors of fit

$$e(t_n) = \hat{s}_n - s - \hat{\beta}t_n - \hat{b}x_S(t_n) \tag{33}$$

can be used to reject wrong classifications. For example, several large $e(t_n)$'s indicate the possibility of a misclassification due to a target crossing. Another diagnostic check is provided by $\hat{R}\hat{\beta}$, which estimates $v_T \sin \alpha_T$. An unreasonable value of $\hat{R}\hat{\beta}$ indicates an improper grouping of bearings.

A development of a multitarget tracking algorithm is beyond the scope of this paper. The results that have been presented for a single target provide a benchmark for the accuracy of any method that determines range from noisy bearings of a distant target.

VII. CONCLUSION

A simple statistical method has been presented for estimating the range, heading, and location of a single target from bearing measurements. The method relies on the critical assumption that the target is moving at a constant speed and heading during the time the bearings are taken. The estimates are approximately unbiased if the target's range is large as compared to distances the tracking ship and the target move during the segment, and the bearings are unbiased. An accurate estimate of range, however, can be obtained from biased bearings if the range is large and the sample size is large relative to the error variance of the bearings. Variances of the estimators are given as a function of the sample size and the coordinates of the tracking ship.

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