

Randomly Modulated Periodicities in Relative Sunspot Numbers

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Abstract

This paper presents a new spectral approach to the study of the periodic variation of relative sunspot cycles and the extent of the randomness in the amplitudes and phases of the harmonic components of the fundamental frequency of the cycles. The new method is called the signal coherence spectrum of a time series that has a randomly modulated periodicity. The data we use is the relative sunspot numbers beginning December 21, 1838 until June 30, 2008 as compiled by the Solar Influences Data Analysis Center (SIDC) at the Royal Observatory of Belgium using the FORTRAN 95 program developed by Hinich (2000). Deterministic sinusoids are often used to model cycles as a mathematical convenience. However, it is time to break away from this simplification in order to model the various periodic signals that are observed in fields ranging from biology, communications, acoustics, astronomy, and the various sciences. We detect a strong coherence at 3966 days (10.86 years) which is consistent with the reported 11-year sunspot cycle. Additionally, we find strongly coherent harmonics at about 20 days (0.8 coherence), 2.75 days (0.87 coherence), and 2.1 days (0.83 coherence). We have no physical

explanation for the randomly modulated periodicities in relative sunspot numbers.

1 INTRODUCTION

Sunspot numbers have been of interest to solar physicists and radio propagation engineers as well as statisticians. The number has been collected into a time series using essentially the same methodology since 1830. This paper presents a new spectral approach to the study of the periodic variation of relative sunspot cycles and the extent of the randomness in the amplitudes and phases of the harmonic components of the fundamental frequency of the cycles. The new method is called the signal coherence spectrum of a time series that has a randomly modulated periodicity. The data we use is the relative sunspot numbers beginning December 21, 1838 until June 30, 2008 as compiled by the Solar Influences Data Analysis Center (SIDC) at the Royal Observatory of Belgium. We employ the concept of a Randomly Modulated Periodicity (RMP) introduced by Hinich (2000) (See Section 3 below) and the FORTRAN 95 program developed by Hinich (2000).¹

Two principle indices of sunspot numbers have been established: the International Sunspot Number and the Boulder Sunspot Number. Both use the same methodology for calculating the number, but use different observatories. Daily International Sunspot data is available from 1818 while American (Boulder) data begins from 1944. In general, the Boulder number is about 25% higher than the International number, but a strong positive correlation exists between the two databases. For the purposes of this paper, we will use the

¹ The data used in this analysis were downloaded from the Solar Influences Data Analysis Center at <http://sidc.oma.be/sunspot-data/dailyssn.php>. Daily numbers have been reported since 1849 and the data from December 21, 1838 to 1849 has some missing days that were interpolated by averaging the days prior and post a missing day.

International Sunspot Number index as compiled by the Solar Influences Data Analysis Center (SIDC).²

Sunspot data is considered by most users to be noisy, demonstrating large changes from day to day and month to month (Hoyt & Schatten 1998; Verdes et al. 2000). As a result, the majority of time series analyses of sunspot data use smoothed numbers, most often a moving monthly or yearly average, the theory being that smoothed numbers are better able to reveal underlying patterns and cycles (Hathaway et al., 1994; Hoyt & Schatten, 1998). The most commonly accepted method for calculating monthly smoothed numbers is the weighted 13-month running average. The calculation centers on a given month and then looks forward and back 6 months from that month with half weight given to the extreme months. All the monthly data provided by the Solar Influences Data Analysis Center are such smoothed numbers.

We use a method for determining the periodic structure of relative sunspot numbers and the modulations in the harmonics without resorting to standard averaging techniques used in the solar physics literature.

Periodic signals in nature have some variability from period to period. While a true sinusoidal time series will have a perfectly deterministic pattern with zero bandwidth, such a pattern does not occur in nature as all signals have some intrinsic variability over time (Hinich & Serletis, 2006). The use of averaging techniques such as those described above to determine cyclic patterns in the time series necessarily forces the loss of actual data. Sunspot data is not noise – it is real data that needs to be fully accounted for in analysis.

² Both International and American data are available from the National Geophysical Data Center website: <http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html>.

2 RELATIVE SUNSPOT NUMBERS

While evidence of tracking sunspots by Chinese astronomers can be found as early as 23 BCE, the first modern documentation is generally considered to be a short article entitled, "Solar Observations during 1843," by Heinrich Schwabe (1844). Schwabe was an amateur astronomer who had observed sunspots between 1826 and 1843, recording them on every clear day. He had actually been attempting to locate Vulcan, a planet then thought to orbit between Mercury and the sun. In his efforts, he carefully drew the spots on the sun in order to detect the planet. He first published his data in 1838 and by 1843, he recognized a pattern of sunspot numbers and postulated a 10 year cycle (Schwabe 1838, 1844; Izenman 1983).

Some years later, Rudolf Wolf, the director of the Berne (Switzerland) Observatory and later director of the Zürich Observatory, came across Schwabe's paper, became interested, and on December 4, 1847 began his own observations (Wolf 1848; Izenman 1983). Additionally he reviewed older records in an attempt to find other sunspot observations and was able to approximate numbers beginning in the year 1610, the year the telescope was invented. Based on that data, in 1852 he calculated a sunspot cycle of 11.1 years between maxima (Wolf 1852; Izenman 1983).³

Realizing that observer conditions were not equal, Wolf developed a methodology to record the number of sunspots, creating a measurement known as the relative sunspot number (R_z), also known as the international sunspot number, Wolf number, or Zürich number. Beginning about 1853, he attempted to compute "relative numbers" by recording the number of sunspot groups observed and adding to that one-tenth of the total individual sunspots. He continued to refine his calculations, finally publishing the method used today: $R_z = k(10g + s)$ where R_z is the relative (Zürich) sunspot number, s is the

³ Some confusion has existed about whether sunspot data was developed by J. R. Wolf or H. A. Wolfer. See Izenman (1983) for a good discussion of the discrepancies in some past literature.

number of individual spots, g is the number of sunspot groups, and k is a factor that varies with location and instrumentation (also known as the *observatory factor*).⁴ He also recorded monthly mean relative sunspot numbers. The Wolf number calculation has remained unchanged since the first publication (Wolf, 1861; Izenman 1983). Despite the apparent arbitrariness of the formula, it has proven to correlate strongly with other, more recently discovered, indices of solar activity such as the 10.7cm solar flux (Hathaway et al. 2002, 359).

The most reliable sunspot data begins in 1849 when Wolf began daily recording and is generally considered the beginning of modern data recording (Waldmeier, 1961; McKinnon, 1987). He reconstructed older data, including that of Schwabe and others, and was able to push the database back to 1818. The period 1818-1848 contained an average of 260 observations per year and, because of some assumptions Wolf made in the reconstruction, it is not as accurate as the later data (Wilson, 1998). Other reconstructions pushed sunspot data back as far as 1749 and some as far back as 1610 when the telescope was invented. For a summary of sunspot cycles, see Appendix A.

Believing that relative sunspot numbers created a database that contained too much variance, Hoyt and Schatten (1998) proposed a new method of recording solar activity. With an understanding that sunspot groups were more important than individual sunspot numbers, something even Wolf recognized with his original relative sunspot number calculation, a monthly relative sunspot group time series, albeit with some gaps in the earlier years, was developed back to 1610 using known data (Hathaway et al., 2002). This time series was designed with more internal self-consistency, i.e., less dependent on the measurement of individual sunspots, and specifically to be less noisy than

⁴ This factor takes into account varying observer variables such as the type of instrument used to observe the sun as well as local atmospheric visibility conditions.

the Wolf relative sunspot number.⁵ The relative sunspot group number is given by: $R_G = \frac{1}{N} \sum_{i=1}^n k_i 12.08^{g_i}$ where N is the number of observers, k_i is the correction factor for observer i, and g_i is the number of sunspot groups reported by observer I (Hoyt and Schatten, 1998).

Sunspot numbers are indicative of solar activity that has impact on global climate, radio frequency propagation, and potential damage to satellites. Predicting sunspot numbers has been of interest for some time. Yule (1927) used his stationary stochastic “autoregressive process” to analyze sunspot numbers using the series from 1749 to 1924 (Moran, 1954.). In the years since, numerous observers have identified certain characteristics of sunspot numbers and cycles. Some of the more important cycles and modulations are summarized in Appendix B.

3 *RANDOMLY MODULATED PERIODICITY*

All signals that appear to be periodic have some sort of variability from period to period regardless of how stable they appear to be in a data plot. A true sinusoidal time series is a deterministic function of time that never changes and thus has zero bandwidth around the sinusoid’s frequency. Bandwidth, a term from Fourier analysis, is the number of frequency components that are needed to have an accurate Fourier sum expansion of a function of time. A single sinusoid has no such expansion. A zero bandwidth is impossible in nature since all signals have some intrinsic variability over time.

⁵ The relative sunspot group number is represented by R_G as opposed to the Relative Zurich number that is represented as R_Z . Sunspot group data is maintained separately by the National Geophysical Data Center. Group data is available for download from: <http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html>.

Deterministic sinusoids are used to model cycles as a mathematical convenience. It is time to break away from this simplification in order to model the various periodic signals that are observed in fields ranging from biology, communications, acoustics, astronomy, and the various sciences.

Hinich (2000) introduced a parametric statistical model, called the Randomly Modulated Periodicity (RMP) that allows one to capture the intrinsic variability of a cycle. A discrete-time random process $x(t_n)$ is an RMP with period $T = N\tau$ if it of the form

$$(3.1) \quad x(t_n) = s_0 + \frac{2}{N} \sum_{k=1}^{N/2} \left[(s_{1k} + u_{1k}(t_n)) \cos(2\pi f_k t_n) + (s_{2k} + u_{2k}(t_n)) \sin(2\pi f_k t_n) \right]$$

where $t_n = n\tau$, τ is the sampling interval, $f_k = \frac{k}{T}$ is the k -th Fourier frequency, and where for each period the $\{u_{11}(t_1), \dots, u_{1,N/2}(t_n), u_{21}(t_n), \dots, u_{2,N/2}(t_n)\}$ are random variables have zero means and a joint distribution that has the following finite dependence property: $\{u_{jr}(s_1), \dots, u_{jr}(s_m)\}$ and $\{u_{ks}(t_1), \dots, u_{ks}(t_n)\}$ are independent if $s_m + D < t_1$ for some D and all $j, k = 1, 2$ and $r, s = 1, \dots, N/2$ and all times s_1, \dots, s_m and t_1, \dots, t_n . Finite dependence is a strong mixing condition Billingsley (1979). These time series are called “modulations” in the signal processing literature.

The process can be written as $x(t_n) = s(t_n) + u(t_n)$ where

$$(3.2) \quad s(t_n) = E[x(t_n)] = s_0 + \frac{2}{N} \sum_{k=1}^{N/2} \left[s_{1k} \cos(2\pi f_k t_n) + s_{2k} \sin(2\pi f_k t_n) \right] \text{ and}$$

$$(3.3) \quad u(t_n) = \frac{2}{N} \sum_{k=1}^{N/2} \left[u_{1k} \cos(2\pi f_k t_n) + u_{2k} \sin(2\pi f_k t_n) \right]$$

Thus, $s(t_n)$ the expected value of the signal $x(t_n)$ is a periodic function. The fixed coefficients s_{1k} and s_{2k} determines the shape of $s(t_n)$.

To produce an example of an RMP we wrote a FORTRAN program to compute a simulation of a RMP whose amplitudes are determined from the sunspot data

analysis presented in Section 6. These amplitudes are computed from the signal coherence spectrum of the sunspot data. The signal coherence spectrum is defined in the next section.

We used the estimated signal coherence of the fundamental of the sunspot data and the coherence estimates of the first 29 harmonics. The period is 125 time ticks. The cosine and sine modulations have the same amplitude and the stochastic components are the realizations of an AR(1) with a lag coefficient of 0.75.

A realization from this simulation is shown in Figure 1. The peak to peak "periods" are 128, 123, 124, 126, 125, and 123. The random components of the modulations produce a variance of the peak to peak "periods" whose mean is the 125 period.

(Figure 1 – 29 Harmonics AMP Example about here)

4 SIGNAL COHERENCE SPECTRUM

To provide a measure of the modulation relative to the underlying periodicity Hinich (2000) introduced a concept called the *signal coherence spectrum* (SIGCOH). For each Fourier frequency $f_k = \frac{k}{T}$ the value of SIGCOH is

$$(4.1) \quad \gamma_x(k) = \sqrt{\frac{|s_k|^2}{|s_k|^2 + \sigma_u^2(k)}}$$

where $s_k = s_{1,k} + is_{2,k}$ is the amplitude of the kth sinusoid written in complex variable form, $i = \sqrt{-1}$, $\sigma_u^2(k) = E|U(k)|^2$ and

$$(4.2) \quad U(k) = \sum_{n=0}^{N-1} u_k(t_n) \exp(-i2\pi f_k t_n)$$

is the discrete Fourier transform (DFT) of the modulation process $u_k(t_n) = u_{1k}(t_n) + i u_{2k}(t_n)$ written in complex variable form.

Each $\gamma_x(k)$ is in the (0, 1) interval. If $s_k = 0$ then $\gamma_x(k) = 0$. If $U(k) = 0$ then $\gamma_x(k) = 1$. The SIGCOH measures the amount of “wobble” in each frequency component of the signal $x(t_n)$ about its amplitude when $s_k > 0$. If the SIGCOH values are not close to one, then the shapes of the periodicity randomly varies in that the times between peaks and between valleys are not constant and the amplitudes vary over time.

The *amplitude-to-modulation standard deviation* (AMS) function is $\rho_x(k) = \frac{|s_k|}{\sigma_u(k)}$

for frequency f_k . Thus $\gamma_x^2(k) = \frac{\rho_x^2(k)}{\rho_x^2(k) + 1}$ is a monotonically increasing function

of this AMP function. Inverting this relationship it follows that $\rho_x^2(k) = \frac{\gamma_x^2(k)}{1 - \gamma_x^2(k)}$.

The relationship between the signal coherence values $\gamma_x(k)$ and the AMS values $\rho_x(k)$ is shown in Table 1.

To estimate the SIGCOH suppose that we know the fundamental period and we observe the signal over M such periods. The m th period is $\{x((m-1)T + t_n), n = 0, \dots, N-1\}$. The estimator of $\gamma(k)$ introduced by Hinich (2000) is

$$(4.3) \quad \hat{\gamma}(k) = \sqrt{\frac{|\bar{X}(k)|^2}{|\bar{X}(k)|^2 + \hat{\sigma}_u^2(k)}}$$

where $\bar{X}(k) = \frac{1}{M} \sum_{m=1}^M X_m(k)$ is the sample mean of the DFT

$X_m(m) = \sum_{n=0}^{N-1} x((m-1)T + t_n) \exp(-i2\pi f_m t_n)$ and $\hat{\sigma}_u^2(k) = \frac{1}{M} \sum_{m=1}^M |X_m(k) - \bar{X}(k)|^2$ is the

sample variance of the residual DFT $X_m(k) - \bar{X}(k)$.

5 DETRENDING THE DATA

There is a subtle trend in the daily data that requires detrending. Suppose that the time series is of the form $y(t_n) = a(t_n) + x(t_n)$ where $a(t_n)$ is a sampled smooth additive trend and $x(t_n)$ is an RMP. Even a simple linear trend $a(t_n) = t_n$ will introduce false coherence peaks in the signal coherence function because there are strong harmonic components of the Fourier transform of a line.

A simple and efficient approach to detrending the time series is to fit it with a set of orthogonal polynomials such as the Legendre polynomials using least squares. The residual of the least squares fit is the detrended process. For most smooth trends, it is sufficient to use the first four Legendre polynomials:

$$(5.1) \quad \begin{aligned} L_1(t_n) &= t_n, \quad L_2(t_n) = \frac{1}{2}(3t_n^2 - 1) \\ L_3(t_n) &= \frac{1}{2}(5t_n^3 - 3t_n), \quad L_4(t_n) = \frac{1}{8}(35t_n^4 - 30t_n^2 + 3) \end{aligned}$$

The adjusted R^2 for the least squares fit of these polynomials to the data is 0.09. The coefficients are $\beta(1) = 15.77$, $\beta(2) = 7.91$, $\beta(3) = -26.21$ and $\beta(4) = -21.03$. The intercept is 55.78.

The power spectrum of the detrended data is shown on Figure 1. If the cycle were nearly deterministic, the spectrum would have large peaks at the fundamental period and at the harmonics that are non-zero in the Fourier expansion of the cycle. But the spectrum does not have large peaks except for the fundamental period of 3966 days as estimated by the use of the SIGCOH method.

(Figure 2 - Power spectrum of the detrended 1838 - data here)

6 **RMP IN RELATIVE SUNSPOT NUMBERS**

We tried a number of fundamental periods around the well known near 11 year (Schwabe) solar cycle period. The primary coherent signal of 3966 days was determined by a search method to permit the data to determine the fundamental rather than using a frequency determined by the authors. The method is sensitive to the frame length used to estimate the signal coherence function (SCF). Coherence is lost if the frame length is different from the fundamental period. In order to find the period, a search over a range of SCF estimates for different fundamental values is performed and the strongest value is taken as the SCF estimate.

The largest signal coherence value for the fundamental is 3966 days (10.86 years). The signal coherence spectrum is shown in Figure 3 for harmonics up to 20.0303 days with a floor for coherencies larger than 0.6. Note that the signal coherence estimate for the fundamental is 0.897, which is an AMS of only 2.03.

(Figure 3 Signal coherence spectrum of the detrended 1838 – data here)

In addition to the fundamental period of 3966 days, the coherent harmonics with a signal coherence greater than 0.7 are 92.2326 days (0.75), 36.7222 days (0.71), 33.6102 days (0.71), and 21.6721 days (0.70). The cycle has a lot of modulation since the standard deviations of the detrended data and the modulation respective are 49.5 and 37.8 and thus the cycle's variance is only 41.7% of the variance of the detrended data.

Since the sunspot numbers between December 21, 1838 to June 30, 2008 that we used has some interpolated values for missing days we also computed the signal coherence and power spectra for the series with no missing days, which starts on December 23, 1848. The adjusted R^2 for the least squares fit of the four polynomials to the data is also 0.09. The coefficients are

$\beta(1) = 17.63$, $\beta(2) = 4.53$ $\beta(3) = -29.83$ and $\beta(4) = -14.40$. The intercept is 55.81.

The power spectrum of the detrended complete time series is shown in Figure 4 and the signal coherence spectrum is shown in Figure 5. The fundamental for this detrended data segment is also at 3966 days with a coherence of 0.901. The coherent harmonics with a signal coherence greater than 0.7 for this segment is at 1983 days (0.712), 92.2326 days (0.718), 33.6102 days (0.766), 32.2439 days (0.701), 22.0333 days (0.7709), and 21.6721 days (0.725). The 36.7222 days harmonic for this signal had a coherence of 90.0.652. The pattern is almost the same for the longer segment but with two additional coherent harmonics, 32.2439 and 22.0333 days. The standard deviations of the detrended data and the modulation respective are 49.8 and 37.4 and thus the cycle's variance is only 41.7% of the variance of the detrended data, which is almost the same as for the longer period.

(Figure 4 Power spectrum of the detrended 1848 – data here)

(Figure 5 Signal coherence spectrum of the detrended 1848 – data here)

7 CONCLUSION

By using signal coherence spectral analysis it is possible to understand the underlying cycles and modulations in relative sunspot numbers without the necessity of resorting to various arithmetic smoothing or averaging techniques.

Solar cycle 23 is at its minimum as this article is being written (November 2008). While a “reverse sunspot” indicative of a new cycle was spotted as early as January 2008, and an additional sighted on July 31, 2008, there is no consensus that the old cycle has ended or the new cycle begun. Indeed, sunspot cycle 23 spots have been spotted as late as March 2008 indicating that the minimum is still with us. It is not unusual for sunspots with the magnetic

polarization of both the old and new cycles to be present simultaneously and is common during the period surrounding the minima.

Appendix A: Minima and Maxima of Sunspot Number Cycles⁶

Sunspot Cycle Number	Year of Min*	Smallest Smoothed Monthly Mean**	Year of Max*	Largest Smoothed Monthly Mean**	Rise to Max (Yrs)	Fall to Min (Yrs)	Cycle Length (Yrs)
-	1610.8	--	1615.5	--	4.7	3.5	8.2
-	1619.0	--	1626.0	--	7.0	8.0	15.0
-	1634.0	--	1639.5	--	5.5	5.5	11.0
-	1645.0	--	1649.0	--	4.0	6.0	10.0
-	1655.0	--	1660.0	--	5.0	6.0	11.0
-	1666.0	--	1675.0	--	9.0	4.5	13.5
-	1679.5	--	1685.0	--	5.5	4.5	10.0
-	1689.5	--	1693.0	--	3.5	5.0	8.5
-	1698.0	--	1705.5	--	7.5	6.5	14.0
-	1712.0	--	1718.2	--	6.2	5.3	11.5
-	1723.5	--	1727.5	--	4.0	6.5	10.5
-	1734.0	--	1738.7	--	4.7	6.3	11.0
-	1745.0	--	1750.3	92.6	5.3	4.9	10.2
1	1755.2	8.4	1761.5	86.5	6.3	5.0	11.3
2	1766.5	11.2	1769.7	115.8	3.2	5.8	9.0
3	1775.5	7.2	1778.4	158.5	2.9	6.3	9.2
4	1784.7	9.5	1788.1	141.2	3.4	10.2	13.6

⁶ "Sunspot Numbers," National Geophysical Data Center, Solar Data Services. Retrieved on 3 April 2007 from

ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/maxmin.new.

5	1798.3	3.2	1805.2	49.2	6.9	5.4	12.3
6	1810.6	0.0	1816.4	48.7	5.8	6.9	12.7
7	1823.3	0.1	1829.9	71.7	6.6	4.0	10.6
8	1833.9	7.3	1837.2	146.9	3.3	6.3	9.6
9	1843.5	10.5	1848.1	131.6	4.6	7.9	12.5
10	1856.0	3.2	1860.1	97.9	4.1	7.1	11.2
11	1867.2	5.2	1870.6	140.5	3.4	8.3	11.7
12	1878.9	2.2	1883.9	74.6	5.0	5.7	10.7
13	1889.6	5.0	1894.1	87.9	4.5	7.6	12.1
14	1901.7	2.6	1907.0	64.2	5.3	6.6	11.9
15	1913.6	1.5	1917.6	105.4	4.0	6.0	10.0
16	1923.6	5.6	1928.4	78.1	4.8	5.4	10.2
17	1933.8	3.4	1937.4	119.2	3.6	6.8	10.4
18	1944.2	7.7	1947.5	151.8	3.3	6.8	10.1
19	1954.3	3.4	1957.9	201.3	3.6	7.0	10.6
20	1964.9	9.6	1968.9	110.6	4.0	7.6	11.6
21	1976.5	12.2	1979.9	164.5	3.4	6.9	10.3
22	1986.8	12.3	1989.6	158.5	2.8	6.8	9.7
23	1996.4***	8.0	2000.3***	120.8	4.0		

Mean Cycle Values: 6.1 113.2 4.7 6.3 11.0

*When observations permit, a date selected as either a cycle minimum or maximum is based in part on an average of the times extremes are reached in the monthly mean sunspot number, in the smoothed monthly mean sunspot number, and in the monthly mean number of spot groups alone. Two more measures are used at time of sunspot minimum: the number of spotless days and the frequency of occurrence of "old" and "new" cycle spot groups.

**The smoothed monthly mean sunspot number is defined here as the arithmetic average of two sequential 12-month running means of monthly mean numbers.

***May 1996 marks the mathematical minimum of Cycle 23. October 1996 marks the consensus minimum determined by an international group of solar physicists. April 2000 marks the mathematical maximum of Cycle 23. However, several other solar indices (e.g., 10.7 cm solar radio flux) recorded a higher secondary maximum in late 2001.

Authors' Note: Cycle 23 is at or near minima as this is being written (February 2008) and some authorities believe cycle 24 may be starting.

Appendix B: Observed Effects in the Sunspot Data

- The *Waldmeier Effect* (Waldmeier, 1935, 1939). A negative correlation between cycle amplitude and the period between cycle minima and maxima. Cycles with large amplitudes rise faster than those with lower amplitudes.
- The *Amplitude – Period Effect* (Chernosky, 1954; Wilson, Hathaway, and Reichmann, 1998). A negative correlation between cycle amplitude and the length of the previous cycle as measured minima to minima.
- The *Amplitude – Minimum Effect* (Wilson, Hathaway, and Reichmann, 1998). A positive correlation between cycle amplitude and the activity level at the previous minima.
- The *Even – Odd “Gnevyshev” Effect* (Gnevyshev and Ohl, 1948; Vitinskii, 1965; Wilson, 1992). Even numbered cycles are smaller than their odd numbered successors.
- The *Secular Trend* (Wilson, 1988). The general increase in cycle amplitude since the Maunder Minimum (1645 – 1715).
- The 11 year *Schwabe Cycle*: The readily observable rise and fall of sunspots over an approximate 11 year period between minima.
- The 22 year *Hale Cycle* (Bray & Laughhead 1964; Priest 1982; Murphy et al. 1994.: At the end of each Schwabe cycle, the magnetic field of the Sun reverses. Two such reversals encompass two Schwabe cycles, or about 22 years.
- The 88 year Gleissberg Cycle: Observed as a 70-100 year amplitude modulation of the Schwabe Cycle (Gleissberg, 1939, 1971; Sonnett and Finney, 1990). Braun, et al, (2005).

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29 Harmonics RMP Example with AR(1) Modulation & Period = 125

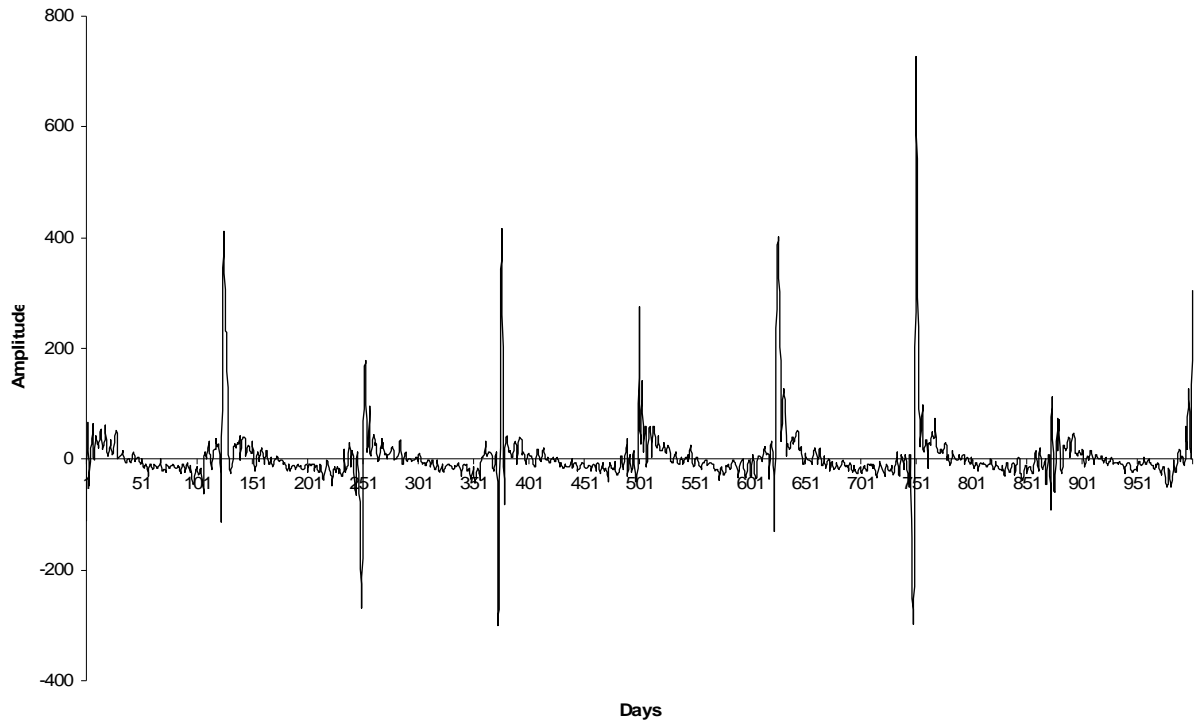


Figure 1

Power Spectrum of Detrended Daily Sunspot Numbers (12-21-1838, 6-30-2008)

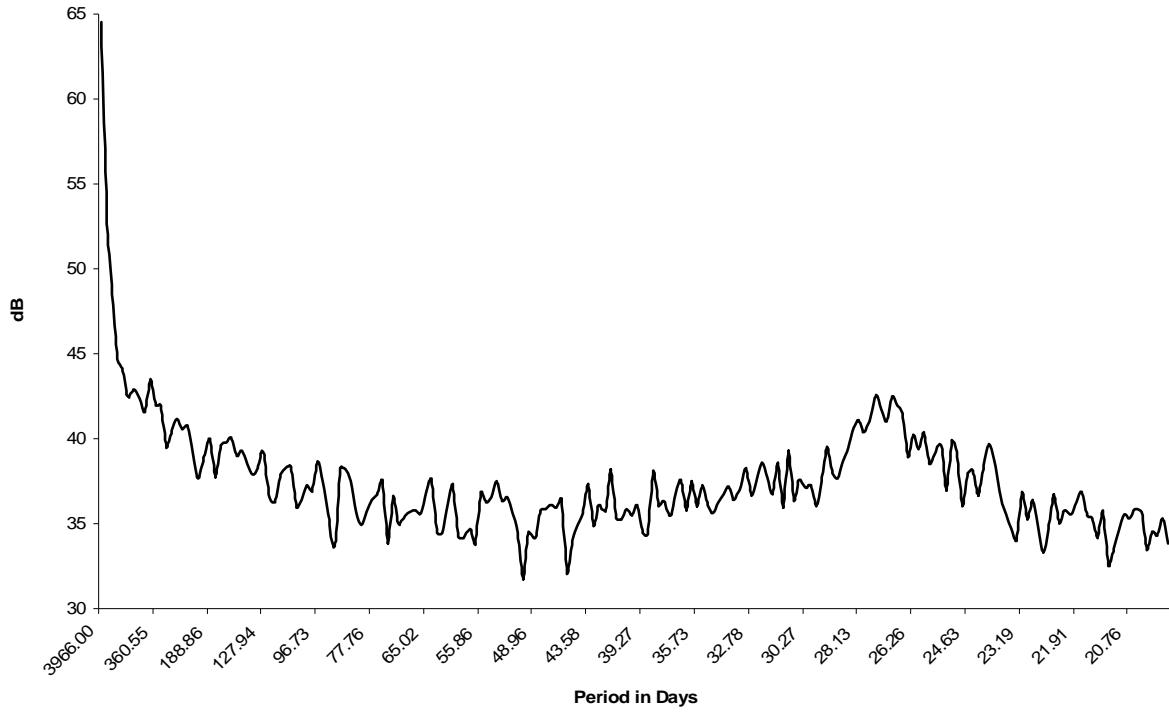


Figure 2

Signal Coherence Spectrum of Detrended Daily Sunspot Numbers (12-21-1838, 6-30-2008)

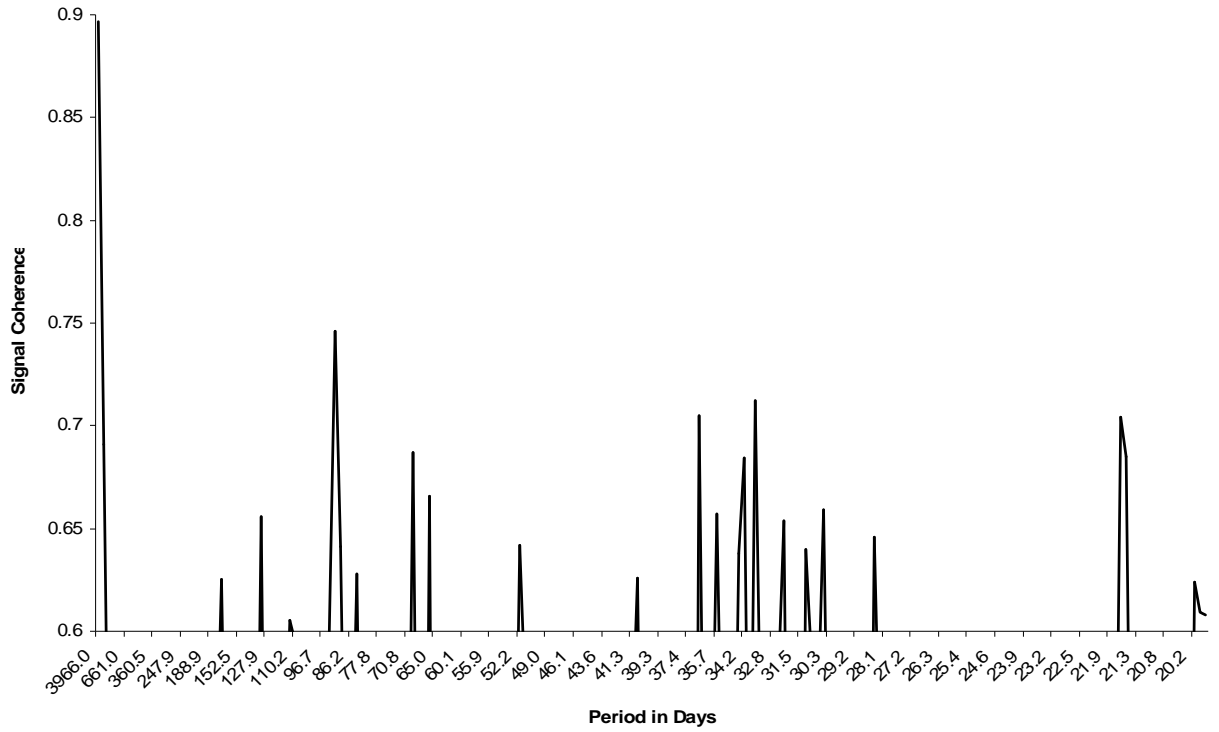


Figure 3

Power Spectrum of Detrended Daily Sunspot Numbers (12-23-1848, 6-30-2008)

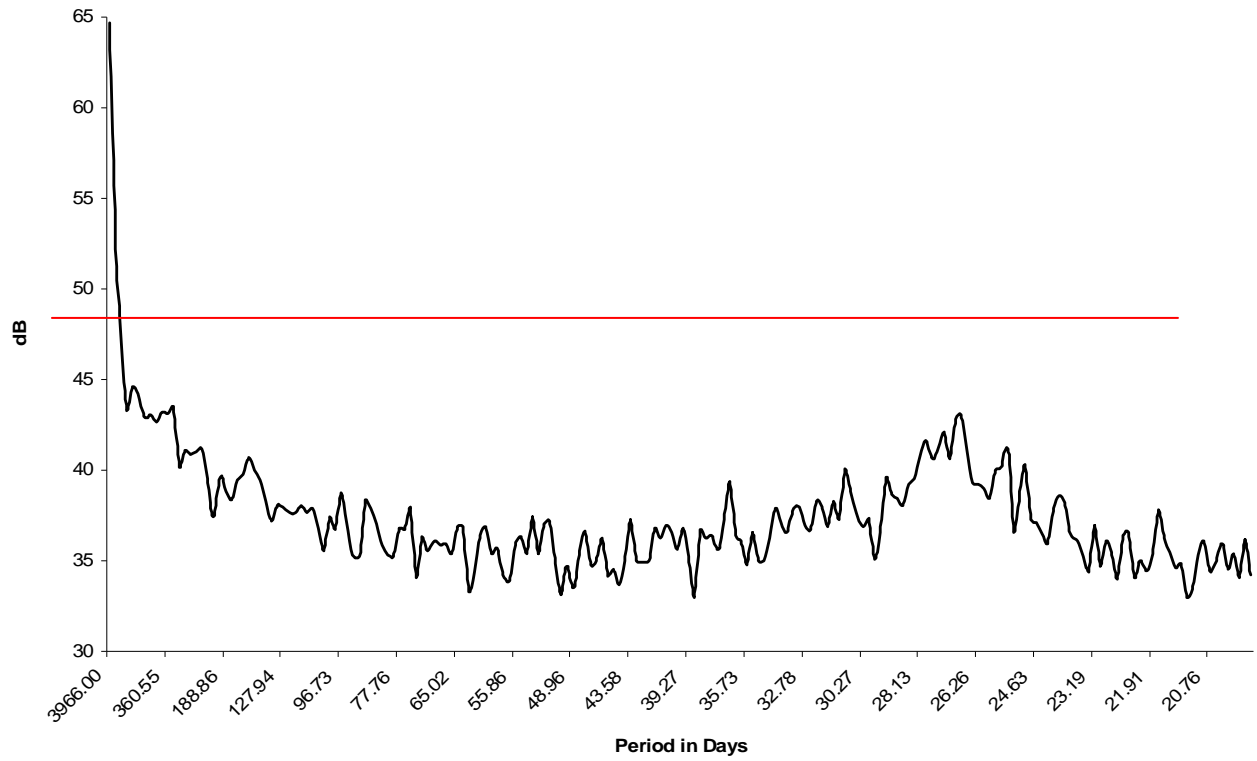


Figure 4

Signal Coherence Spectrum for Detrended Sunspot Numbers (12-23-1848, 6-30-2008)

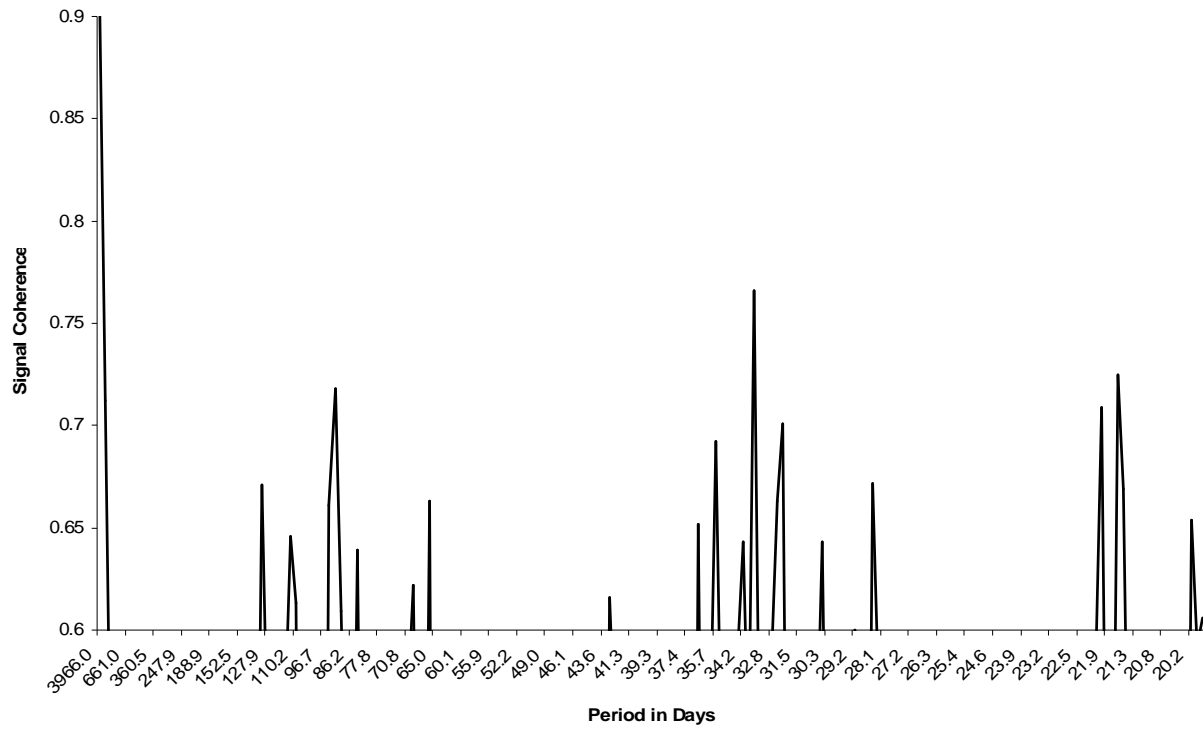


Figure 5

Signal Coherence	AMS
0.95	9.26
0.90	4.26
0.85	2.60
0.80	1.78
0.75	1.29
0.70	0.97
0.60	0.56
0.50	0.33
0.40	0.19

Table 1 – Relationship between SIGCOH and AMS