Detection of EM Pulses Modeled as Randomly Modulated Periodicity

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Abstract

Detection of underwater electromagnetic pulses at low frequencies is complicated due to waveform distortion even at relatively short propagation distances mainly caused by the conductivity of the sea water. Additionally, the individual pulses exhibit variability in shape, amplitude and time of arrival caused by random variations in the propagating media. The detection performance of a estimator-correlator detector is highly dependent on the quality of the signal estimator in order to form the correlator replica. Here we introduce an alternative approach for the signal estimation by means of signal coherence. The replicas based on the coherent part of the mean frame demonstrate potential to serve as a representative candidate to the waveform of the pulses to be detected. The detection performance for both simulations and real data show promising results.

1 Introduction

There is an increased interest towards applications in underwater active electromagnetic systems. The attenuation and propagation velocity of electromagnetic (EM) pulses in sea water are highly frequency dependent. The waveform distortion of the pulses is high even at a moderate distance due to the conductivity of the water. Therefore, the shape of the transmitted EM-pulse can be optimized for a specific propagation distance in order to maximize the received pulse amplitude.

Song and Chen [6] found a time domain solution for the optimum antenna current at a particular distance with a maximum intensity. However, often in real situations the propagation distance varies. One possibility to circumvent this problem is to generate a bank of different pulses within a certain range. The less distorted pulse or the mean frame, based on stacking several pulses, is used for detection. However, this is difficult if the pulse-to-pulse random variation is large both in time and amplitude. Also, it may be impractical in underwater surveillance. Another way to increase detection and estimation performance is to introduce an improved signal model. The EM signal model we use here belongs to a recently introduced class of signals known as Randomly Modulated Periodicity (RMP), e.g. [2]. With RMP a periodic signal is modeled as a sum of complex sinusoids with an additive random modulation. This modulation accounts for a broadening of the spectrum around the fundamental frequency and a small time jitter that is inevitable in many real-world applications. If this jitter cause large random variations the conventional mean frame estimate is not the best correlator replica to be used in detection.

The RMP model can be described by two components; harmonic and modulation. We estimate the signal coherence, which is a measure of the jitter at the particular frequency. The signal coherence is a generalization of the concept of signal-to-noise ratio (SNR) referring to the (statistical) degree of deviation from a pure sinusoid in a harmonic signal. After thresholding the coherence spectra and applying the inverse Fourier transform, the remaining coherent signal is used as a EM pulse replica in a estimator-correlator detector.

The authors would like to thank PhD. P. Sigray who supplied us with experimental data. This work was kindly supported by grant from T. Sturesson, FMV.

In fig.(1) the signal generating system under consideration is sketched with the EM source, receiver and the three layered environment depicted. If the water depth d is shallow the three layered

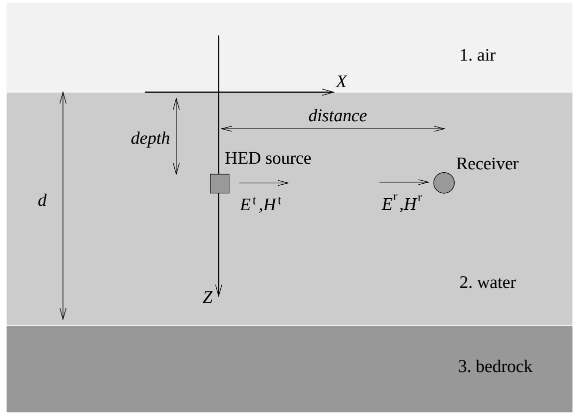

Figure 1: *The source transmitted EM pulses represented by E^t and H^t and the receiver picked up the propagated pulses here denoted by E^r and H^r.*

environment significantly affects the characteristic of the propagation and has to be accounted for in the construction of the optimized pulses. The solution to wave propagation of EM pulses in a three layer environment can be found in [5].

2 Signal Detection

The basic problem under consideration is to detect the presence or absence of a signal, $\mathbf{x}$, in a recorded sequence, $\mathbf{y}$, corrupted by an additive noise, $\mathbf{v}$. This problem can be treated mathematically as a statistical hypothesis test with the following two hypotheses

$$\begin{array}{ll} H_0: & \mathbf{y} = \mathbf{v} \\ H_1: & \mathbf{y} = \mathbf{x} + \mathbf{v} \end{array} \tag{1}$$

where $\mathbf{y} = [y(0), \ldots, y(T-1)]^T$ is the received and digitized EM signal, $\mathbf{v} = [v(0), \ldots, v(T-1)]^T$ is the ambient EM noise with the probability distribution $p_{\mathbf{v}}$ and $\mathbf{x} = [x(0), \ldots, x(T-1)]^T$ is the signal to be detected representing the propagated EM pulses. The shape of each received pulse varies due to random variations in the sea water. Hence, the signal, $\mathbf{x}$, can be considered as stochastic with the multivariate probability distribution $p_{\mathbf{x}}$. In this work the signal, $\mathbf{x}$, is modeled as an RMP, which is presented in the preceding section. The final objective is to formulate a decision rule for the hypothesis testing problem. A more complete theory of signal detection can be found in [4]. In a general framework the decision rule can be expressed as

$$\delta(\mathbf{y}) = \begin{cases} H_0 & \text{if } L(\mathbf{y}) < \tau, \\ H_1 & \text{if } L(\mathbf{y}) > \tau. \end{cases} \tag{2}$$

where $L(\mathbf{y})$ is a detection statistic and τ is a threshold. In order to formulate an optimal detector for the hypothesis testing problem in eq.(1) it is necessary to theoretically derive a detection statistic and threshold τ. There are several strategies which can be adopted when selecting the statistic and threshold, e.g. Neyman-Pearson, Minimax. In this paper, we have adopted the Neyman-Pearson criteria

$$\max_{\delta} \{P_D(\delta)\} \quad \text{under the constraint} \quad P_F \leq \alpha, \tag{3}$$

where P_D is the probability of detection, P_F is the probability of false alarm and α is a user defined maximal probability of false alarm. The probability of detection is

$$P_D = \int_{\Gamma_1} p(\mathbf{y}|H_1) d\mathbf{y}, \tag{4}$$

and the probability of false alarm is

$$P_F = \int_{\Gamma_0} p(\mathbf{y}|H_0)d\mathbf{y}, \tag{5}$$

where $p_{\mathbf{y}}(\mathbf{y}|H_j)$ for $j = 0, 1$ is the conditional probability distribution for the received signal, $\mathbf{y}$, under the two hypothesis, $\Gamma_1 = \{\mathbf{y} \in \mathbb{R}^N | L(\mathbf{y}) > \tau\}$ is the acceptance region and $\Gamma_0 = \{\mathbf{y} \in \mathbb{R}^N | L(\mathbf{y}) < \tau\}$ is the rejection region. Hence, the Neyman-Pearson criterion ensures a decision rule which maximizes the probability of detection for a given maximal probability of false alarm. Further, the Neyman-Pearson lemma indicates that a sufficient statistic for the hypothesis testing problem in eq.(1) is the likelihood ratio [1]

$$L(\mathbf{y}) = \frac{p_{\mathbf{y}}(\mathbf{y}|H_1)}{p_{\mathbf{y}}(\mathbf{y}|H_0)}. \tag{6}$$

By using eq.(1) and noting that $p_{\mathbf{y}}(\mathbf{y}|H_0) = p_{\mathbf{v}}(\mathbf{y})$ and $p_{\mathbf{y}}(\mathbf{y}|H_1) = E_{\mathbf{x}}\{p_{\mathbf{v}}(\mathbf{y} - \mathbf{x})\}$, where $E\{\cdot\}$ is the expectation operation, eq.(6) can be rewritten as

$$L(\mathbf{y}) = \int_{\mathbb{R}^N} \frac{p_v(\mathbf{y} - \mathbf{x})}{p_v(\mathbf{y})} p_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} = \frac{p_v(\mathbf{y} - \hat{\mathbf{x}})}{p_v(\mathbf{y})} \tag{7}$$

where the second equality holds for some $\hat{\mathbf{x}} \in \mathbb{R}^N$ and relies on the assumption that p_v is sufficiently regular. By using the assumption of i.i.d. Gaussian ambient noise, $v \sim N(0, \sigma_v^2)$, eq.(7) can be expressed

$$L(\mathbf{y}) = \exp\left\{\hat{\mathbf{x}}^T \sigma_v^{-2} \mathbf{y} - \frac{1}{2} \hat{\mathbf{x}}^T \sigma_v^{-2} \hat{\mathbf{x}}\right\}. \tag{8}$$

This expression is known as the estimator-correlator form of the likelihood ratio. In an interpretation of eq.(7) and eq.(8), the signal $\hat{\mathbf{x}}$ can be seen as an estimate of the most representative member of the stochastic signal $\mathbf{x}$. Further, $\hat{\mathbf{x}}$ is treated as a coherent signal and is used as the correlator replica in a matched filter $\hat{\mathbf{x}}^T \sigma_v^{-2}$.

In order to assess the performance of the detector we have followed the commonly used approach in line with the Neyman-Person strategy where the probability of detection is presented versus the probability of false alarm for a specific SNR. This representation goes under the name; receiver-operating-characteristic (ROC). The SNR serves as a measure of the difficulty of the detection scenario at hand. In this paper we use following SNR definition

$$SNR = 10 \log_{10} \left(\frac{E\{\mathbf{x}^T \mathbf{x}\}}{E\{\mathbf{v}^T \mathbf{v}\}} \right). \tag{9}$$

3 The RMP Signal Model

A signal $x(t)$ can be called a randomly modulated periodicity RMP with period N if it is of the form [2]

$$x(t) = \frac{1}{K} \sum_{k=-K/2}^{K/2} [\mu_k + u_k(t)] \, e^{i2\pi f_k t} \quad \text{for} \quad f_k = \frac{k}{N} \tag{10}$$

where $\mu_{-k} = \mu_k^*$, $u_{-k}(t) = u_k^*(t)$, and $E\{u_k(t)\} = 0$ for each k and * denotes the complex conjugate. The $K/2 + 1$ $\{u_k(t)\}$ are jointly dependent random processes with finite moments. The signal can also be separated into two parts as

$$x(t) = s(t) + u(t) \tag{11}$$

where

$$s(t) = \frac{1}{K} \sum_{k=-K/2}^{K/2} \mu_k e^{i2\pi f_k t} \quad \text{and} \quad u(t) = \frac{1}{K} \sum_{k=-K/2}^{K/2} u_k(t) e^{i2\pi f_k t} \tag{12}$$

Due to the periodicity of the narrow-band signal $x(t)$ the covariance between $x(t)$ and a delayed version $x(t+\Delta t)$ does not reach zero as Δt increase to a large number, i.e. the contributions are not decorrelated even at a fairly large time delay, Δt. The mean of $x(t)$ is equal to the periodic component of $s(t)$. The additive term $u(t)$ in eq.(11) is a random variation of $x(t)$ from the mean periodic signal $s(t)$. A RMP signal is created by some physical mechanism, which has a more or less stable inherent periodicity. For example, the radiated sound field from a low frequency transducer in shallow water [3].

The discrete samples in the mth frame is $x^m(0), \ldots, x^m(N-1)$. Its discrete Fourier transform (DFT) at frequency $f_r = r/N$ for each $r = 1, \ldots, N/2$ is

$$\begin{aligned} X^m(r) &= \sum_{t=0}^{N-1} x^m(t) e^{-i2\pi f_r t} \\ &= \sum_{t=0}^{N-1} \frac{1}{K} \sum_{k=-K/2}^{K/2} \left[\mu_k^m + u_k^m(t)\right] e^{i2\pi kt/N} e^{-i2\pi rt/N} \\ &= \mu_r^m + U^m(r) \quad \text{where} \quad U^m(r) = \sum_{t=0}^{N-1} u^m(t) e^{-i2\pi f_r t} \end{aligned} \tag{13}$$

We assume the joint distribution to be the same for each frame, thereby the index m will be omitted. The variability of $X(r)$ about its mean μ_r depends on the complex variance of $U(r)$ and the covariances of $U(r_1)$ and $U(r_2)$ is $E\{U^*(r_1)U(r_2)\} = \sigma_U(r_1, r_2)$, where

$$\sigma_U(r_1, r_2) = \sum_{t_1=0}^{N-1} \sum_{t_2=0}^{N-1} c_u(t_1, t_2) e^{i2\pi \frac{r_1 t_1 - r_2 t_2}{N}} \tag{14}$$

and

$$c_u(t_1, t_2) = E\{u(t_1)u(t_2)\} = E\{u(t_1)u^*(t_2\} = c_u^*(t_1, t_2). \tag{15}$$

If $u(t)$ is weakly stationary then $c_u(t_1, t_2) = c_u(\Delta t)$ where $\Delta t = t_1 - t_2$. It then follows that the variance of $U(r)$ is $\sigma_U(r_1, r_2) = \sigma_U^2(r)$ since $r_1 = r_2 = r$. For a more detailed discussion of the properties of $\sigma_U^2(r)$ see [2]. We are now able to introduce a measure called signal coherence $\gamma_x(r)$ defined by

$$\gamma_x(r) = \sqrt{\frac{|\mu_r|^2}{|\mu_r|^2 + \sigma_U^2(r)}}. \tag{16}$$

Suppose we have recorded M frames, each of length N, of $y(t) = x(t) + v(t)$ where $x(t)$ is defined in eq.(10) and $v(t)$ is the ambient noise used in eq.(8). Then the mean frame can expressed as

$$\bar{y}(t) = \frac{1}{M} \sum_{m=1}^{M} y_m(t) \tag{17}$$

which is an unbiased estimator of the signal $s(t)$ if v is white and has zero mean. Asymptotically the mean frame, $\bar{y}(t)$, is normally distributed with mean $s(t)$ and variance $M^{-1}(\sigma_x^2(t) + \sigma_v^2)$. If M is sufficiently large then $M^{-1}(\sigma_x^2(t) + \sigma_v^2)$ is negligible. Further, $\bar{Y}(r)$ is a consistent estimator of μ_r with an negligible error for large M [2]. Since $y_m(t) = s_m(t) + u_m(t) + v_m(t)$ we can define

$$\Delta y_m(t) = y_m(t) - \bar{y}(t), \tag{18}$$

and let $\Delta Y_m(r)$ denote the rth DFT component of $[\Delta y_m(0), \ldots, \Delta y_m(N-1)]$. It follows that $\Delta Y_m(r)$ is the rth DFT component of $[u_m(0) + v_m(0), \ldots, u_m(N-1) + v_m(N-1)]$ plus an error of order $O_p(M^{-1/2})$. The estimate of the variance $\sigma_U^2(r)$ is

$$\hat{\sigma}_U^2(r) = \left[\frac{1}{M} \sum_{m=1}^{M} |\Delta Y_m(r)|^2\right] - \sigma_v^2. \tag{19}$$

This estimate is approximately normally distributed with mean $\sigma^2_U(r)$ and variance decreasing with $O_p(M^{-1})$. In order to acquire an estimate of the signal coherence, $\gamma_x(r)$, based on real data we can use $\bar{Y}(r)$ as an estimate of μ_r and $\hat{\sigma}^2_U(r)$ as an estimate of $\sigma^2_U(r)$ to obtain

$$\hat{\gamma}_x(r) = \sqrt{\frac{|\bar{Y}(r)|^2}{|\bar{Y}(r)|^2 + \hat{\sigma}^2_U(r)}}. \tag{20}$$

This is a consistent estimator of the signal coherence at frequency f_r with an error of order $O_p(M^{-1/2})$. To obtain a correlator replica for the detector in eq.(8) this coherence spectra is transformed to the time domain via an inverse Fourier transform. To further suppress the influence of the noise and random modulation in the construction of the correlator replica we applied a threshold where all coherence spectral components bellow 0.95 were set to zero before transformation to time domain. This time domain replica is called the coherent part of the mean frame (CPMF).

The assumptions for connecting the RMP model and the EM pulses are; at a very short source-receiver distance, the signal is received with a fairly low level of time-of-arrival jitter with a modulation only caused by the amplitude pulse-to-pulse variation. This gives similar properties for both the mean frame (MF) and CPMF value in eq.(20). At larger source-receiver distances the EM pulses are strongly affected by the wave propagating medium introducing both temporal and amplitude modulations. The MF is not necessarily able to estimate the best replica for the individual pulses. The CPMF is less sensitive to the temporal modulation and therefore keep the waveform of the replica less distorted as long as frame synchronisation is possible, i.e. only one pulse in each frame.

4 Simulations and Experiment

The experimental setup is sketched in fig.(1), where the transmitter consisted of a $6m$ long rod attached with two $1m$ long titanium cylinders, the later constituting the transmitter electrodes. The receiver antenna was made out of Ag/AgCl electrodes $1m$ apart. Both the receiver and transmitter were at a depth of approximately $8m$. The received EM signals were transformed to voltage and digitized with a sampling rate of $50kHz$. Signals were recorded with the receiver located at three different distances, $70m$, $115m$ and $140m$ from the source. At each distance an optimized pulse were transmitted. In the location of the experiment the water depth were approximately $15m$. Due to the shallow depth the three layered environment significantly affects the characteristic of the propagation and has to be accounted for in the construction of the optimized pulses [5].

In order to set up a fairly realistic simulation of the detector performance, in terms of ROC curves, the measured EM noise signal were analyzed. In fig.(2) a histogram of the EM noise and a Gaussian probability distribution function are presented. The close agreement between the

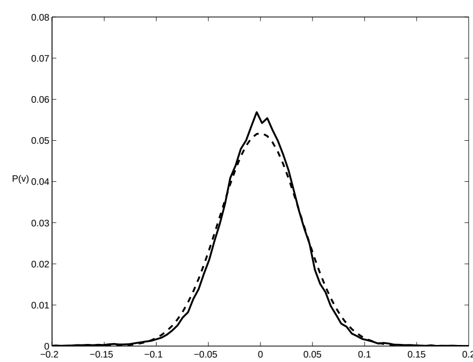

Figure 2: Histogram of the recorded EM noise (solid) and a Gaussian probability distribution function with the same mean and variance as the recorded EM noise (dashed).

curves indicates that the amplitude distribution of the EM noise can be modeled as a Gaussian distribution. Further, in order to investigate if there exist dependence between the recorded EM noise samples the auto-correlation coefficient were computed. The result is depicted in fig.(3). The

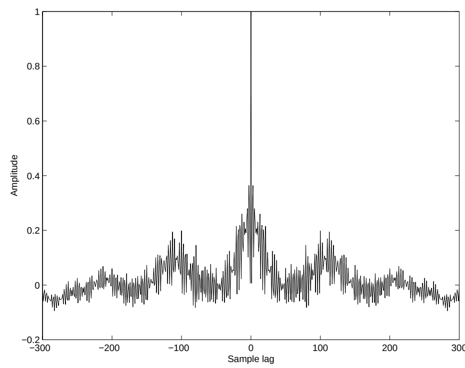

Figure 3: Auto correlation coefficient of the recorded EM noise

results shows that there is some correlations in the recorded EM noise, although relatively small. These small correlations could also be artifacts caused by the anti-aliasing and notch filtering at 50[Hz] that were used during the experiment. Nevertheless, the assumption of an i.i.d. Gaussian noise distribution for the real EM noise is a realistic model.

During the experiment 100 consecutive pulses were transmitted at each distance. The pulses were separated by $20ms$ which corresponds to a frame length of 1000 samples. Fig.(4) shows one frame of a real received pulse. From fig.(4) it is clear that the ambient noise is dominating. Based

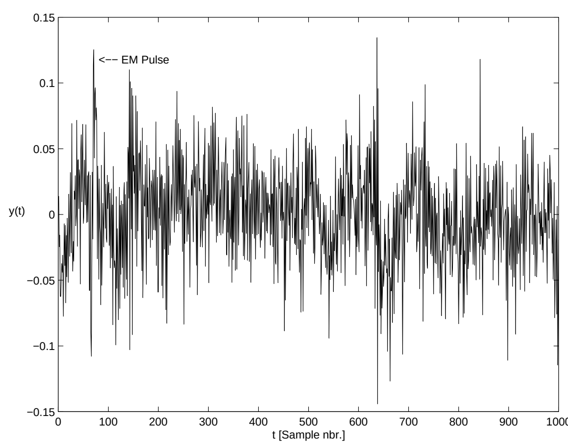

Figure 4: Time series of the real signal plus noise recorded at a distance of $115m$ from the transmitter.

on the recorded pulses the coherent part of the mean frame (CPMF) were estimated. An example of the estimated CPMF signal is shown in fig.(5), where the pulse is clearly visible.

The test model for the signal used for the simulations is

$$x(t) = \sum_{k=1}^{K} [s_k + u_1(t)] \cos(2\pi f_k t) + [s_k + u_2(t)] \sin(2\pi f_k t) \tag{21}$$

where $s_k = (2 - (k-1)/4K)/\sqrt{2L}$, $f_k = k/L$, $K = 47$ and $L = 1000$ is the frame length. The modulations $u_1(t)$ and $u_2(t)$ were generated by two AR models

$$u_1(t) = \rho u_1(t-1) + e_1(t) \quad \text{and} \quad u_2(t) = \rho u_2(t-1) + e_2(t), \tag{22}$$

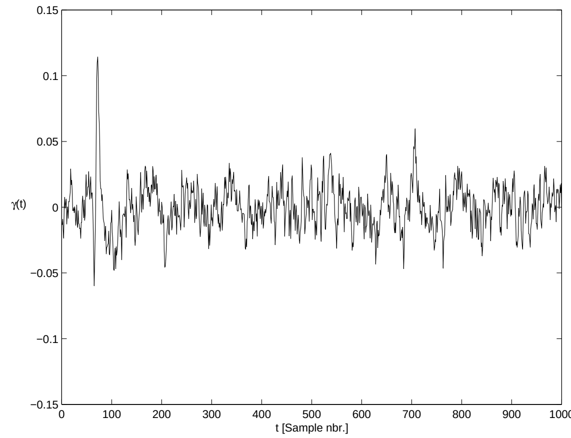

Figure 5: Estimate of the coherent part of the mean frame (CPMF) for the real signal recorded at a distance of $115m$ from the transmitter.

where $\rho = 0.9$, $e_1(t)$ and $e_2(t)$ are independent i.i.d Gaussian noise processes i.e. $N(0, \sigma_e)$ where we chose $\sigma_e = 5$ to account for the pulse-to-pulse variability. The parameter values for the simulated signal were chosen to roughly resemble the received real EM pulses in terms of time duration, shape and pulse-to-pulse variability.

ROC curves for different SNR's were computed to obtain an indication of the performance of this new way of constructing an estimator-correlator detector. The ROC curves were generated by estimating P_D and P_F by feeding simulated signals to the estimator-correlator detector and counting the number of detections and false alarms. The simulated signals consisted of 600 frames containing noise only or signals with an additive noise. These results are presented in fig.(6). From

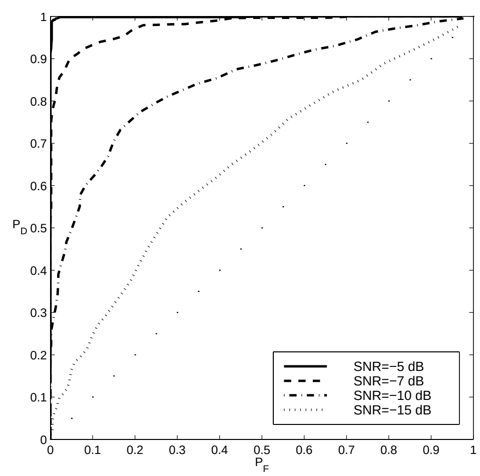

Figure 6: ROC curves based on simulated signals for the SNRs $-5dB$ (solid), $-7dB$ (dashed), $-10dB$ (dash-dotted) and $-15dB$ (dotted).

fig.(6) it is clear that a estimator-correlator detector based on the coherent part of the mean frame perform quite well.

The detector performance in terms of ROC curves were also estimated based on the recorded signals. In order to relate the ROC curves based on the real data to the simulated ROC curves the SNR of the recorded signals had to be estimated. The SNR estimate were computed as

$$SNR_{est} = 10 \log_{10} \left\{ \frac{E\{\mathbf{y}^T \mathbf{y}\} - E\{\mathbf{v}^T \mathbf{v}\}}{E\{\mathbf{v}^T \mathbf{v}\}} \right\}, \tag{23}$$

where $\mathbf{y} = \mathbf{x} + \mathbf{v}$. This SNR estimate reduces to eq.(9) if $E\{\mathbf{v}\} = 0$ and $\mathbf{x}$ and $\mathbf{v}$ are independent. The ROC curves computed on real data shown in fig.(7) indicates an even better performance than for the simulation.

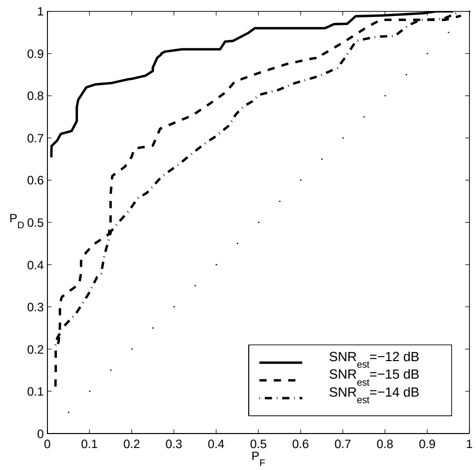

Figure 7: ROC curves based on real signals recorded at distances of $70m$ (dashed), $115m$ (solid) and $140m$ (dash-dotted)

5 Conclusions and Discussion

The waveform variability of low frequency underwater EM pulses are well modeled by the class of signals called randomly modulated periodicity (RMP). By using the RMP model we estimate the signal coherence for both simulated and real EM pulses. The signal coherence of the pulses are high even at a quite large randomness in amplitude and time-of-arrival. By thresholding the signal coherence we are able to find a replica called coherent part of the mean frame (CPMF) to be used in the detector. The detection performance for this replica are able to perform well even if the variability of the pulses are large. Another important issue in the field of active underwater EM is target feature estimation. If the CPMF is a robust replica for correlator detection it can be a strong candidate for increased target identification.

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