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Detecting randomly modulated pulses in noise

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Abstract

All signals that are normally called "periodic" have some amplitude and phase variation from period to period. For example an active sonar system transmits a periodic pulse train to detect targets. The received pulses are not perfectly periodic due to random modulation of the pulses from scattering and attenuation. Target scattering and propagation distortion of the transmitted and received signals produce seemingly random variation from pulse to pulse. A stochastic nonparametric model of period-to-period variation is presented in this paper. This model is used to derive an asymptotic likelihood-ratio test for wide bandwidth pulses that undergo a random modulation from pulse to pulse. It is shown that the test statistic is a linear combination of the matched filter using the expected value of the received signal as the matching vector and the signal's periodogram.

Keywords: Randomly modulated pulses; Periodic; Likelihood ratio; Periodogram

1. Introduction

An active sonar system transmits a periodic pulse train to detect objects in the water column or on the bottom. If the object is a perfect reflector and the medium is homogeneous then the reflected pulses will have the same form as the transmitted pulses plus additive ambient noise. For this idealized case the detector with the highest probability of detection for a fixed false alarm probability is the matched filter provided that the additive noise is gaussian [2, p. 390–392]. If the noise is not gaussian but its joint density is known then the statistically most powerful detector is the classic likelihood-ratio test. The optimality of the matched filter follows from the Neyman–Pearson lemma [6, pp. 251, 252].

For most active sonar environments there is considerable pulse-to-pulse variation in the received pulse train. As the sonar moves the transmitted pulses scatter from a changing water column. Target scattering and propagation distortion of the transmitted and received signals produce variation from pulse to pulse. This is especially true for sonar in a shallow water environment where the medium is usually inhomogeneous and the boundaries scatter and absorb part of the sound energy [3, Chapter 6]. The pulses reflect from a changing bottom and a stochastic water surface as the sonar moves. The physics of scattering from an object much larger than the mean wavelength of the pulse results in variation from pulse to pulse of the received pulses.

The considerable pulse-to-pulse variation is not discussed in standard books such as Burdic [2] nor in Clay and Medwin [3] and thus it is not well known. The author has analyzed classified active sonar data

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and found that the modulation of the pulses is a complicated random process.

There is pulse variation for certain types of radars. The classical radar processing texts present parametric random processes to model this variation. The approach used in this paper employs a nonparametric modulation model.

The organization of this paper is as follows. A stochastic nonparametric model of pulse-to-pulse variation is introduced. This model is used to derive the likelihood-ratio test for pulses with random modulation. It will be shown that the test statistic is a linear combination of the matched filter using the expected value of the received signal as the matching vector and the signal's periodogram.

2. Randomly modulated pulses

There is some random variation in any signal that is normally called "periodic" [5]. Consider the following model for the random variation of an observed signal whose mean is periodic with period $T = N\tau$, τ is the time unit, and $f_k = k/T$ is the kth Fourier frequency:

$$s(t) = \frac{1}{N}\sum_{k=1}^{N/2}[(a_k + u_k(t))\cos(2\pi f_k t) + (b_k + v_k(t))\sin(2\pi f_k t)], \qquad (2.1)$$

where for each period the $\{u_1(t),\dots,u_{N/2}(t), v_1(t), \dots, v_{N/2}(t)\}$ are random variables with a zero mean vector and a joint distribution that has finite cumulants. The realizations from pulse to pulse may be dependent but the random modulation is periodic. An example of such dependence across pulses is given by the models $u_k^{p+1}(t+T) = 0.9u_k^p(t) + \alpha_k^p(t)$ and $v_k^{p+1}(t+T) = 0.7v_k^p(t) + \beta_k^p(t)$ where $u_k^p(t)$ is the kth modulation process for each t in pulse p $(k = 1,\dots,N/2)$ and $\alpha_k^p(t)$ and $\beta_k^p(t)$ are independently distributed white stochastic processes. Fig. 1 shows an example of three randomly modulated pulses generated using artificial data.

Let

$$p(t) = \frac{1}{N}\sum_{k=1}^{N/2}[a_k(t)\cos(2\pi f_k t) + b_k(t)\sin(2\pi f_k t)], \qquad (2.2)$$

$$u(t) = \frac{1}{N}\sum_{k=1}^{N/2}[u_k(t)\cos(2\pi f_k t) + v_k(t)\sin(2\pi f_k t)]. \qquad (2.3)$$

Thus $s(t) = p(t) + u(t)$. Suppose that we observe one frame of length T of either the randomly modulated pulse plus noise $x(t) = p(t) + u(t) + e(t)$ or noise alone where $e(t)$ is a noise process that is bandlimited at $f_{N/2} = 1/2\tau$. Both $u(t)$ and $e(t)$ noise may be *non-gaussian*. Assume that $e(t)$ and $u(t)$ are independently distributed.

To simply exposition assume that the discrete-time modulations are independently and identically distributed (*pure white*) noise processes that are mutually independent. Then $\{u(n\tau)\}$ is a pure white noise. Also assume that $e(n\tau)$ is pure white noise.

The received signal is sampled and digitized at the rate $1/2\tau$ and thus there are N digitized observations in the pulse frame. The assumption of a large bandwidth for the system ensures that N is a large integer.

For the active sonar example $u(t)$ is the noise imparted to the reflected pulse by the scattering of the wave in the medium and the target. The additive noise $z(t)$ is the ambient noise picked by the receiving array.

When considering a single pulse it is common to set the time origin at the time of first observation. The *scaled* discrete Fourier transform (DFT) for the frequencies $f_k = k/T$ $(k = 0, 1, \dots, N/2)$ of one pulse starting at time zero is

$$X(f_k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x(n\tau)\exp(-\mathrm{i}2\pi f_k n\tau) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x(n\tau)\exp\left(-\mathrm{i}2\pi\frac{kn}{N}\right) = N^{-1/2}A_k + Z(f_k), \qquad (2.4)$$

where $A_k = a_k + \mathrm{i}b_k$ and $Z(f_k) = N^{-1/2}\sum_{n=0}^{N-1}(u(n\tau) + e(n\tau))\exp(-\mathrm{i}2\pi kn/N)$ is the DFT of the sum of the modulation noise and the ambient noise.

The full statement of the theorem and its proof is presented in the appendix.

Let σ_z^2 denote the variance of $z(n\tau) = u(n\tau) + e(n\tau)$. Since the discrete-time noise measurements $z(n\tau)$ are

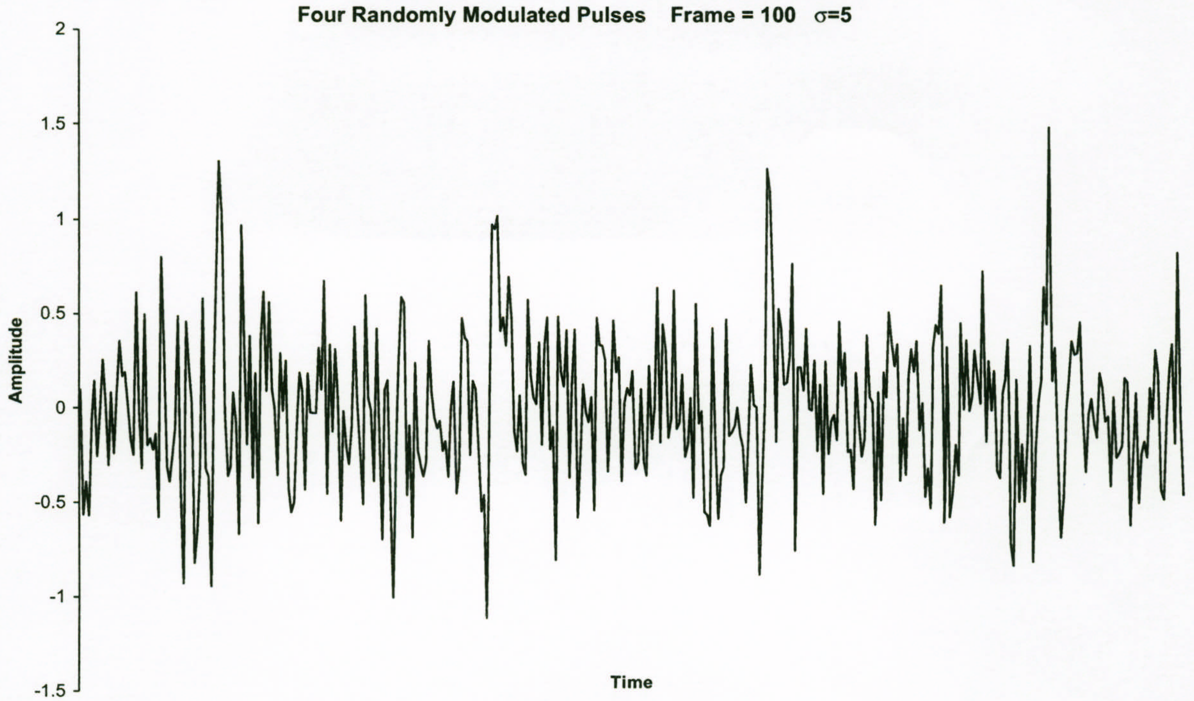

Fig. 1.

independently and identically distributed random variables then it follows from the theorem presented in the appendix that as $N \to \infty \{Z(f_1), Z(f_2), \dots, Z(f_{N/2})\}$ are *asymptotically independent complex gaussian variables* with mean zero and variance σ_z^2.

The standard large N application of this asymptotic result states that the joint density of $\mathbf{X} = (X(f_1), X(f_2), \dots, X(f_{N/2}))'$ is

$$(2\pi)^{-N/4}\sigma_z^{N/2} \times \exp\left(-\frac{1}{2\sigma_z^2}\sum_{k=1}^{N/2}|X(f_k) - N^{-1/2}A_k|^2\right). \quad (2.5)$$

This complex gaussian density will then replace the true finite sample density in the likelihood ratio test. Using such asymptotic results for hypothesis testing is common in statistical theory. From now on the likelihood will be understood to be the above large N gaussian approximation of the true likelihood.

The variance of $z(n\tau)$ is $\sigma_z^2 = E(x(n\tau) - p(n\tau))^2$ and it can be estimated from the observed signal since $\{p(n\tau)\}$ is the mean frame. Since the two noises are independent then $\sigma_z^2 = \sigma_u^2 + \sigma_e^2$. If the ambient noise variance σ_e^2 is known then the estimate of the variance σ_u^2 is $\hat{\sigma}_z^2 - \sigma_e^2$ where $\hat{\sigma}_z^2$ is the estimate of σ_z^2.

Then the likelihood ratio test statistic for the null hypothesis that $x(n\tau) = e(n\tau)$ for $n = 0, 1, \dots, N-1$ versus the alternative hypothesis that $x(n\tau) = p(n\tau) + u(n\tau) + e(n\tau)$ is

$$T = \sum_{k=1}^{N/2}\left[2\mathrm{Re}\,A_k^* X(f_k) + \frac{\sigma_u^2}{\sigma_e^2}|X(f_k)|^2\right], \quad (2.6)$$

where in this case the statistic is not normalized to have unit variance. The component $\sum_{k=1}^{N/2} \mathrm{Re}\,A_k^* X(f_k)$ is the frequency domain equivalence of the matched filter detector of the mean signal. The second term is proportional to the variance of the observed pulse since $(2/N)\sum_{k=1}^{N/2}|X(f_k)|^2 = (1/N)\sum_{n=0}^{N-1} x^2(n\tau)$. Thus, the optimal statistic is a linear combination of a matched filter and the variance of the received pulse. The weight factor σ_u^2/σ_e^2 can be viewed as a noise-to-noise ratio. The weight of the variance in the statistic relative to the matched filter is a linear function of the variance of the modulation noise relative to the ambient noise.

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Appendix

The following central limit result is a special case of Theorem 4.4.1 in [1]. This result can be generalized to the case where $\{z(n\tau)\}$ is not pure noise.

Theorem. *Assume that the discrete-time noise measurements* $\{z(n\tau)\}$ *is pure white noise (independently and identically distributed random variables). As* $N \to \infty \{Z(f_1), Z(f_2), \ldots, Z(f_{N/2})\}$ *are* asymptotically independent complex gaussian variables *with mean zero and variance* σ_z^2 *of* $z(n\tau)$.

Proof. Let $\kappa_m(z)$ denote the mth cumulant of $z(n\tau)$. Choose any integer K and integers $m_1, m_2, \ldots, m_K$ where $m = \sum_{k=1}^{K} m_k$. The DFT of the noise is $A(f_k) = \sum_{n=0}^{N-1} z(n\tau)\exp(-i2\pi kn/N)$. The joint cumulant of $\{A^{m_1}(f_1), A^{m_2}(f_2), \ldots, A^{m_k}(f_K)\}$ is $N\kappa_m(z)$ if $m_1k_1 + m_2k_2 + \cdots + m_kk_K = 0 \bmod (N)$ and is zero otherwise [4, p. 394]. This implies that the joint cumulant of $\{N^{-1/2}A^{m_1}(f_1), \ldots, N^{-1/2}A^{m_K}(f_K)\}$ is

$$\left(\prod_{k=1}^{K} N^{-(m_k/2)}\right) N\kappa_m(z) = N^{-(m/2)} N\kappa_m(z)$$

$$= N^{(1-(m/2))}\kappa_m(z).$$

This joint cumulant goes to zero as $N \to \infty$ if $m \geqslant 3$. Note that the rate of convergence increases with the cumulant's order increases. The gaussian distribution is the only one that has zero third and higher cumulants and joint cumulants. Thus the joint distribution of $\{Z(f_1), Z(f_2), \ldots, Z(f_{N/2})\}$ is asymptotically gaussian.

In addition the complex covariance of $Z(f_{k_1})$ and $Z(f_{k_2})$ is zero if $k_1 \neq -k_2$ and thus the Z's are asymptotically independent since they are asymptotically gaussian.

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