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Cross-correlations and cross-bicorrelations in Sterling exchange rates

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Abstract

This paper proposes two new tests for linear and nonlinear lead/lag relationships between time series based on the concepts of cross-correlations and cross-bicorrelations, respectively. The tests are then applied to a set of Sterling-denominated exchange rates. Our analysis indicates that there existed periods during the post-Bretton Woods era where the temporal relationship between different exchange rates was strong, although these periods have become less frequent over the past 20 years. In particular, our results demonstrate the episodic nature of the nonlinearity, and have implications for the speed of flow of information between financial series. The method generalises recently proposed tests for nonlinearity to the multivariate context. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Researchers in economics and finance have been interested in testing for nonlinear dependence in time series for almost a decade now. Following relatively early work by Brock (1986), Hsieh (1989a; b), and Scheinkman and LeBaron (1989a; b), the number of applications has increased dramatically. There appear to

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be at least two reasons for the popularity of this line of research. First, if evidence of nonlinearity is found in the residuals from a linear model applied to a financial time series, this must cast doubt on the adequacy of the linear model as an adequate representation of the data. Second, if the nonlinearity is present in the conditional first moment, it may be possible to devise a trading strategy based on nonlinear models which is able to yield higher returns than a buy-and-hold rule.

The most popular portmanteau tests for nonlinearity employed have been the BDS test of Brock et al. (1987), now published as Brock et al. (1996), and the bispectrum test of Hinich (1982). The vast majority of researchers to use these tests have found strong evidence for nonlinearity (see Brock et al. (1991) and Brooks (1996) for surveys and applications), although the usefulness of nonlinear time series models for yielding superior predictions of asset returns is still undecided (see LeBaron, 1993; Nachane and Ray, 1993; Weigend and Gershenfeld, 1993; etc.). Although Baek and Brock (1992), Gallant et al. (1993) and Hiemstra and Jones (1994) provide contradictory results, the majority of studies to date examining the issue of nonlinearity have been entirely univariate in nature, considering each series in isolation. This is highly restrictive, since relationships between variables over time are clearly of importance.

There also exists a parallel literature which seeks to determine whether observed nonlinearities in financial time series are due to the existence of stochastic nonlinear relationships or fully deterministic (chaotic) dynamics. Although there is almost no evidence in favour of the latter (see Ramsey et al., 1990; Cecen and Erkal, 1996a,b; Brooks, 1998), it appears that most of the nonlinearity can be explained by reference to the GARCH family of models (e.g., see Baillie and Bollerslev, 1989; Hsieh, 1989a,b).

This paper attempts to draw the two somewhat disparate areas of research into nonlinearity and multivariate time series analysis together proposing a new test for nonlinearity which allows for cross-bicorrelations between pairs of series. Tests of simple cross-correlations are also considered. These tests can be viewed as natural multivariate extensions of the Hinich (1996) portmanteau bicorrelation and whiteness statistics which search for nonlinear cofeatures between time series. The method is more general than the tests for common features that are proposed by Engle and Kozicki (1993), since no knowledge of the kind of dynamics purported to be present in the data is required to detect the dependence. [1] The present paper hopefully provides an additional tool to the nonlinear Granger causality tests employed in the t literature by Baek and Brock (1992) and Hiemstra and Jones (1994). The test proposed in this paper is able to pick up any form of nonlinear dependence of the third-order statistic between two series and might also help researchers to determine the functional form of the nonlinear relationship between

[1] Although generality can be viewed as a virtue of a test, one might also reasonably argue that it reduces the test's power.

the two series by determining in which directions the bicorrelations flow and which of the lags are significant.

The remainder of this paper is organised as follows. Section 2 outlines the testing methodology used; Section 3 describes the data employed, while Section 4 offers some analysis and concluding remarks.

2. Testing methodology

Let the data be a sample of length N, from two jointly covariance stationary time series $\{x(t_k)\}$ and $\{y(t_k)\}$ which have been standardised to have a sample mean of zero and a sample variance of one by subtracting the sample mean and dividing by the sample standard deviation in each case. Since we are working with small subsamples of the whole series, stationarity is not a stringent assumption. The null hypothesis for the test is that the two series are independent pure white noise processes, against an alternative that some cross-covariances, $C_{xy}(r) = E[x(t_k)y(t_k+r)]$ or cross-bicovariances $C_{xxy}(r,s) = E[x(t_k)x(t_k+r)y(t_k+s)]$ are nonzero. As a consequence of the invariance of $E[x(t_1)x(t_2)y(t_3)]$ to permutations of $(t_1,\ t_2)$, stationarity implies that the expected value is a function of two lags and that $C_{xxy}(-r,s) = C_{xxy}(r,s)$. If the maximum lag used is $L < N$, then the principal domain for the bicovariances is the rectangle $\{1 \le r \le L,\ -L \le s \le L\}$.

Under the null hypothesis that $\{x(t_k)\}$ and $\{y(t_k)\}$ are pure white noise, then $C_{xy}(r)$ and $C_{xxy}(r,s) = 0 \forall r,s$ except when $r = s = 0$. This is also true for the less restrictive case when the two processes are merely uncorrelated, but the theorem given below to show that the test statistic is asymptotically normal requires independence between the two series. If there is second or third order lagged dependence between the two series, then, $C_{xy}(r)$ or $C_{xxy}(r,s) \neq 0$ for at least one r value or one pair of r and s values, respectively. The following statistics give the r sample xy cross-correlation and the r,s sample xxy cross-bicorrelation, respectively:

$$C_{xy}(r) = (N-r)^{-1} \sum_{t=1}^{N-r} x(t_k)\,y(t_k+r), \qquad r \neq 0, \tag{1}$$

and

$$C_{xxy}(r,s) = (N-m)^{-1} \sum_{t=1}^{N-m} x(t_k)\,x(t_k+r)\,y(t_k+s)$$
$$\text{where } m = \max(r,s). \tag{2}$$

The cross-bicorrelation can be viewed as a correlation between the current value of one series and the value of previous cross-correlations between the two series. Note that the summation in the second-order case (1) does not include contemporaneous terms, and is conducted on the residuals of an autoregressive fit to filter out the univariate autocorrelation structure so that contemporaneous correlations will not cause rejections. For the third-order test, we estimate the test on the residuals of a bivariate vector autoregressive model containing a contemporaneous term in one of the equations. The motivation for this prewhitening step is to remove any traces of linear correlation or cross-correlation so that any remaining dependence between the series must be of a nonlinear form. It can then be shown that:

$$E\left[C_{xy}(r)\right] = 0, \tag{3}$$

$$E\left[C_{xxy}(r,s)\right] = 0, \tag{4}$$

$$E\left[C_{xy}^2(r)\right] = (N-r)^{-1}, \tag{5}$$

$$E\left[C_{xxy}^2(r,s)\right] = (N-m)^{-1}. \tag{6}$$

under the null hypothesis. Let $L = N^c$ where $0 < c < 0.5$. [2] The test statistics for nonzero cross-correlations and cross-bicorrelations are given by

$$H_{xy}(N) = \sum_{r=1}^{L} (N-r) C_{xy}^2(r), \tag{7}$$

and

$$H_{xxy}(N) = \sum_{s=-L}^{L}{}' \sum_{r=1}^{L} (N-m) C_{xxy}^2(r,s), \qquad ('-s \neq -1,1,0), \tag{8}$$

respectively. These tests are joint or composite tests for cross-correlations and cross-bicorrelations (in a similar vein to the Ljung–Box Q^* test for autocorrelation), where the number of correlations tested for is L and the number of cross-bicorrelations tested for is $L(2L-1)$. We use theorem 1 from Hinich (1996), namely:

Theorem 1. H_{xy} and H_{xxy} are asymptotically χ^2 with L and $L(2L-1)$ degrees of freedom, respectively, as $N \to \infty$,

which is proved in the appendix to Hinich (1996) for the univariate bicorrelation test statistic. An extension of this theorem to the multivariate test proposed in this paper is presented in abbreviated form in Appendix A. The full version is available from the authors upon request.

3. The data and preliminaries

The analysis presented here is based on 5192 daily mid-price spot exchange rates of the Austrian schilling, the Danish krone, the French franc, the German

[2] In this application, we use $c = 0.25$, although the results and the null distribution of the test are not very sensitive to changes in this parameter.

mark, the Italian lira, the Japanese yen, and the U.S. dollar data, denominated against the UK pound. The sample period taken covers the whole of the post-Bretton Woods era, specifically from January 2, 1974 until July 2, 1994 inclusive. We analyse the differences of the log of the exchange rates, which can be interpreted as continuously compounded daily returns. The cross-correlations and cross-bicorrelations are examined via pairwise comparisons between all combinations of two of the exchange rates from the set of seven (21 pairs). The three currencies with the largest world turnover [3] denominated against the pound are the U.S. dollar/pound (8.5% of average daily world turnover), the German mark/pound (4.9%), and the Japanese yen/pound ($<1\%$). These three exchange rates are considered together with a number of less frequently traded European currencies [4] to consider whether these smaller-volume currencies returns follow those of the other European currencies, or whether they take their lead from the larger (mostly non-European) currencies.

The data are split into a set of 148 non-overlapping windows of length 35 observations (i.e., about 7 trading weeks). Samples of this size suggest a use of $L=35^{0.25}$, which is rounded to 2. The reason for using many short windows is that potential arbitrage opportunities induced by non-contemporaneous cross-correlations or cross-bicorrelations are not likely to last long. Hence, the use of long data series would probably yield very little, [5] and hence nonlinearities which persist only for short periods of time would remain hidden. This is a major advantage of the testing approach used here relative to many of its competitors which require large volumes of data to have sufficient power, and which have poor small sample properties.

The results of a small Monte Carlo study to determine size of the test for samples of the length used here are given in Table 1. Two series, each of length 35 are generated using a Gaussian, uniform or Student's t distribution with 5 or 10 *degrees of freedom*. The two series drawn from the same distribution are then tested for cross-correlations or cross-bicorrelations. This procedure is repeated 6000 times.

The results of the simulation clearly demonstrate that the tests are conservative at small samples for the uniform distribution, and the empirical sizes of the tests are close to their nominal values for the Gaussian data. The last two columns of Table 1 also show the empirical size of the test when data are drawn from a

[3] All figures quoted in this section refer to the year 1992, and are taken from *International Capital Movements and Foreign Exchange Markets: A Report to the Ministers and Governors by the Group of Deputies*, Rome, April 23, 1993.

[4] Excluding the German mark/pound, all other intra-EMS currency pairs make up only 7% of world daily average turnover.

[5] Indeed, an application of an identical procedure to that used here for the whole data series used as one single window gave no significant cross-correlations or cross-bicorrelations for any of the currencies, even at the 10% level.

Table 1
Size of the cross-correlation and cross-bicorrelation test statistics for small samples

Test under study	Nominal size of test (%)	Actual size of test for Gaussian data (%)	Actual size of test for uniform data (%)	Actual size of test for Student's t with 5 *degrees of freedom* (%)	Actual size of test for Student's t with 10 *degrees of freedom* (%)
xxy	5	3.8	2.3	4.9	4.5
	1	1.4	0.4	2.2	1.4
	0.1	0.4	0.1	0.8	0.3
yyx	5	4.1	2.7	5.4	4.4
	1	1.6	0.6	2.2	1.4
	0.1	0.6	0.1	0.9	0.4
xy	5	3.3	3.7	4.1	3.9
	1	0.4	0.8	0.6	0.7
	0.1	0.1	0.1	0.2	0.2

t-distribution with 5 and 10 *degrees of freedom*, respectively; these distributions are more likely to be representative of financial asset return series since they are fat-tailed. The simulation shows that the test is only modestly over-sized for the t with 5 *degrees of freedom*, and is appropriately sized for the slightly less fat-tailed distribution. Thus, the test statistic is well behaved with respect to the asymptotic theory, even for rather small samples. One should also be able to obtain similar results for x and y being drawn from different distributions (e.g., one set of Gaussian draws and one set of uniform), so long as the two were independent processes with finite first six moments.

4. Results

The p-values for the cross-correlations that are significant at the 1% level are shown in Table 2 together with the dates of the windows in which this occurred.

These cross-correlation statistics are calculated on the residuals of an AR(3) fit to each series to filter out any linear autoregressive dependence. [6] Many signifi-

[6] The test is asymptotically invariant to linear filtering, and so may be applied to the residuals of a linear model, or to the raw data. It is important that linear dependence in the data is removed, for its presence could lead to spurious rejections of the null hypothesis. In theory, it would also be possible to apply the tests to the residuals from a nonlinear model, for example, the MA(1)-GARCH(1,1) model is often used to summarise the first two moments of financial returns. However, such a step is unnecessary with the correlation and bicorrelation tests since the presence or otherwise of ARCH-effects will not cause a rejection of the null hypotheses. This arises from the fact that the tests are effectively tests for cross-relationships in the conditional mean rather than in the conditional variance.

Table 2
Dates and p-values for test statistics for cross-correlations, and values of most significant cross-correlations

Series (x/y)	Dates (star–end)	p-value for xy statistic	Most significant correlation (at lag[a])
Austrian schilling/Danish krone	No significant cross-correlations		
Austrian schilling/French franc	9/10/74–10/28/74	0.0043	0.49 (−1)
	6/20/85–8/7/85	0.0098	0.52 (−1)
	3/27/92–5/19/92	0.0082	0.53 (1)
Austrian schilling/German mark	8/2/77–9/20/77	0.0000	0.66 (−2)
	1/15/81–3/4/81	0.0038	0.48 (2)
	6/20/84–8/7/85	0.0070	0.52 (1)
Austrian schilling/Italian lira	8/2/77–9/20/77	0.0003	0.39 (−2)
Austrian schilling/Japanese yen	12/15/78–2/6/79	0.0047	0.46 (−2)
Austrian schilling/U.S. dollar	3/17/80–5/7/80	0.0078	0.35 (2)
Danish krone/French franc	4/30/85–6/19/85	0.0070	0.33 (1)
	3/13/89–5/3/89	0.0032	0.43 (2)
Danish krone/German mark	8/28/75–10/15/75	0.0054	0.49 (1)
	1/15/81–3/4/81	0.0093	0.49 (−2)
	4/30/85–6/19/85	0.0065	0.31 (−1)
	3/13/89–5/3/89	0.0002	0.42 (2)
Danish krone/Italian lira	9/13/82–10/29/82	0.0018	0.62 (2)
	3/13/89–5/3/89	0.0011	0.44 (2)
Danish krone/Japanese yen	12/15/78–2/6/79	0.0011	0.48 (−2)
Danish krone/U.S. dollar	No significant cross-correlations		
French franc/German mark	9/10/74–10/28/74	0.0009	−0.56 (2)
	1/15/81–3/4/81	0.0009	0.58 (2)
	10/26/87–12/11/87	0.0049	−0.55 (1)
	3/13/89–5/3/89	0.0065	0.34 (−2)
French franc/Italian lira	10/27/78–12/14/78	0.0037	0.39 (2)
French franc/Japanese yen	8/20/84–10/8/84	0.0075	0.29 (2)
French franc/U.S. dollar	8/20/84–10/8/84	0.0031	0.31 (2)
German mark/Italian lira	9/10/74–10/28/74	0.0020	0.44 (−1)
	8/2/77–9/20/77	0.0031	0.46 (2)
	1/15/81–3/4/81	0.0091	0.50 (−2)
	3/13/89–5/3/89	0.0043	0.33 (2)
German mark/Japanese yen	No significant cross-correlations		
German mark/U.S. dollar	No significant cross-correlations		
Italian lira/Japanese yen	12/15/78–2/6/79	0.0054	0.45 (0)
Italian lira/U.S. dollar	8/20/84–10/8/84	0.0036	−0.63 (1)
Japanese yen/U.S. dollar	8/28/92–10/15/92	0.0076	0.49 (1)

[a] x leads for positive lags, y leads for negative lags.

cant test statistics are caused by contemporaneous cross-correlations, but there are also many that are not contemporaneous. The former is hardly surprising, and could be interpreted as arising from Sterling-related news which affected two bilateral exchange rates against sterling in a similar fashion. The lead/lag cross-correlations are, however, of considerably greater interest, and indicate that for some currencies, there may have been a degree of predictability at certain times over the past 20 years. For example, there was a correlation of 0.66 between the Austrian schilling/pound lagged two periods, and the and the German mark/pound, indicating that if the German mark rises one day during that period, we would have expected the Austrian schilling to rise two trading days later. Many such relationships exist between the currencies, although there are many more cross-correlations between the intra-European currency pairs than between pairs containing the Japanese yen or U.S. dollar.

The number and percentage of significant cross-bicorrelation windows for each pair of exchange rates are given in column 2 of Table 3.

The results for cross-bicorrelations outlined in the ensuing analysis are estimated on the residuals of a bi-variate vector autoregression of order 3 in each equation (a BVAR(3,3)). The proportion of significant cross-bicorrelation windows is much larger than the nominal 1% threshold used, indicating that significant nonlinear lead/lag relationships existed between currencies. Correlations between the values of the *xxy* and *yyx* statistics are given in column 3 of Table 3. On the whole, they show a very high degree of correlation, indicating that the

Table 3
Number and percentage of significant (at the 1% level) cross-bicorrelation windows and correlations between *xxy* and *yyx*, and between *xxy* and the simple cross-correlation for all windows

Series (*x* / *y*)	No. (%) sig. cross-bicorrelation windows	Corr (*xxy*, *yyx*)	Corr (*xxy*, *xy*)
Austrian schilling/Danish krone	40 (27.0)	0.646	0.205
Austrian schilling/French franc	40 (27.0)	0.680	0.285
Austrian schilling/German mark	44 (29.7)	0.643	0.238
Austrian schilling/Italian lira	37 (25.0)	0.490	0.160
Austrian schilling/Japanese yen	36 (24.3)	0.438	0.132
Austrian schilling/U.S. dollar	29 (19.6)	0.452	0.320
Danish krone/French franc	42 (28.4)	0.695	0.275
Danish krone/German mark	44 (29.7)	0.695	0.271
Danish krone/Italian lira	46 (31.1)	0.620	0.118
Danish krone/Japanese yen	29 (19.6)	0.446	0.202
Danish krone/U.S. dollar	28 (18.9)	0.365	0.281
French franc/German mark	43 (29.1)	0.728	0.322
French franc/Italian lira	47 (31.8)	0.652	0.069
French franc/Japanese yen	31 (20.9)	0.406	0.334
French franc/U.S. dollar	29 (19.6)	0.395	0.201
German mark/Italian lira	42 (28.4)	0.478	0.207
German mark/Japanese yen	32 (21.6)	0.415	0.256
German mark/U.S. dollar	26 (17.6)	0.394	0.197
Italian lira/Japanese yen	34 (23.0)	0.413	0.187
Italian lira/U.S. dollar	36 (24.3)	0.413	0.072
Japanese yen/U.S. dollar	30 (20.3)	0.521	0.239

nonlinear relationships may be bidirectional. The correlation between the values of the cross-correlation (xy) and the cross-bicorrelation (xxy) statistics are much lower, however, indicating that linear and nonlinear relationships between the series need not occur at the same time. A more detailed analysis of the significant cross-bicorrelation is given in Table 4.

Only bicorrelations with xxy or yyx values that are greater than 0.5 in absolute value are shown in Table 4 due to space constraints, so that we concentrate on only the very largest bicorrelations. It is evident that there are many more significant cross-bicorrelations than cross-correlations, although the former are much more difficult to interpret. The majority of the significant cross-bicorrelations occur for the smaller-volume European exchange rates, particularly the Austrian schilling/pound and the Italian lira/pound. The p-values associated with the test statistics are typically much smaller than would be generated by a fat-tailed distribution if the data were i.i.d. (such as those given in the Monte Carlo study outlined above). It is also evident that there are more significant cross-bicorrelations during the earlier part of the series. The significant windows appear to occur in clusters; the most recent prolonged period of dependence was during late 1992, around the time of Sterling's departure from the European Exchange Rate Mechanism (ERM).

A recent paper by Karolyi and Stulz (1996) has shown that cross-correlations between the shares of U.S. and Japanese companies trading in the U.S. are not significantly affected by macroeconomic announcements, or interest rate shocks. They show that co-movements between the series are high when the individual markets are volatile, or when ''the markets move a lot'' (p. 984). The cross-correlation framework proposed here provides a natural testing ground for this conjecture. If the markets do indeed move closely together, this will imply that the cross-correlation and cross-bicorrelation statistics (the latter being calculated after prewhitening using a VAR), should have small values when the individual variances of the series are high. In other words, we would expect Corr $(xy, \mathrm{Var}X)$, Corr $(xy, \mathrm{Var}Y)$, Corr $(xxy, \mathrm{Var}X)$, Corr $(xxy, \mathrm{Var}Y)$, Corr $(yyx, \mathrm{Var}X)$, Corr $(yyx, \mathrm{Var}Y)$ to be negative and fairly large. The results of Table 5 show, however, that this hypothesis is not borne out, with no strong relationship (either positive or negative) between the test statistics and the variances, except in the case of the Danish krone/U.S. dollar, where the simple cross-correlation statistics are negatively correlated with the individual variances. These results contrast with those of Karolyi and Stulz (1996), where co-movements and variances did tend to be positively related. However, Karolyi and Stulz considered only linear cross-correlations, and they examined stock returns rather than exchange rates.

Our findings have important implications for the ability of investors to internationally diversify portfolios, since strong contemporaneous co-movements between series coupled with high individual variances imply that fewer apparently country-specific risks are internationally diversifiable, so that the riskiness of the

Table 4
Dates and p-values for cross-bicorrelation tests statistics together with values of most significant bicorrelations

Series (x / y)	Dates (start–end)	p-value for xxy statistic	p-value for yyx statistic	Most significant xxy bicorrelations (at lags)	Most significant yyx bicorrelations (at lags)
Austrian schilling/Danish krone	No significant cross-bicorrelations				
Austrian schilling/French franc	4/1/75–5/19/75	0.0121	0.0003	0.41 (2, 0)	0.78 (2, 1)
	1/27/76–3/15/76	0.0003	0.0302	0.56 (2, 1)	0.47 (1, 1)
	6/25/76–8/12/76	0.0200	0.0029	0.61 (2, 2)	0.48 (1, 2)
	2/26/86–4/17/86	0.0301	0.0671	0.54 (1, 1)	0.26 (1, 2)
Austrian schilling/German mark	10/29/74–12/16/74	0.0499	0.0005	0.51 (2, 1)	0.28 (1, 1)
	9/24/81–11/11/81	0.0076	0.0001	0.35 (1, 2)	0.51 (1, 0)
	10/9/84–11/26/84	0.7268	0.0019	–	0.53 (1, 2)
	8/8/85–9/27/85	0.0001	0.4955	0.75 (1, 2)	–
	11/15/85–1/7/86	0.3863	0.0001	–	0.58 (2, 1)
Austrian schilling/Italian lira	4/23/90–6/12/90	0.0047	0.7926	0.62 (2, 2)	–
	1/1/91–2/18/91	0.7781	0.0033	–	0.55 (1, 2)
	8/17/93–5/10/93	0.4963	0.0001	–	0.80 (1, 2)
Austrian schilling/Japanese yen	No significant cross-bicorrelations				
Austrian schilling/U.S. dollar	9/10/74–10/28/74	0.0239	0.0005	0.64 (2, 1)	0.69 (1, 0)
	4/15/82–3/6/82	0.6949	0.0098	–	0.59 (1, 1)
Danish krone/French franc	8/28/75–10/16/75	0.0000	0.0303	0.44 (2, 2)	0.56 (2, 2)
	1/27/76–3/15/76	0.0000	0.0066	0.58 (2, 1)	0.59 (1, 1)
Danish krone/German mark	9/21/77–4/8/77	0.0001	0.9981	0.62 (1, 1)	–
	9/13/82–10/29/82	0.0045	0.9714	0.64 (2, 2)	–
	11/1/82–12/17/82	0.0000	0.2576	0.87 (2, 0)	–
	6/10/86–7/28/86	0.0068	0.4884	0.62 (1, 0)	–

Danish krone/Italian lira	5/6/76–6/24/76	0.6116	0.0000	–	0.75 (1, 1)
	8/13/76–10/1/76	0.0693	0.0000	0.48 (1, 2)	0.63 (1, 0)
	6/20/85–8/7/85	0.0001	0.7395	0.69 (1, 1)	–
	8/28/92–10/15/92	0.0008	0.9582	0.56 (2, 1)	–
Danish krone/Japanese yen	5/24/83–7/12/83	0.8143	0.0032	–	0.65 (1, 2)
Danish krone/U.S. dollar	9/21/77–8/11/77	0.0000	0.4406	0.54 (1, 2)	–
	4/15/82–6/3/82	0.8218	0.0044	–	0.52 (1, 1)
	3/8/85–4/29/85	0.0003	0.1219	0.60 (2, 1)	–
French Franc/German mark	1/27/76–3/15/76	0.0230	0.0000	0.52 (1, 1)	0.35 (2, 1)
	8/2/77–9/20/77	0.0000	0.0021	0.61 (1, 2)	0.56 (1, 2)
	1/15/81–3/4/81	0.0006	0.8692	0.77 (2, 2)	–
	11/15/85–1/7/86	0.5314	0.0008	–	0.56 (1, 2)
	2/26/86–4/17/86	0.0018	0.8432	0.51 (1, 1)	–
French Franc/Italian lira	5/6/76–6/24/76	0.0038	0.0061	0.59 (2, 1)	0.53 (1, 1)
	8/13/76–10/1/76	0.3210	0.0000	–	0.59 (1, 0)
	6/4/82–7/22/82	0.0092	0.4661	0.77 (1, 1)	–
	11/1/82–12/17/82	0.0049	0.5174	0.56 (1, 1)	–
	12/20/82–2/9/83	0.0000	0.8099	0.59 (2, 2)	–
	3/13/89–5/3/89	0.0054	0.0971	0.53 (1, 1)	0.32 (1, 1)
	8/28/92–10/15/92	0.0006	0.9862	0.70 (2, 1)	–
	8/17/93–10/5/93	0.5460	0.0064	–	0.55 (1, 1)
	3/7/94–4/26/94	0.0202	0.0084	0.52 (2, 1)	0.45 (2, 1)
French franc/Japanese yen	8/13/76–10/1/76	0.4363	0.0001	–	0.60 (1, 2)
French franc/U.S. dollar	1/27/76–3/15/76	0.0479	0.0000	0.54 (1, 1)	0.50 (2, 1)
	4/22/77–6/13/77	0.0000	0.88660	0.53 (2, 2)	–
	2/10/83–3/30/83	0.0421	0.0008	0.32 (1, 1)	0.68 (1, 1)

Table 4 (continued)

Series (*x* / *y*)	Dates (start–end)	*p*-value for *xxy* statistic	*p*-value for *yyx* statistic	Most significant *xxy* bicorrelations (at lags)	Most significant *yyx* bicorrelations (at lags)
German mark/Italian lira	9/10/74–10/28/74	0.0068	0.9694	0.71 (1, 1)	–
	5/6/76–6/24/76	0.3530	0.0057	–	0.59 (1, 1)
	8/13/76–10/1/76	0.0864	0.0006	0.37 (1, 2)	0.58 (1, 0)
	9/24/81–11/11/81	0.0142	0.0013	0.54 (1, 2)	0.30 (1, 0)
	12/20/82–2/9/83	0.0000	0.5858	0.54 (2, 2)	–
	12/26/86–2/13/87	0.00660	0.6078	0.60 (1, 1)	–
	8/28/92–10/15/92	.0002	0.9992	0.73 (2, 1)	–
	8/19/93–5/10/93	0.7725	0.0024	–	0.57 (1, 1)
German mark/Japanese yen	No significant cross-bicorrelations				
German mark/U.S. dollar	9/10/74–10/28/74	0.0023	0.0377	0.56 (1, 1)	0.44 (1, 0)
Italian lira/Japanese yen	1/27/76–3/15/76	0.8231	0.0051	–	0.53 (1, 2)
	8/13/76–10/1/76	0.0000	0.0000	0.71 (1, 0)	0.55 (1, 2)
	9/8/78–10/26/78	0.0072	0.7623	0.69 (1, 1)	–
	10/9/84–11/26/84	0.0000	0.2005	0.65 (1, 1)	–
	1/18/85–3/7/85	0.0001	0.3889	0.57 (2, 0)	–
Italian lira/U.S. dollar	12/4/75–1/26/76	0.0000	0.5887	0.72 (2, 0)	–
	5/6/76–6/24/76	0.0041	0.0249	0.52 (1, 0)	0.44 (2, 1)
	8/13/76–10/1/76	0.9434	0.0021	–	0.69 (1, 2)
	11/22/76–1/11/77	0.0000	0.9870	0.66 (1, 1)	–
	2/20/78–4/10/78	0.0006	0.8740	0.58 (2, 2)	–
	1/18/85–3/7/85	0.0057	0.0299	0.55 (2, 0)	–
	8/8/85–9/26/85	0.9151	0.0075	–	0.66 (1, 2)
Japanese yen/U.S. dollar	No significant cross-bicorrelations				

Table 5
Correlation of the correlation and bicorrelation test statistics with the individual variances of the series

Series (x / y)	Corr (xy, Var X)	Corr (xy, Var Y)	Corr (xxy, Var X)	Corr (xxy, Var Y)	Corr (yyx, Var X)	Corr (yyx, Var Y)
Austrian schilling / Danish krone	0.06	0.04	0.01	−0.03	0.05	−0.09
Austrian schilling / French franc	0.05	0.04	0.05	0.04	0.12	0.16
Austrian schilling / German mark	0.01	−0.21	0.14	0.14	0.07	0.12
Austrian schilling / Italian lira	−0.08	−0.12	0.18	0.22	0.16	0.16
Austrian schilling / Japanese yen	−0.01	0.00	0.10	−0.09	−0.01	−0.02
Austrian schilling / U.S. dollar	0.08	−0.03	0.14	0.05	0.01	−0.03
Danish krone / French franc	0.00	0.02	−0.14	0.02	0.00	0.11
Danish krone / German mark	−0.03	−0.01	0.05	0.04	−0.05	0.03
Danish krone / Italian lira	−0.08	−0.04	0.15	0.28	0.11	0.18
Danish krone / Japanese yen	−0.09	−0.06	0.13	0.03	0.03	−0.03
Danish krone / U.S. dollar	−0.37	−0.37	0.03	0.19	−0.01	−0.01
French franc / German mark	−0.05	−0.05	−0.03	0.11	0.00	0.12
French franc / Italian lira	−0.05	−0.01	0.13	0.01	0.20	0.15
French franc / Japanese yen	−0.08	0.00	0.02	−0.02	0.02	0.04
French franc / U.S. dollar	−0.04	−0.05	0.03	−0.16	0.11	0.00
German mark / Italian lira	−0.05	0.00	0.23	0.21	0.16	0.19
German mark / Japanese yen	0.01	0.10	0.16	−0.05	0.06	0.04
German mark / U.S. dollar	−0.02	0.03	0.05	−0.03	0.06	0.11
Italian lira / Japanese yen	−0.08	−0.09	0.10	−0.02	0.02	0.08
Italian lira / U.S. dollar	−0.03	−0.14	0.11	−0.08	−0.06	−0.01
Japanese yen / U.S. dollar	0.18	0.14	0.04	−0.04	0.01	0.07

portfolio overall increases. This issue is becoming increasingly important following increases in capital mobility and the openness of trade. Also, countries which are part of the European ERM coordinating fiscal and monetary policies more closely in order to meet the ''convergence criteria'' for forming a single currency mans that the correlations between currencies within Europe are likely to become stronger over the next few years.

5. Conclusions

In this paper, we have examined a new approach to testing for nonlinear interactions between series, and we have illustrated the method on a set of exchange rates. The method provides a complement to Granger causality analysis, and is general enough to detect many types of nonlinear dependence between series in their conditional means. We find a much larger number of significant cross-correlations and cross-bicorrelations than one would expect if the data were generated by independent white noise processes. Moreover, this type of structure cannot be generated by one of the GARCH family of models, so long as the GARCH model is a Martingale difference sequence. A Martingale difference has zero bicorrelations except for $E[x(t)x(t+r)y(t+r)]$, which is not included in the sum for the bicorrelation statistics. Therefore, GARCH models should give rise to third-order statistics that are not significantly different from zero.

The episodic nature of the observed linear and nonlinear co-dependence should be noted. We find that, in common with the analysis of Ramsey and Zhang (1997) of the univariate case, multivariate activity in financial markets are relatively short-lived and surrounded by longer periods of apparent randomness. It is, perhaps, also not surprising that the cross-correlations and cross-bicorrelations all feature a small-volume European exchange rate on at least one side, and that there is little dependence between, for example, the Japanese yen and the U.S. dollar. The currencies which are less frequently traded and which are likely to be less closely scrutinised by dealers, are also likely to be slower to respond to new information. So, for example, the return on one of these currencies today may still be reflecting information that was fully incorporated into the ''bigger'' currencies yesterday. Thus, the return of the ''smaller'' exchange rate today will be correlated with the return of the larger exchange rate yesterday. This will manifest itself as a nonzero cross-correlation *or* cross-bicorrelation for the relationship between the two need not necessarily be linear. This argument was first suggested by Fisher (1966) to explain serial correlation in stock market indices and portfolios containing the stocks of small firms (see Perry (1985) or Chelley-Steeley and Steeley (1995) for more recent applications of this logic). This argument has been played down in much of the recent literature, which argues that the effects of this phenomenon will be small for data sampled at daily or lower frequencies. Boudoukh et al. (1994), however, argue that this nonsynchronous trading effect has been understated in the literature, and that most of the apparent predictability observed by, for example, Cohen et al. (1986), can be explained by this effect.

The dependencies observed in this paper must, by definition, be present for more than a few days to be detected. Hence, we conjecture that the observed level of cross-correlation and cross-bicorrelation between currencies cannot be entirely attributed to nonsynchronous trading, and their existence must be considered evidence inconsistent with the weak form of the efficient markets hypothesis. Although further research is required to determine whether profitable trading strategies could be developed from this analysis, and building an appropriate multivariate nonlinear model of the switching type is not a simple task, our results are encouraging, and suggest that further investigation is worth while. The cross-bicorrelation test is, however, suggestive of an appropriate functional form for a nonlinear model since the cross-bicorrelation is essentially a test of $E[x(t)x(t+r)y(t+s)]$. If we restrict ourselves to consider the case where r and s are negative, then one might be able to predict $x(t)$ on the basis of the lags of $x(t+r)y(t+s)$; a brief description of one method of implementing such models is given by Brooks and Hinich (1998).

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Appendix A. Proof of Theorem 1

The null hypothesis is that $\{x(t_k)\}$ and $\{y(t_k)\}$ are mutually independent i.i.d. zero mean series. Set $\sigma_x = \sigma_y = 1$. Redefine the three time points in the triple product $x(t_k)x(t_k+r)y(t_k+s)$ for a given (r,s) as follows: $t_{k_1} = t_k, t_{k_2} = t_k + r, t_{k_3} = t_k + s \quad (k = 1,\dots,l)$. Then (a) $E[x(t_{k_1})x(t_{k_2})y(t_{k_3})] = 0$, and (b) $E[x(t_{k_1})x(t_{k_2})y(t_{k_3})]^2 = 1$.

The nth-order cumulant of a product of variates can be related to the joint cumulants of the variates, but the relationship is more complicated than the one between the moments and cumulants stated above. There is no simple approach to deal with the combinatorial relationships between the nth-order joint cumulants of the triple product $P(t_{k_1}t_{k_2}t_{k_3}) = x(t_{k_1})x(t_{k_2})y(t_{k_3})$ for various values of t_k, r, and s, and the cumulants of $u(t)$ even though the $x(t_k)$'s and $y(t_k)$'s are independent. The relationships rest on a definition of *indecomposable partitions* of two dimensional tables of subscripts of the t's (see Leonov and Shiryaev (1959) and

Section 2.3 of Brillinger (1975)). We display the table of the t's next to the table of their subscripts which Brillinger uses in his exposition.

Consider the following $l \times 3$ table of $t_{k_1} = t_k, t_{k_2} = t_k + r_k, t_{k_3} = t_k + s_k (k = 1,\dots,l)$:

Times			Using delay notation		
t_{11}	t_{12}	t_{13}	t_1	$t_1 + r_1$	$t_1 + s_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t_{l1}	t_{l2}	t_{l3}	t_l	$t_l + r_l$	$t_l + s_l$

Let $\nu = \nu_1 \cup \cdots \cup \nu_M$ denote a partition of the k_{ji} in this table into M sets where $j = 1,\dots,l$ and $i = 1,2,3$. There are many partitions of the $l \times 3$ times from the single set of all the elements to $l \times 3$ sets of one element.

The mth set in the partition is denoted $v_m = (k_{j_{1(m)} i_{1(m)}},\dots,k_{j_{\vartheta(m)} i_{\vartheta(m)}})$ where ϑ is the number of elements in the set. The cumulant of $[x(k_{j_{1(m)} i_{1(m)}}),\dots,x(k_{j_{\vartheta(m)} i_{\vartheta(m)}})]$ is $\kappa[x(k_{j_{1(m)} i_{1(m)}}) \cdots x(k_{j_{\vartheta(m)} i_{\vartheta(m)}})]$. The symbol $\kappa[\nu(m)]$ will be used for this joint cumulant.

If no two ji are equal for a set $\nu(m)$, then $\nu(m)$ is called a *chain*. A partition is called *indecomposable* if there is a set with at least one chain going through each row of the table (all the rows are chained together). A partition is *decomposable* if one set or a union of some set in ν equals a subset of the rows of the table. Consider, for example, the following 2×3 table:

$$\begin{array}{ccc} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{array} \tag{A.1}$$

The decomposable partitions are: $(k_{11},k_{12},k_{13}) \cup (k_{21},k_{22},k_{23})$, which is the union of the two rows and all its subpartitions. Three indecomposable partitions of this 2×3 table are $(k_{11}, k_{21}) \cup (k_{12},k_{22}) \cup (k_{13},k_{23})$, $(k_{11},k_{22}) \cup (k_{12},k_{21}) \cup (k_{13},k_{23})$, and $(k_{11},k_{21},k_{12},k_{22}) \cup (k_{13},k_{23})$. Each pair of these partitions are chains.

Let $\nu_r = \nu_1 \cup \cdots \cup \nu_{M_r}$ denote the rth indecomposable partition of Eq. (A.1) into M_r sets. The joint cumulant of $[x(t_{k_{11}})x(t_{k_{12}})y(t_{k_{13}}),\dots,(x(t_{k_{\vartheta 1}})x(t_{k_{\vartheta 2}})y(t_{k_{\vartheta 3}})]$ is the sum over r of the products of the M_r cumulants $\kappa[\nu(m)]$ of the ν_m in each indecomposable ν_r.

It is easy to check that $\kappa[x(t_{k_1})x(t_{k_1}+r)y(t_{k_1}+s),\dots,x(t_{k_1})x(t_{k_1}+r)y(t_{k_1}+s)] = 0$ unless $t_{k_1} = \cdots t_{k_1} = t$. It then follows by an enumeration of each of the cumulants of the sets in the indecomposable partitions $\nu = \nu_1 \cup \cdots \cup \nu_p$ that most of the products of cumulants are zero for a given partition. A summary of the second-order joint cumulants of the triples is as follows: $\kappa[x(t_1)x(t_1+r_1)y(t_1+s_1)x(t_2)x(t_2+r_2)y(t_2+s_2)] = 0$ unless $t_1 = t_2 = t, r_1 = r_2 = r$, and $s_1 = s_2 = s$. If so, then $\kappa[p^2(t,t+r,t+s)] = 1$.

The covariance of $C_{xxy}(r_1,s_1)$ and $C_{xxy}(r_2,s_2)$ is $[(N-s_1)(N-s_2)]^{-1/2}$ times a double sum of covariances of the P's. There are $N-s$ nonzero terms (all equal to one) in the double sum of covariances. Then from the theorem. $\mathrm{Var}[C_{xxy}(r,s)]=(N-s)/(N-s_2)=1$ and $\mathrm{Cov}[C_{xxy}(r_1,s_2),\ C_{xxy}(r_2,s_2)]=0$.

To obtain the third-order joint cumulants, consider the following 3×3 table of $0<r_k<s_k$:

$$\begin{array}{cccccc} t_{11} & t_{12} & t_{13} & t_1 & t_1+r_1 & t_1+s_1 \\ t_{21} & t_{22} & t_{23} & t_2 & t_2+r_2 & t_2+s_2 \\ t_{31} & t_{32} & t_{33} & t_3 & t_3+r_3 & t_3+s_3 \end{array}$$

Using the delay notation for indices, first consider the following indecomposable partition: $\nu_1=(t_1,t_2,t_3)\cup(t_1+r_1,t_2+r_2,t_3+r_3)\cup(t_1+s_1,t_2+s_2,t_3+s_3)$. If (1) $t_1=t_2=t_3$, (2) $r_1=r_2=\ r_3$, (3) $s_1=s_2=s_3$, then the third-order cumulants of the three columns equal γ and thus the product of the cumulants is γ^3. If any one of these equalities in (1), (2), or (3) do not hold then the product is zero.

Now consider the indecomposable partition: $\nu_2=(t_1,t_2,t_3)\cup(t_1+r_1,t_2+r_1,t_3+r_3,t_1+s_1)\cup(t_2+s_2,t_3+s_3)$. The only nonzero product of cumulants holds for $t_1=t_2=t_3=t$, $r_1=r_2=r_3=r$, and $s_1=s_2=s_3=s$, which yields $\kappa[\,p^3(t,t+r_1,t+s_1)]=\gamma^3$.

Suppose that $s_3\neq r_1$. Consider the indecomposable partition (a subpartition of ν_2) $\nu_3(t_1,t_2,t_3)\cup(t_1+r_1,\ t_2+r_1)\cup(t_3+r_3,t_1+s_1)\cup(t_2+s_2,t_3+s_3)$. Then if $t_1=t_2=t_3=t$ and $r_1=r_2$, $s_2=s_3$, $r_3=s_1$, the product of the cumulants is near zero since $x(t+s_1)$ and $y(t+s_1)$ are independent. The pattern should be clear. All the other indecomposable partitions have at least one zero cumulant. From the theorem, the third-order joint cumulant of the C_{xxy} are zero.

We also require an understanding of the higher order joint cumulants to prove the asymptotic properties of our test statistic. The general form can be deduced from the fourth-order case by enumerating the sets in the indecomposable partitions of the 4×3 table:

$$\begin{array}{cccccc} t_{11} & t_{12} & t_{13} & t_1 & t_1+r_1 & t_1+s_1 \\ t_{21} & t_{22} & t_{23} & t_2 & t_2+r_2 & t_2+s_2 \\ t_{31} & t_{32} & t_{33} & t_3 & t_3+r_3 & t_3+s_3 \\ t_{41} & t_{42} & t_{43} & t_4 & t_4+r_4 & t_4+s_4 \end{array}$$

The major term in the error of the approximation is a function of the nonzero products of the following two types of indecomposable partitions of this table: $\nu_4=(t_1,t_3)\cup(t_2,t_4)\cup(t_1+r_1,t_2+r_2)\cup(t_3+r_3,t_4+s_4)\cup(t_1+s_1,t_2+s_2)\cup(t_3+s_3,t_4+s_4)$, $\nu_5=(t_1,t_2,t_3,t_4)\cup(t_1+r_1,t_2+r_2)\cup(t_3+r_3,t_4+s_4)\cup(t_1+s_1,t_2+s_2)\cup(t_3+s_3,t_4+s_4)$. If (4) $t_4=t_3=t_2=t_1=t$, (5) $r_1=r_3\neq r_2=r_4$, and (6) $s_1=s_3\neq s_2=s_4$ then the cumulants of all the pairs in ν_4 are one and the product then one, and the cumulant of the first column in ν_5 is κ, which is the product of the cumulants of ν_5. The cumulant products of the other indecomposable partitions of the table are all zero given constraints (4), (5), and (6). Thus $\kappa[p^2(t,t+r,t+s)p^2(t,t+r_2,t+s_2)] = (1+\kappa)$.

For each (r_1,s_1) and (r_2,s_2), these nonzero cumulant products equalities hold for at most Nt's. Thus the fourth-order joint cumulant $\kappa[c^2_{xxy}(r_1,s_2)c^2_{xxy}(r_2,s_2)]$ is $[(N-s_1)(N-s_2)(N-s_3)(N-s_4)]^{-1/2}O(N)(1+\kappa) = O(N^{-1})(1+\kappa)$. There are of order $O(L^4)$ such pairs of indices where $r_1 \neq r_2$ and $s_1 \neq s_2$ which have these joint cumulants. If $r_1 = r_2$ or $s_1 = s_2$, then there are a lot more fourth-order nonzero cumulants. An enumeration of indecomposable partitions with nonzero cumulant products yields the following two results:

$$\kappa\left[p^4(t,t+r,t+s)\right] = \kappa^3 + 9\kappa^2 + 27\kappa + 24 \tag{A.2}$$

and

$$\kappa\left[p^2(t,t+r,t+s_1)\,p^2(t,t+r,t+s_2)\right] = \kappa^2 + 6\kappa + 8.$$

The same pattern holds of partitions of the general $l \times 3$ table of subscripts into pairs with identical indices. The major nonzero cumulants are $\kappa[p^2(t,t+r_1,t+s_1)\cdots \mathrm{p}^2(t,t+r_{l/2},t+s_{l/2})] = O(N^{-1})$ when l is even, and $\kappa[p^2(t,t+r_1,t+s_1)\cdots p^2(t,t+r_{(l-1)/2},t+s_{(l-1)/2})p(r_l,s_l)] = O(N^{-1})$ for a restricted set of $(r_k - s_k)$ when l is odd. Thus the lth joint cumulant of the C_{xxy}'s is of order $\kappa O(N^{1-l/2})$.

These results will now be applied to prove that the test statistic H_N is asymptotically normal. It has already been shown that $\mathrm{Var}[C_{xxy}(r_m,s_m)] = E[C^2_{xxy}(r_m,s_m)] = 1$ and thus $E(H_N) = 0$ under the null hypothesis. From the relationship between the covariances and the fourth-order cumulants, $\mathrm{Var}[C^2_{xxy}(r_1,s_1)C^2_{xxy}(r_2,s_2)] = \kappa[c^2_{xxy}(r_1,s_1)c^2_{xxy}(r_2,s_2)] + 2$, and $\mathrm{Var}\,[C^2_{xxy}(r_1,s_1)\,C^2_{xxy}(r_2,s_2)] = \kappa[c^2_{xxy}(r_1,s_1)c^2_{xxy}(r_2,s_2)] + 2\kappa^2\,[c_{xxy}(r_1,s_1)$-$c_{xxy}(r_2,s_2)]$. Suppose that $r_4 = r_3 = r_2 = r_1 = r$, and $s_4 = s_3 = s_2 = s_1 = s$. Then from Eq. (A.2), $\kappa[c^4_{xxy}(r,s)] = O(\mu(\kappa)/N)$ where $\mu(\kappa) = \kappa^3 + 9\kappa^2 + 27\kappa + 24$. Thus $\mathrm{Var}[C^2_{xxy}(r,s)] = 2 + O(\mu(\kappa)/N)$.

If $r_1 = r_3 \neq r_2 = r_4$ and $s_1 = s_3 \neq s_2 = s_4$ (constraints (5) and (6)), then $\kappa[c^2_{xxy}(r_1,s_1)c^2_{xxy}(r_2,s_2)] = O((1+\kappa)/N)$. If $r_1 = r_2$, then the joint cumulant is $O(\nu(\kappa)/N)$, where $\nu(\kappa) = \kappa^2 + 6\kappa + 8$. Thus $\mathrm{Cov}[C^2_{xxy}(r_1,s_1)C^2_{xxy}(r_2,s_2)] = O(N^{-1})$. Since the number of $C^2_{xxy}(r_m,s_m) - 1$ terms in the sum is $L^2/2$ $(L = N^c)$, $\mathrm{Var}(H_N) = 1 + O(L^2/N) \to 1$ as $N \to \infty$ since $0 < c < 1/2$. There are approximately L^3 such (r_1,s_1), (r_2,s_2), (r_3,s_3), and (r_4,s_4) in the double sum which satisfies constraints (5) and (6) and thus the error in the variance of H_N due to these covariances is of the order $L^{-2}L^3N^{-1} = N^{c-1}$.

To complete the proof, we will now demonstrate that the cumulants of H_N of order $l \geq 3$ go to zero as $N \to \infty$. The lth cumulant of H_N depends on the $2l$ order joint cumulant of the $C^2_{xxy}(r_k,s_k)$ for $k = 1,\ldots,l$. From above, these cumulants are of order $L^2L^{-l}N^{1-l/2}$ which goes to zero as $N \to \infty$ for $l \geq 3$.

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