Estimating bearing when the source is endfire to an array

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Consider the problem of estimating the direction of arrival of a plane wave using a linear array when the source is endfire to the array. An estimator of θ, the direction of arrival with respect to the array axis, is presented with a small mean square error when θ is small and the number of sensors or the array gain is large.

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INTRODUCTION

Consider the problem of estimating the direction of arrival of a plane wave using a linear array of length L with M sensors. Let θ denote the direction with respect to the array axis. Assume that we know that $\theta > 0$. The sign of θ is ambiguous, given data from a linear array. The approximations to the rms errors of the maximum-likelihood[1-3] and the least-squares[4] estimators of θ are proportional to $|\sin\theta|^{-1}$. The Cramer–Rao bound also has this property.[5] This seems to imply that these estimators have infinite variances as $\theta \to 0$. This is not the case, as I will now show.

I. ESTIMATING BEARING WHEN θ IS SMALL

Assume that the signal at each sensor is a plane wave plus Gaussian noise with energy SNR ρ. To simplify exposition, let the wave be narrow-band with wavelength λ. If each sensor is simultaneously sampled for a period $T = N\delta$, then the maximum-likelihood estimator $\hat{a}$ of $a = \cos\theta$ (using either the frequency–wavenumber approach or time delay) has the property that $\hat{a} = a + \epsilon$, where the distribution of ϵ is Gaussian with mean *zero* and variance $\sigma^2 = 3\lambda^2/2MNL^2\rho\pi^2$ if M *or* N is large.

The maximum-likelihood estimator is $\hat{\theta} = \cos^{-1}\hat{a}$, which has a discontinuous derivative at $\hat{a} = 1$. Thus we must select the appropriate Taylor's formula approximation to derive the approximate mean-square error (mse) of $\hat{\theta}$ when $\theta \approx 0$. As a first step, note that $1 - a = \frac{1}{2}\theta^2 + O(\theta^4)$ and thus $\hat{a} < 1$ if and only if $\epsilon < \theta^2/2$ when $\theta \approx 0$. More precisely,

$$\Pr(\hat{a} < 1) = \Phi(\theta^2/2\sigma) + O(\theta^4\sigma^{-1}), \tag{1}$$

where $\Phi(y)$ is the cumulative distribution of a Gaussian $N(0,1)$ variate.[6]

Assume that $\sigma\theta^{-2}$ is sufficiently small so that $\Pr(\hat{a} \geq 1)$ is negligible. Then

$$\cos^{-1}\hat{a} = [2(1-\hat{a})]^{1/2} + O(\theta^3). \tag{2}$$

Expanding (2) in terms of ϵ,

$$\cos^{-1}\hat{a} = \theta + \theta^{-1}\epsilon + O(\sigma^2\theta^{-3}) + O(\theta^3). \tag{3}$$

From (3), $\text{mse}(\hat{\theta}) \simeq \sigma^2\theta^{-2}$, which is *small* when $\sigma\theta^{-2}$ and θ are *small*.

When $\sigma\theta^{-2}$ is not small, $\Pr(\hat{a} \geq 1)$ is not negligible, and $\hat{\theta}$ must be modified due to the constraint $a \leq 1$. Define the estimator of θ,

$$\tilde{\theta} = \cos^{-1}\hat{a} \quad \text{if} \quad \hat{a} < 1$$
$$= 0 \quad \text{if} \quad \hat{a} \geq 1 \tag{4}$$

If $\theta \to 0$, $\tilde{\theta} \to (-2\epsilon)^{1/2}$ if $\epsilon < 0$ and $\tilde{\theta} \to 0$ if $\epsilon \geq 0$. Since $\epsilon \sim N(0, \sigma^2)$,

$$\lim_{\theta \to 0} \text{mse}(\tilde{\theta}) = -E(\epsilon \mid \epsilon < 0)$$
$$= \sigma/\sqrt{2\pi}\ . \tag{5}$$

II. ARTIFICIAL DATA RESULTS

The bias, rms error and mean absolute error of $\tilde{\theta}$ were estimated for $\theta = 1°$, $3°$, $5°$, and $10°$ and four apertures: $L/\lambda = 10$, 20, 40, and 60. In each trial, 400 independent $a + \epsilon$ were computed using the same pseudorandom Gaussian generator that is programmed in the TI58 calculator. In each case $M = 50$ and $\rho = 10/3N$ (a 5.2-dB post-filtering SNR). For example $\sigma = 0.17°$ when $L/\lambda = 10$. The results, given in Table I, show that large relative apertures are needed to obtain precise

TABLE I. Statistical properties of θ.

Relative aperture L/λ	Mean bias $\langle\tilde{\theta} - \theta\rangle$	rmse $\langle(\tilde{\theta} - \theta)^2\rangle^{1/2}$	mae $\langle|\tilde{\theta} - \theta|\rangle$
	$\theta = 1°$		
10	0.81°	2.3°	1.8°
20	0.34°	1.5°	1.3°
40	0.18°	1.1°	1.0°
60	0.04°	0.9°	0.8°
	$\theta = 3°$		
10	−0.14°	2.3°	2.1°
20	−0.28°	1.6°	1.3°
40	−0.17°	1.0°	0.7°
60	−0.07°	0.6°	0.4°
	$\theta = 5°$		
10	−0.36°	2.3°	1.8°
20	−0.16°	1.1°	0.8°
40	−0.02°	0.5°	0.4°
60	−0.01°	0.3°	0.2°
	$\theta = 10°$		
10	−0.08°	1.0°	0.8°
20	0.03°	0.5°	0.4°
40	0.01°	0.3°	0.2°
60	0.01°	0.2°	0.1°

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estimates of θ when $\theta = 1°$ or $3°$, $M = 50$, and $\rho = 10/3N$. For a fixed aperture, M or ρ must be very large. These results are not surprising.

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[6]Since $\epsilon \sim N(0, \sigma^2)$, $Y = \epsilon/\sigma$ has a $N(0, 1)$ distribution. If $\sigma < \theta^2/6$, then $1 - \Phi(\theta^2/2\sigma) \leq 10^{-3}$.

Fatigue and recovery of the human acoustic stapedius reflex in industrial noise

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The fatiguability of the acoustic stapedius muscle reflex in an actual noisy industrial environment was investigated in normal-hearing subjects. In a laboratory situation a small depression was found with a considerable individual variability . The stapedius reflex recovered slowly, approximately as a linear function of time. In a field study on an entire day of exposure in a ship-building yard the reflex depression was on the average 4 dB in response to a stimulation of 2000 Hz 10 min after the end of the workday. This corresponds to less than 8 dB immediately at the end of the exposure.

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INTRODUCTION

It is well known from laboratory experiments that the stapedius reflex decays rapidly during exposure to a steady narrow-band sound, especially high-frequency signals (Kato, 1913; Lüscher, 1930, and others). It is also known that a short pause or a shift in the frequency of the stimulus sound lead to reactivation of the reflex (Metz, 1951; Wersäll, 1958; Gjaevenes and Söhoel, 1966; Borg and Ödman, 1979). With these observations in mind, it would seem important to study the decay and recovery of the human stapedius reflex following exposure to industrial noise. Except for the findings that movements of the stapedius tendon persisted for long time in a variable sound environment (Lüscher, 1930; Kobrak *et al.*, 1941), we have no data at present which would enable us to put forth an opinion on whether or not the muscle of the middle ear is active throughout a workday in industry. The aim of the present work is to gain more information on the fatiguability of the stapedius reflex during normal conditions in a noisy industry. This report gives preliminary presentation of results from two studies.

Since it was not possible to monitor the muscle activity directly while the worker performed his normal tasks in the noisy environment (e.g., by chronically implanted electrodes and telemetry), two other kinds of experiments were conceptualized. Experiment 1 was a laboratory experiment, where test subjects were exposed monaurally trough earphone to a tape for 30 min containing industrial noise while the stapedius activity was continuously recorded. The aims was to follow short-term fatiguability and time course of recovery after the end of the exposure. Experiment 2 was a field experiment, where workers were unilaterally exposed to noise in their ordinary environment throughout their entire workday. Their ipsilateral and contralateral reflexes were recorded in both ears before and within 10 min after the end of the workday. The results of experiment 1 were used to extrapolate from the earliest recordings obtained after the end of the workday to the actual end of the exposure.

The study was performed on workers at Götaverken, Arendalsvarvet, a major Swedish ship-building yard, and a complete presentation is in preparation.

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