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Voting One Issue at a Time: The Question of Voter Forecasts

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When issues (i.e., dimensions) are voted on one at a time, a voter whose preferences are not separable across issues must forecast the outcome of later issues in order to know how to vote in the present. This is the problem of expectations. In this article, we develop a general theory designed to handle this problem. Assuming that voters are risk averse and maximize expected utility, we demonstrate that a random variable forecast of how later issues will be decided reduces to a point forecast, which is the mean of the multidimensional random variable. We also show that single-peaked preferences are induced on each issue, and consequently there exists an equilibrium across issues.

There are many settings in which voting takes place serially across a set of issues. Congressmen vote on bills and amendments to bills one at a time. Supreme Court justices vote on cases one at a time. Members of regulatory bodies such as the Federal Election Commission and the Securities and Exchange Commission vote on issues brought before them one at a time. In each case, matters are decided serially by some form of majority rule.

It is often true that what a voter most desires on one issue depends on decisions on other issues. This dependence has been recognized at least since Black and Newing (1951) and more recently by Shepsle (1979) in his work on structure-induced equilibrium. If a congressman must vote on a $178 billion defense authorization bill, his vote may depend on whether he expects new taxes or spending cuts will be approved to help reduce the federal deficit. A Supreme Court justice may worry about how the disposition of one First Amendment case, such as the Pentagon Papers case, may affect the disposition of future First Amendment cases involving national security. A member of a regulatory body such as the Federal Trade Commission may worry about how one enforcement action like a truth-in-advertising rule may affect related rulings. In short, it is commonplace for voters to condition their votes on past decisions or on expectations regarding future ones.

It is the purpose of this article to construct a model of how voters cast their votes under such circumstances. As we will point out, it may be very difficult in political settings for voters to establish the linkage between present alternatives and future decisions. This is particularly true when the same issues are not decided repeatedly over time or important changes occur in the voting environment after current voting has taken place. This difficulty forces each voter to make simplifying assumptions about how each alternative presently being voted on is linked to a decision on each future issue of concern to the voter. We will show that a reasonable method exists whereby voters can condition present votes on expectations regarding future decisions, even when uncertainty exists concerning how present alternatives and future decisions are related. This method induces symmetric, single-peaked preferences for each voter on each issue. Thus, an equilibrium decision exists on each issue. In the following section, we will discuss the difficulty of linking present alternatives to future decisions in a politically charged environment. This discussion will motivate the model to be developed in subsequent sections of the paper.

Relating Present to Future Decisions

It may appear that a sufficient condition for predicting future decisions on the basis of present alternatives is complete knowledge of all voters' preferences. Denzau and Mackay (1981) have shown that this conjecture is incorrect. Instead of simplifying the voter's decision problem, complete preference information can complicate it. Representing each of a set of issues with a single Euclidean dimension, complete preference information allows each voter to calculate the median position on each future issue, conditional on each alternative of the present issue. A contest over one-dimensional alternatives is thereby trans-

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formed into a contest over multidimensional alternatives, opening up the possibility of majority rule cycles even though votes are nominally being taken over one-dimensional alternatives. Endless cycling may preclude consideration of future issues, in which case no linkage between present alternatives and future decisions exist. Or, if a majority rule cycle over one-dimensional alternatives is terminated at some arbitrary point and the next issue considered, voters will be unsure of where this stopping point will be, in which case they can only predict future decisions probabilistically. In either case, predicting decisions on future issues is complicated by the uncertainty concerning the final decision on the present issue.

It is clear, however, that although information about the preferences of other voters may be insufficient to predict future decisions with complete accuracy, this information is necessary to do so. Here it is important to keep in mind that preference information is not always easy to obtain, particularly when preferences are interrelated across issues. What the voter is trying to assess is how other voters make trade-offs between each pair of issues as well as the relative importance of each issue. For n issues, this means estimating $(n(n+1))/2$ parameters for *each* voter —a mind-boggling feat and one hardly conceivable given a limited number of votes as a basis for estimation. Votes, of course, are usually all that the voter observes and are the only hard evidence on which such estimates can be based. In addition, it is possible that if enough time separates different votes, preferences may change.

If the same issues were voted on repeatedly, there might be some way to establish how present alternatives condition future decisions. For example, authorization and appropriations bills for federal agencies and cabinet departments are voted on in the Congress every year. Thus, a congressman may learn something about how alternative Defense Department budgets will affect the final budget for the Department of Health and Human Services. However, critical features of the voting environment rarely remain the same; membership turnover, varying degrees and types of lobbying pressure, and other important changes in the atmosphere surrounding each vote all combine to make apparently similar voting situations different in important respects. In 1975 a watered-down, common site picketing bill was passed by the House but vetoed by President Ford. Two years later, an amendment was added to another common site picketing bill, making it nearly identical to the 1975 bill. The bill was then rejected by the House on final passage. Past experience proved an unreliable guide to the future.

More recently, numerous flip-flops have been observed in voting on spending and revenue-cutting measures. Although the Senate recently voted to approve a constitutional amendment mandating a balanced budget, this decision did not prevent it from voting down a cut in dairy price supports the following day. Cutting individual income taxes by 25% has not dampened recent bipartisan enthusiasm for expensive public jobs programs. In short, there is no straightforward linkage between various spending and revenue-cutting measures that are approved by the Congress.

The same point applies to the Supreme Court, appellate courts, and regulatory agencies. Although guided by precedent, the Supreme Court has frequently broken with its own traditions in response to changes in public opinion and its own membership. The 1973 landmark case, *Roe v. Wade,* which legalized abortion, is an uncertain guide to the disposition of several abortion cases currently scheduled for argument before the court. The recent used-car ruling by the FTC is an uncertain guide to future consumer protection rules by the Commission.

Predicting decisions on future issues as a function of alternatives presently being voted on is a hazardous enterprise. The only recourse seemingly open to the voter is to assume that future decisions are independent of present alternatives. This approach is taken by Denzau and Mackay (1981). Their other approach is to assume that each voter can calculate the median position on each future issue, conditional on each alternative of the present issue: a very restrictive assumption.

In the next section, we will develop an alternative approach that is explicitly informed by political realities. We will assume that the voter uses a probability density function to characterize his beliefs about what the final decision on each future issue will be. The mean and variance of this density function will be assumed to be independent of the alternatives of the issue presently being voted on. Otherwise, an unspecified dependence may exist between present alternatives and future decisions. Future and past decisions may also be linked in the voter's mind. In other words, we assume some independence between present alternatives and future decisions, but we do so because voter uncertainty about the future precludes further knowledge of what this dependence is.

The Model

We exposit our results for the case of three issues voted on one at a time, although our results are fully general and are proved for any finite number of issues in the Appendix. We assume that the alternatives to be voted on are legislative appropriations merely to facilitate the exposition of our results. Alternatively, we may have assumed each issue to be a ruling by a regulatory agency or a case to be decided by a panel of judges.

Consider a set of voters $N = \{1, \ldots, n\}$ that must decide the level of spending on each of a set of issues $I = \{1,2,3\}$. Issue 1 might be spending on the B-1 bomber, issue 2 spending on the stealth bomber, and issue 3 spending on the MX missile. Such issues can be voted separately, for example, as amendments to a defense authorization bill. Or, the three issues might be three amendments to a given budget resolution, each seeking to restore funds that have been cut from various programs. Another possibility is three authorization bills reported by three different committees and voted on at differing points in time. It is not necessary for a voter actually to vote on all the issues; all that is required is that his or her preferences be defined over alternative decisions taken over all three issues. The issues also may not be voted on consecutively. It is even possible for preferences to change after one issue has been decided and before the next issue is considered. This last possibility is particularly important to keep in mind from the standpoint of the assumptions we will make about voter forecasting. Changing preferences certainly make forecasting more difficult, particularly if the nature of such changes is hard to predict.

Each issue is decided by majority rule and is voted on separately in numerical order. The issues may not be voted on in unbroken order. However, if intervening issues exist, we assume that voter preferences are independent of these issues. We assume also that a status quo spending level exists on each issue, which is the amount appropriated for the current fiscal year. Any spending level can be proposed as the amount for the upcoming fiscal year and is accepted in place of the status quo if a majority of voters—$(n+1)/2$ for n odd or $(n/2) + 1$ for n even—prefers the new proposal to the status quo. A finite set of proposals is voted on, each vote being taken between two proposals in this set. The agenda by which voting takes place is not predetermined. Once all proposals in the set are voted on, the issue is finally decided. The last accepted proposal on the prior issue is the final decision on that issue. Voting on succeeding issues takes place in similar fashion. Once a final decision is reached on an issue, it is not voted on again.

Each voter i has preferences defined over the space of possible outcomes, where an outcome is a triple, consisting of an appropriation for issues 1, 2, and 3. In keeping with classical spatial theory (Davis, Hinich, & Ordeshook, 1970), we assume that preferences are based on weighted Euclidean distance. Thus, voter i prefers outcome $\underline{y} = (y_1, y_2, y_3)'$ to outcome $\underline{z} = (z_1, z_2, z_3)'$ if and only if

$$\|\underline{y} - \underline{x}_i\|_{A_i} < \|\underline{z} - x_i\|_{A_i} \tag{1}$$

where $\underline{x}_i = (x_{i1}, x_{i2}, x_{i3})'$ is i's ideal package of appropriations for issues 1, 2, and 3 and $A_i = \begin{pmatrix} a_{i11} & a_{i12} & a_{i13} \\ a_{i12} & a_{i22} & a_{i23} \\ a_{i13} & a_{i23} & a_{i33} \end{pmatrix}$ is i's symmetric, positive definite matrix of salience weights, describing how important each issue is to i and how i makes trade-offs between issues.

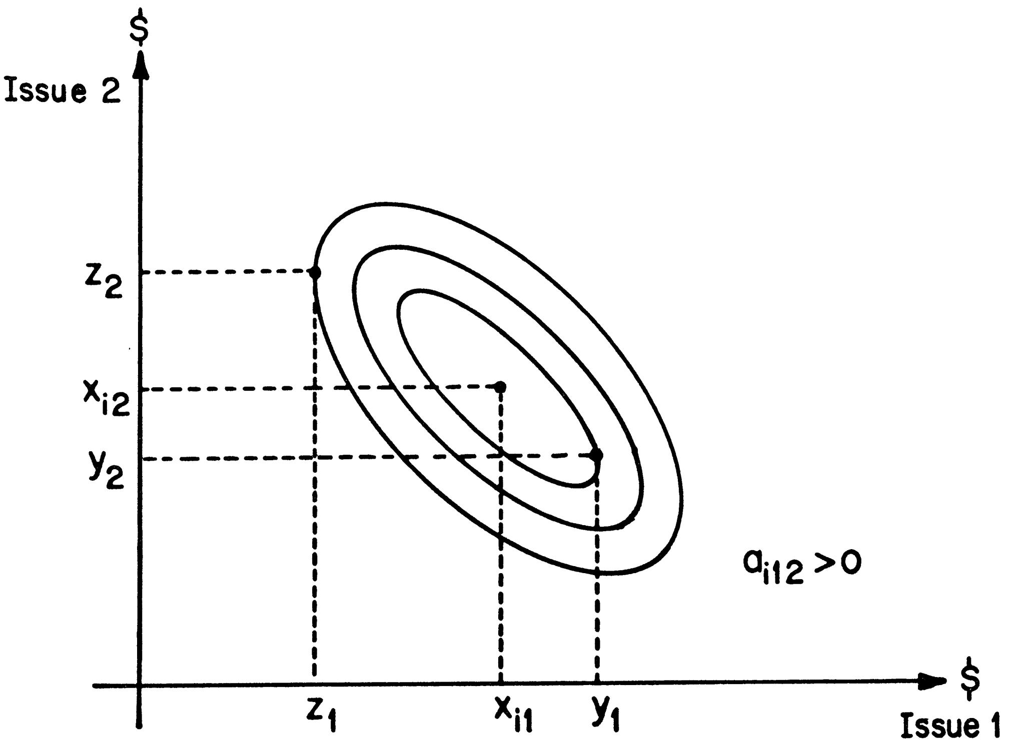
If $a_{ijk} = 0$ for $j \neq k$, then i's preferences are separable across issues, which means that what i most prefers on issue j is independent of alternatives on issue k. If $a_{ijk} \neq 0$ for $j \neq k$, then what i most prefers on issue j depends on the level of spending on issue k. To see this, refer to Figure 1, where preferences are defined over two issues, so that $\underline{x}_i = (x_{i1}, x_{i2})'$ and $A_i = \begin{pmatrix} a_{i11} & a_{i12} \\ a_{i12} & a_{i22} \end{pmatrix}$. Assuming $a_{i12} > 0$, it is clear that what i most prefers on either issue depends on the level of spending on the other issue. If y_1 is the level set on issue 1, y_2 is the level most preferred on issue 2; if z_1 is the level set on issue 1, z_2 is the most preferred level on issue 2. Thus, if issue 1 is spending on the B-1 bomber and issue 2 spending on the stealth bomber, the more that is spent on the B-1, the less i wishes to see spent on the stealth program. In other words, if i must vote on issue 1 *before* voting on issue 2, he will have to make some prediction about what the spending level on issue 2 will be.

Returning to the case of three issues, suppose i must vote on issue 1 without knowing how issues 2 and 3 will be decided. Assuming that a_{i12}, a_{i13}, and a_{i23} are not all zero, i must make some type of forecast concerning how issues 2 and 3 will be decided if he is to know what he most prefers on issue 1.

As we have argued, this is a very difficult problem in most political settings. Accordingly, suppose the voter's forecasts of the final decisions on issues 2 and 3 are a pair of random variables $\bar{\theta}_2$ and $\bar{\theta}_3$. These forecasts may be idiosyncratic. $\bar{\theta}_2$ is a random variable representing the range of final decisions on issue 2 that i believes possible as well as the probability density associated with each. $\bar{\theta}_2$ may be discrete or continuous and has mean μ_2 and

Figure 1

Nonseparable Voter Preferences



variance σ_2^2. $\overline{\theta}_3$ is defined in the same way with mean μ_3 and variance σ_3^2. Following Davis, Dempster, and Wildavsky (1966), if z_j is the current appropriation on issue j and z_j' is the executive's budgetary request on the same issue, $\overline{\theta}_j$ takes its values in the interval $[z_j\, z_j']$, where $z_j' > z_j$. A legislator who believes that the final appropriation on issue j has a 50% chance of being z_j and a 50% chance of being z_j' has a mean forecast μ_j of $(z_j + z_j')/2$.

The forecasts $\overline{\theta}_2$ and $\overline{\theta}_3$ may not be independent of each other or of alternatives on issue 1. In the presence of uncertainty about the future, the form of this dependence is *unknown.* In fact, critical changes in the voting environment that occur between votes may significantly alter this unknown dependence. A mid-term loss of 26 Republican seats in the House of Representatives may totally change the voting climate for public jobs programs. A growing federal deficit may or may not lead to significant cuts in defense spending. Predictions about defense spending can be made, but any attempt to condition these predictions, for example, on alternative federal debt ceiling levels is equivalent to assuming that politics is practiced under laboratory controls.

In the face of this fundamental uncertainty about future decisions, a reasonable approach for voter i is to adopt some type of independence condition as a working hypothesis. The condition we will invoke is that μ_2, μ_3, σ_2^2, and σ_3^2 are independent of alternative spending levels on issue 1. To put it more fully, since voter i does not know how alternative spending levels on issue 1 condition decisions on later issues (or even all the factors that eventually will influence these decisions), he invokes the *minimal assumption* that the means and variances of his forecasts on later issues are independent of the spending level set on issue 1. He is not myopic in the sense of expecting complete independence of decisions across issues; nor is he perfectly informed about the precise form of this dependence: Denzau and Mackay's (1981) two assumptions. Rather, he is aware of the interdependence of decisions across issues, but in the absence of perfect information about the nature of this interdependence and how it may change over time, he assumes the independence of the mean and variance of each forecast. We will later discuss the consequences of weakening this minimal assumption.

The Maximization Problem

How, then, can voter i incorporate his forecasts $\bar{\theta}_2$ and $\bar{\theta}_3$ into his voting decision on issue 1? A simple solution arises if the voter has the following risk averse utility for an outcome $\underline{y}$:

$$u(\underline{y}\,|\underline{x}_i, A_i) = c_i - \|\underline{y} - \underline{x}_i\|^2_{A_i} \tag{2}$$

where c_i is an arbitrary constant, measuring the maximum utility obtainable by i. Assume, then, that θ_1 is paired against ψ_1 as two possible spending levels on issue 1. If i is an expected utility maximizer, he will vote for θ_1 if and only if

$$E\|\underline{\hat{\theta}} - \underline{x}_i\|^2_{A_i} < E\|\underline{\hat{\psi}} - \underline{x}_i\|^2_{A_i} \tag{3}$$

where $\underline{\hat{\theta}} = (\theta_1, \bar{\theta}_2, \bar{\theta}_3)'$ and $\underline{\hat{\psi}} = (\psi_1, \bar{\theta}_2, \bar{\theta}_3)'$.

Expanding both sides of equation (3), we obtain

$$\begin{aligned}
&E[a_{i11}(\theta_1 - x_{i1})^2 + 2a_{i12}(\bar{\theta}_2 - x_{i2})(\theta_1 - x_{i1}) \\
&+ 2a_{i13}(\bar{\theta}_3 - x_{i3})(\theta_1 - x_{i1}) + a_{i22}(\bar{\theta}_2 - x_{i2})^2 \\
&+ 2a_{i23}(\bar{\theta}_3 - x_{i3})(\bar{\theta}_2 - x_{i2}) + a_{i33}(\bar{\theta}_3 - x_{i3})^2] \\
&< E[a_{i11}(\psi_1 - x_{i1})^2 + 2a_{i12}(\bar{\theta}_2 - x_{i2})(\psi_1 - x_{i1}) \\
&+ 2a_{i13}(\bar{\theta}_3 - x_{i3})(\psi_1 - x_{i1}) + a_{i22}(\bar{\theta}_2 - x_{i2})^2 \\
&+ 2a_{i23}(\bar{\theta}_3 - x_{i3})(\bar{\theta}_2 - x_{i2}) + a_{i33}(\bar{\theta}_3 - x_{i3})^2].
\end{aligned}$$

Gathering terms, we have that voter i will vote for θ_1 if and only if

$$\begin{aligned}
&\theta_1[a_{i11}(\frac{\theta_1}{2} - x_{i1}) + a_{i12}(\mu_2 - x_{i2}) \\
&+ a_{i13}(\mu_3 - x_{i3})] < \psi_1[a_{i11}(\frac{\psi_1}{2} - x_{i1}) \\
&+ a_{i12}(\mu_2 - x_{i2}) + a_{i13}(\mu_3 - x_{i3})].
\end{aligned} \tag{4}$$

If both sides of equation (4) are multiplied by 2, dividing both sides by a_{i11} gives

$$\begin{aligned}
&\theta_1^2 - 2\theta_1 x_{i1} + 2\theta_1 [\frac{a_{i12}}{a_{i11}}(\mu_2 - x_{i2}) \\
&+ \frac{a_{i13}}{a_{i11}}(\mu_3 - x_{i3})] < \psi_1^2 - 2\psi_1 x_{i1} \\
&+ 2\psi_1 [\frac{a_{i12}}{a_{i11}}(\mu_2 - x_{i2}) + \frac{a_{i13}}{a_{i11}}(\mu_3 - x_{i3})].
\end{aligned} \tag{5}$$

Completing the square and taking the square root, we obtain from equation (5)

$$\begin{aligned}
&|\theta_1 - x_{i1} + \frac{a_{i12}}{a_{i11}}(\mu_2 - x_{i2}) + \frac{a_{i13}}{a_{i11}}(\mu_3 - x_{i3})| \\
&< |\psi_1 - x_{i1} + \frac{a_{i12}}{a_{i11}}(\mu_2 - x_{i2}) \\
&+ \frac{a_{i13}}{a_{i11}}(\mu_3 - x_{i3})|.
\end{aligned} \tag{6}$$

Thus, we have that θ_1 is preferred to ψ_1 if and only if θ_1 is closer than ψ_1 to $x_{i1} - \frac{a_{i12}}{a_{i11}}(\mu_2 - x_{i2}) - \frac{a_{i13}}{a_{i11}}(\mu_3 - x_{i3})$. But what is this later quantity? If $E\|\underline{\hat{\theta}} - \underline{x}_i\|^2_{A_i}$ is differentiated with respect to θ_1 and the result set equal to zero, we obtain

$$\theta_1^* = x_{i1} - \frac{a_{i12}}{a_{i11}}(\mu_2 - x_{i2}) - \frac{a_{i13}}{a_{i11}}(\mu_3 - x_{i3}). \tag{7}$$

Since the second derivative $(2a_{i11})$ is positive (A_i is positive definite and, thus, $a_{i11} > 0$), the right-hand side of equation (7) is i's conditional ideal point on issue 1, based on his forecasts $\bar{\theta}_2$ and $\bar{\theta}_3$ for issues 2 and 3. Thus, we have that i will vote for θ_1 over ψ_1 if and only if θ_1 is closer to i's conditional ideal point on issue 1. This condition means that i has symmetric, single-peaked preferences on this issue.

What we have established is that if i forecasts the random variables $\bar{\theta}_2$ and $\bar{\theta}_3$ for issues 2 and 3 and is an expected-utility maximizer, symmetric, single-peaked preferences are induced on issue 1. Further, the conditional ideal point on issue 1 is identical to that which would exist if i were making the point forecasts μ_2 and μ_3 on the two succeeding issues. To see this, differentiate $\|(\theta_1, \mu_2, \mu_3)' - \underline{x}_i\|^2_{A_i}$ with respect to θ_1, set the result equal to zero and (7) will be obtained.

Let issue 1 be voted on. If the median θ_1^* is unique, we have from Black's (1958) well-known median voter result that the proposal closest to the median θ_1^* will have the votes of a majority against any other proposal. But even if the median θ_1^* is not unique, ties are not permitted under congressional rules, and so prediction of the final decision is still possible. For convenience, however, assume that the median θ_1^* is unique and is one of the proposals that is voted on. The median θ_1^* will then be the final decision on issue 1. If issue 2 is then voted on, voter i will vote for θ_2 instead of ψ_2 if and only if

$$E\|(med\,\theta_1^*, \theta_2, \bar{\theta}_3)' - \underline{x}_i\|^2_{A_i}$$
$$< E\|(med\,\theta_1^*, \psi_2, \bar{\theta}_3)' - \underline{x}_i\|^2_{A_i}$$

For convenience, assume that A_i, $\underline{x}_i$, and $\bar{\theta}_3$ are the same as in equation (3). But this is not necessary. So long as these parameters are fixed while issue 2 is under consideration, symmetric, single-peaked preferences will exist on issue 2. Repeating the same steps used to derive i's voting rule on issue 1, we obtain the rule for issue 2 that i will vote for θ_2 over ψ_2 if and only if

$$|\theta_2 - x_{i2} + \frac{a_{i12}}{a_{i22}}(med\,\theta_1^* - x_{i1})$$
$$+ \frac{a_{i23}}{a_{i22}}(\mu_3 - x_{i3})| < |\psi_2 - x_{i2}$$
$$+ \frac{a_{i12}}{a_{i22}}(med\,\theta_1^* - x_{i1}) + \frac{a_{i23}}{a_{i22}}(\mu_3 - x_{i3})|. \qquad (8)$$

In short, i will vote for θ_2 if and only if θ_2 is closer than ψ_2 to i's conditional ideal point on issue 2, based on what has been decided on issue 1 and the mean of his forecast on issue 3. This mean, μ_3, and variance of σ_3^2 are assumed to be independent of θ_2 and ψ_2.

Since symmetric, single-peaked preferences are induced on issue 2, the equilibrium on issue 2 is *med* θ_2^*, where θ_2^* *is the conditional ideal* point on issue 2 based on *med* θ_1^* and μ_3. Assuming that *med* θ_2^* is unique and is one of the proposals that is voted on, *med* θ_2^* will be the final decision on issue 2. This means that on issue 3, i will vote for θ_3 over ψ_3 if and only if

$$E\|(med\,\theta_1^*, med\,\theta_2^*, \theta_3)' - \underline{x}_i\|^2_{A_i}$$
$$< E\|(med\,\theta_1^*, med\,\theta_2^*, \psi_3)' - \underline{x}_i\|^2_{A_i},$$

or if and only if

$$|\theta_3 - x_{i3} + \frac{a_{i13}}{a_{i33}}(med\,\theta_1^* - x_{i1})$$
$$+ \frac{a_{i23}}{a_{i33}}(med\,\theta_2^* - x_{i2})| < |\psi_3 - x_{i3}$$
$$+ \frac{a_{i13}}{a_{i33}}(med\,\theta_1^* - x_{i1}) + \frac{a_{i23}}{a_{i33}}(med\,\theta_2^* - x_{i2})|. \qquad (9)$$

Again, $\underline{x}_i$ and A_i must be fixed while issue 3 is under consideration, but they may be different from their values when earlier issues were voted on. In either case, symmetric, single-peaked preferences are also induced on issue 3, conditioned on *med* θ_1^* and *med* θ_2^*. Assuming *med* θ_3^* is the unique median of this set of conditioned ideal points and is one of the proposals that is voted on, (*med* θ_1^*, *med* θ_2^*, *med* θ_3^*) will be the equilibrium outcome and final set of decisions for all three issues.

It is important to point out that coalition formation does not upset this equilibrium so long as each voter votes according to his induced preferences on each issue. In other words, we have identified a strong equilibrium.

An Example

A simple example illustrates the above results and will aid in clarifying some additional points.

Assume five voters labelled *A, B, C, D,* and *E* with preferences defined over three issues. Each issue can be seen as the level of spending for a given program. Table 1 lists the ideal spending package for each voter on these three issues, the current appropriation for each, and the budgetary request from the executive branch.

We will assume that the preferences of voters *A, C,* and *D* are based on simple Euclidean distance. Thus $A_a = A_c = A_d = I$, the 3 × 3 identity matrix, and preferences are separable across issues. However, for voter *B* we will set $a_{b11} = a_{b22} = a_{b33} = 1$, but $a_{b12} = -.9$, $a_{b13} = -.4$, and $a_{b23} = -.6$. Thus, the more that is spent on each issue, the more *B* wishes to see spent on remaining issues. For voter *E*, we will set $a_{e11} = a_{e22} = a_{e33} = 1$ and $a_{e12} = .9$, $a_{e13} = .6$, $a_{e23} = .5$. Thus, the less that is spent on each issue, the more *E* would like to see spent on remaining issues.

Since *A, C,* and *D* have separable preferences, forecasts are unnecessary for them to decide how

Table 1. Ideal Spending Packages (in millions).

Voter	Issue 1	Issue 2	Issue 3
A	$120	$100	$200
B	100	50	20
C	30	30	30
D	15	200	100
E	60	60	60
Current appropriation	100	100	80
Budgetary request	20	30	30
Mean forecast	65	65	65

to vote. As inspection of equations (6), (8), and (9) shows, each will vote for the spending level on each issue that is closer to the spending level in his ideal spending package. However, B and E's preferences on each issue are conditioned by what has already been decided on previous issues and what, on average, each thinks will be the final decisions on later issues. B knows only his own preferences with certainty, as does E. How, then, does each of these two voters arrive at an average forecast for issues yet to be voted on?

Let us assume that B and E are not even sure of the ideal points of the other voters. However, they do know the current appropriation z_j and the budgetary request z_j' on each issue $j = 1, 2, 3$. Given our earlier assumption that each forecast variable takes its values in the interval between z_j and z_j', what is a reasonable mean forecast on issues 2 and 3? This is not a simple question, and we do not wish to suggest any final answers. However, the voter's task is greatly simpified if we focus on a small set of proposals. Let {$30, $50, $80, $100} be the set of proposals that will be voted on in the form of substitute amendments for each of the three issues. Then, it is not unreasonable to suppose that the forecasting difficulties we have described earlier leave the voters with the conclusion that each of these proposals is equally likely to be the final decision. If this is so, $65 is the mean forecast on each issue. Given these mean forecasts, the conditional ideal point of B on issue 1 (in millions of dollars) is $100 + .9(65–50) + .4(65–20) = 131.5$, and the conditional ideal point of E on issue 1 (in millions of dollars) is $60 - .9(65–60) - .6(65–60) = 52.5$. The revised set of ideal points on issue 1 is, then, {$120, $131.5, $30, $15, $52.5}, and the median of this set is $52.5. Since $50 is the proposal closest to $52.5, $50 will be the final decision on issue 1.

Using this information in order to vote on issue 2, B's conditional ideal point on the second issue is $50 + .9(50–100) + .6(65–20) = 32$, and E's conditional ideal point on issue 2 is $60 - .9(50–60) - .5(65–60) = 66.5$, so that the revised set of ideal points on issue 2 is {$100, $32, $30, $200, $66.5), with $66.5 the median on the second issue. Since $80 is the proposal closest to $66.5, $80 will be the final decision on issue 2. Notice that $80 is the actual decision reached on issue 2, whereas $65 is the mean forecast. What are B and E to make of this discrepancy? Remember that $65 is not the actual forecast on issue 2, but is, instead, the mean of the forecast. If it were voted on, $65 would win against any of the other four alternatives. What is most important here is that it is impossible to learn very much from the way the votes are cast about how present alternatives condition future decisions. In fact, although we do not assume it for this example, voters may anticipate that preferences will change in some unknown way with respect to some future issue between the time the present issue is voted on and the time the future issue is taken up. Thus, if a fourth issue were being voted on, it is hard to imagine how B or E might alter their forecasts on issue 4 based on the discrepancy between their mean forecast on issue 2 and the final decision.

Continuing with our three-issue example, since $50 and $80 are the final decisions reached on issues 1 and 2, B's conditional ideal point on issue 3 is $20 + .4(50–100) + .6(80–20) = 36$, and E's conditional ideal point on issue 3 is $60 - .6(50–60) - .5(80–60) = 56$. Thus, {$200, $36, $30, $100, $56} is the revised set of ideal points on issue 3, and so $56 is the median point on this issue. Since $50 is closest to $56, {$50, $80, $50} will be the set of final decisions reached on issues 1, 2, and 3.

We assume that each issue is voted on once and not voted on again. If the issues were voted on again, after the first set of final decisions were reached (something which happens in Congress usually only after at least a year has elapsed), the new set of final decisions may not be the same. But there is no reason why they have to differ. The final decisions on both issues 2 and 3 are equally balanced around the mean forecasts on these issues, offering voters B and E a good hedge against uncertainty. Considering how limited B's and E's information is, neither voter has any way of knowing for sure whether the second round of voting will increase or decrease the spending levels set in the first round. There is, consequently, no compelling reason why the same forecasts should not be used in a second round of voting.

A point worth stressing is that the median position on each issue is sensitive to the mean forecasts of the voters. Thus, observation of final decisions that differ from the set of issue-by-issue median positions does not necessarily refute the median voter result. A possible approach to testing the model developed in this article is to obtain mean forecasts from voters in an experimental (admittedly nonpolitical) setting and then compare final decisions with the revised median positions.

Weakening the Independence Condition

The question we wish to consider in this section is how far can we weaken the condition we have imposed that the mean and variance of each forecast are independent of alternative spending levels on other issues and yet still obtain our stability results?

Assume when voting on issue 1, for example, that the means but not the variances of $\bar{\theta}_2$ and $\bar{\theta}_3$ are seen as depending on θ_1 and ψ_1. Then, we can write the two conditional means $\mu_{2|\theta_1}$ and $\mu_{2|\psi_1}$ for $\bar{\theta}_2$ and the two conditional means $\mu_{3|\theta_1}$ and $\mu_{3|\psi_1}$ for θ_3. How does this dependence affect the results previously derived?

Interestingly, repeating the steps used to derive expression (6) from expression (3), the only change in the later expression is that $\mu_{2|\theta_1}$ and $\mu_{3|\theta_1}$ occur on the left-hand side of expression (6) in place of μ_2 and μ_3 and $\mu_{2|\psi_1}$ and $\mu_{3|\psi_1}$ occur on the right-hand side of expression (6) in place of μ_2 and μ_3. Otherwise, expression (6) is unchanged. This is also true for expressions (8) and (9) and holds true in the general case. As we will point out shortly, however, there is an important change in the meaning of these expressions.

Why have we sacrificed this additional generality for our results by assuming a lack of dependence between mean forecasts and alternatives on other issues? We have done so because we assume imperfect information about future decisions. For voters to condition μ_2 and μ_3 on alternative spending levels on issue 1 (or for them to condition μ_3 on alternative spending levels on issues 1 and 2) is to presume that they know the form of this conditioning. Since we have explained the difficulties in connecting present with future decisions (even in the unlikely case that all voter preferences are known), it is self-contradictory to build this assumption into the model. Thus, the independence of the μ's (or σ's) from alternatives on other issues is based on the idea that *voters do not possess sufficient information to operate on any other basis.*

It should also be mentioned that there are two ways of weakening the independence condition. The first way is to allow the mean or variance of each forecast to depend on alternatives presently being voted on. The second way is to allow the mean or variance of each forecast to depend on factors other than the alternatives of the present issue, such as decisions on past issues. The first type of dependence destroys the general existence of equilibrium, whereas the second does not. Referring to expression (8), for example, it is clear that if μ_3 differs, depending on θ_2 or ψ_2, the last term on both sides of the inequality will no longer be identical. Thus, there can be two conditional ideal points on issue 2, one based on θ_2 and one based on ψ_2. Preferences on issue 2 will then be multipeaked and so, in general, equilibrium will not exist. The same reasoning applies if σ_3^2 is conditioned on alternatives of issue 2.

If, however, μ_3 or σ_3^2 were dependent only on *med* θ_1^*, a single conditional ideal point on issue 2 would exist. Thus, if the mean or variance of each forecast is dependent on factors other than the alternatives presently being voted on, equilibrium across issues is preserved. As stated earlier, preferences may change after one issue is decided and before the next issue is voted on. The mean or variance of the voter's forecast of future decisions may then also change, and as long as these changes are not conditioned on alternatives of the present issue, equilibrium will be preserved.

Conclusion

A new approach to the problem of forecasting has been developed in this article. What characterizes this approach is that voters possess limited information about voter preferences and future decisions, but use this information to derive revised preferences on each issue. Owing to uncertainty, the forecast on each future issue is a random variable. The central result of this article is that if voters are expected-utility maximizers, using a random variable forecast is equivalent to using a point forecast, this point being the mean of the random variable. The form of the random variable makes no difference. The voter need only estimate its mean to obtain all the information he needs to incorporate future issues into his present voting decision.

Further, under this approach, each voter possesses symmetric, single-peaked preferences on each issue. Thus, an equilibrium exists across issues which cannot be upset even by a coalition of voters, as long as votes are cast according to preferences.

Imperfect information about future decisions is the justification for assuming the independence of the mean and variance of each forecast from alternatives of other issues. Dropping this assumption leads to very small changes in our results. Unless the means or variances are conditioned on the alternatives of the present issue, single-peakedness (and thus equilibrium) is preserved on each issue.

Finally, we wish to stress that our model of voter forecasts is a *political* model. It may be wondered whether opportunities for learning exist that will allow voters to condition forecasts on present alternatives. Certainly, this is not impossible. However, in politics we regard this event as a special case. To say that political environments are complex is not to say that voters are illogical. Instead, the appropriate conclusion to draw is that voters must rely on what they know, or at least think they know. To build a general model based on information that voters generally do not possess does not increase our understanding of politics: in fact, it works in the opposite direction.

Appendix

First, we will show what voter i's revised ideal point is on issues $n+1, \dots p$ when issues $1, \dots, n$ have already been decided. Let $\underline{\theta} = (\theta_1, \dots, \theta_n)'$ represent the outcome on these first n issues. Voter i is uncertain about how issues $n+1, \dots, p$ will be decided. Let $\underline{\overline{\theta}} = (\overline{\theta}_{n+1}, \dots, \overline{\theta}_p)'$ represent the $(p-n)$-dimensional random variable that describes i's forecast on the remaining $p-n$ issues. Let $\Theta = (\theta_1, \dots, \theta_n, \overline{\theta}_{n+1}, \dots, \overline{\theta}_p)'$.

Let $\underline{\overline{\mu}} = (\mu_{n+1}, \dots, \mu_p)'$ denote i's forecast of the mean of $\underline{\overline{\theta}}$, $\underline{\overline{\sigma}}^2 = (\sigma_{n+1}^2, \dots, \sigma_p^2)'$ the variance of $\underline{\overline{\theta}}$, and let $\underline{x} = (\underline{x}_n, \underline{x}_{p-n})'$ denote i's unrevised ideal point on all p issues, where $\underline{x}_n = (x_1, \dots, x_n)'$ and $\underline{x}_{p-n} = (x_{n+1}, \dots, x_p)'$. For simplicity, the voter subscript is dropped. Finally A is i's $p \times p$ salience matrix on all p issues (A is symmetric, positive definite). Let A be partitioned into four submatrices as follows:

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{12}' & A_{22} \end{pmatrix},$$

$$A_{11} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{1n} & \cdots & a_{nn} \end{pmatrix},$$

$$A_{12} = \begin{pmatrix} a_{1n+1} & \cdots & a_{1p} \\ \vdots & & \vdots \\ a_{nn+1} & \cdots & a_{np} \end{pmatrix},$$

$$A_{22} = \begin{pmatrix} a_{n+1n+1} & \cdots & a_{n+1p} \\ \vdots & & \vdots \\ a_{n+1p} & \cdots & a_{pp} \end{pmatrix},$$

We will now derive i's revised ideal point on issues $n+1, \ldots p$.

$\| \Theta - \underline{x} \|^2_A$ is the weighted Euclidean distance between Θ and $\underline{x}$. Given the above notation,

$$\| \Theta - \underline{x} \|^2_A = ((\underline{\theta} - \underline{x}_n)', (\underline{\bar{\theta}} - \underline{x}_{p-n})') \begin{pmatrix} A_{11} & A_{12} \\ A'_{12} & A_{22} \end{pmatrix} \begin{pmatrix} \underline{\theta} - \underline{x}_n \\ \underline{\bar{\theta}} - \underline{x}_{p-n} \end{pmatrix} = (\underline{\theta} - \underline{x}_n)' A_{11} (\underline{\theta} - \underline{x}_n)$$

$$+ 2(\underline{\theta} - \underline{x}_n)' A_{12} (\underline{\bar{\theta}} - \underline{x}_{p-n}) + (\underline{\bar{\theta}} - \underline{x}_{p-n})' A_{22} (\underline{\bar{\theta}} - \underline{x}_{p-n}). \tag{A1}$$

If we differentiate (A1) with respect to $\underline{\bar{\theta}}$ and set the result equal to 0, we can solve for i's revised ideal point on issues $n+1, \ldots, p$.

$$\frac{\partial \| \Theta - x \|^2_A}{\partial \underline{\bar{\theta}}} = 2A'_{12} (\underline{\theta} - \underline{x}_n) + 2A_{22} (\underline{\bar{\theta}} - \underline{x}_{p-n}) = 0, \text{ so that}$$

$\underline{x}_{p-n} - A_{22}^{-1} A'_{12} (\underline{\theta} - \underline{x}_n)$ is i's revised ideal point (observe that the second-order condition for a minimum is met).

We can now use this result to derive a rule whereby i can determine how to vote on issue $n+1$, given $\underline{\theta}$ on issues $1, \ldots, n$, his revised ideal point on issues $n+1, \ldots, p$ and his forecast $(\bar{\theta}_{n+2}, \ldots, \bar{\theta}_p)$ on the remaining issues. Assume that $(\mu_{n+2}, \ldots, \mu_p)$ and $(\sigma^2_{n+2}, \ldots, \sigma^2_p)$ are independent of the alternatives on the n+1st issue. This assumption is reasonable since voters are uncertain about the outcomes on later issues.

Suppose i must decide whether to vote for θ_{n+1} or ψ_{n+1} on issue $n+1$. If he is an expected-utility maximizer, he will vote for θ_{n+1} if and only if

$$E \| \underline{\hat{\theta}} - \underline{\hat{x}} \|^2_{A_{22}} < E \| \underline{\hat{\psi}} - \underline{\hat{x}} \|^2_{A_{22}} \tag{A2}$$

where $\underline{\hat{\theta}} = (\theta_{n+1}, \bar{\theta}_{n+2}, \ldots, \bar{\theta}_p)'$, $\underline{\hat{\psi}} = (\psi_{n+1}, \bar{\theta}_{n+2}, \ldots, \bar{\theta}_p)'$, and $\underline{\hat{x}} = \underline{x}_{p-n} - A_{22}^{-1} A'_{12} (\underline{\theta} - \underline{x}_n)$ is his revised ideal point. Thus, (A2) can also be written

$$E[(\underline{\hat{\theta}} - \underline{x}_{p-n} + A_{22}^{-1} A'_{12} (\underline{\theta} - \underline{x}_n))' A_{22} (\underline{\hat{\theta}} - \underline{x}_{p-n} + A_{22}^{-1} A'_{12} (\underline{\theta} - \underline{x}_n))]$$

$$< E[(\underline{\hat{\psi}} - \underline{x}_{p-n} + A_{22}^{-1} A'_{12} (\underline{\theta} - \underline{x}_n))' A_{22} (\underline{\hat{\psi}} - \underline{x}_{p-n} + A_{22}^{-1} A'_{12} (\underline{\theta} - \underline{x}_n))]. \tag{A3}$$

Multiplying through and gathering terms, we can rewrite (A3) as (A4).

$$E(\underline{\hat{\theta}}' A_{22} \underline{\hat{\theta}}) - 2E(\underline{\hat{\theta}}' A_{22} \underline{x}_{p-n}) + 2(\underline{\theta} - \underline{x}_n)' A_{12} \underline{\hat{\mu}}_\theta$$

$$< E(\underline{\hat{\psi}}' A_{22} \underline{\hat{\psi}}) - 2E(\underline{\hat{\psi}}' A_{22} \underline{x}_{p-n}) + 2(\underline{\theta} - \underline{x}_n)' A_{12} \underline{\hat{\mu}}_\psi \tag{A4}$$

where $\underline{\hat{\mu}}_\theta = (\theta_{n+1}, \mu_{n+2}, \ldots, \mu_p)'$ and $\underline{\hat{\mu}}_\psi = (\psi_{n+1}, \mu_{n+2}, \ldots, \mu_p)'$.

Taking the first term on the left-hand side of (A4),

$$E(\underline{\hat{\theta}}' A_{22} \hat{\theta}) = E[(\theta_{n+1}, \bar{\theta}_{n+2}, \ldots, \bar{\theta}_p) \begin{pmatrix} a_{n+1,n+1} & \cdots & a_{n+1,p} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ a_{n+1,p} & \cdots & a_{pp} \end{pmatrix} \begin{pmatrix} \theta_{n+1} \\ \bar{\theta}_{n+2} \\ \cdot \\ \cdot \\ \cdot \\ \bar{\theta}_p \end{pmatrix}],$$

and it is clear that the only part of this term not identical to its counterpart in the first term on the right-hand side of (A4) is

$$\theta_{n+1} (a_{n+1,n+1}, \ldots, a_{n+1,p}) (\theta_{n+1}, 2\mu_{n+2}, \ldots, 2\mu_p)'.$$

In the same way, the only part of term 2 on the left-hand side of (A4) not identical to its counterpart in the second term on the right-hand side is

$$-2\,\theta_{n+1}(a_{n+1,n+1},\ldots,a_{n+1,p})\underline{x}_{p-n}.$$

Finally, the only part of term 3 on the left-hand side of (A4) which does not drop out is

$$2\,\theta_{n+1}(\underline{\theta}-\underline{x}_n)'(a_{1,n+1},\ldots,a_{n,n+1})'$$

and thus (A4) can finally be rewritten as (A5)

$$\theta_{n+1}\underline{a}'_{n+1}(\theta_1-x_1,\ldots,\theta_n-x_n,\theta_{n+1}/2-x_{n+1},\mu_{n+2}-x_{n+2},\ldots,\mu_p-x_p)'$$
$$<\psi_{n+1}\underline{a}'_{n+1}(\theta_1-x_1,\ldots,\theta_n-x_n,\psi_{n+1}/2-x_{n+1},\mu_{n+2}-x_{n+2},\ldots,\mu_p-x_p)' \qquad \text{(A5)}$$

where $\underline{a}_{n+1}=(a_{n+1,1},\ldots,a_{n+1,p})'$ is the $(n+1)$st column of A.

(A5) can be better understood by way of an example. Suppose there are a total of 3 issues. The outcome of issue 1 is θ_1, the forecast on issue 3 is $\overline{\theta}_3$. Issue 2 is being voted on, and the choice is between θ_2 and ψ_2. Then, by (A5) i will vote for θ_2 if and only if

$$\theta_2\,(a_{12},a_{22},a_{23})(\theta_1-x_1,\theta_2/2-x_2,\mu_3-x_3)'$$
$$<\psi_2\,(a_{12},a_{22},a_{23})(\theta_1-x_1,\psi_2/2-x_2,\mu_3-x_3)'.$$

Multiplying both sides by 2, and dividing by a_{22} gives

$$\theta_2^2-2\theta_2X_2+2\theta_2\left(\frac{a_{12}}{a_{22}}(\theta_1-x_1)+\frac{a_{23}}{a_{22}}(\mu_3-x_3)\right)$$
$$<\psi_2^2-2\psi_2x_2+2\psi_2\left(\frac{a_{12}}{a_{22}}(\theta_1-x_1)+\frac{a_{23}}{a_{22}}(\mu_3-x_3)\right).$$

Completing the square and taking the square root, we obtain

$$\left|\theta_2-x_2+\frac{a_{12}}{a_{22}}(\theta_1-x_1)+\frac{a_{23}}{a_{22}}(\mu_3-x_3)\right|<\left|\psi_2-x_2+\frac{a_{12}}{a_{22}}(\theta_1-x_1)+\frac{a_{23}}{a_{22}}(\mu_3-x_3)\right|.$$

But, $x_2-\frac{a_{12}}{a_{22}}(\theta_1-x_1)-\frac{a_{23}}{a_{22}}(\mu_3-x_3)$ is i's revised ideal point on issue 2, given θ_1 as the decision on issue 1 and μ_3 as the decision forecast on issue 3. So, i will vote for θ_2 if and only if it is closer than ψ_2 to this revised ideal point.

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