A New Approach to Voter Uncertainty in the Downsian Spatial Model

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A new model of voter uncertainty about candidate positions is presented in which voters simplify the issue positions of the candidate by representing them as a random variable on an underlying evaluative dimension. It is further assumed that the degree of voter uncertainty depends upon the mean location of this random variable. It is demonstrated that this type of spatially dependent uncertainty results in a shift of each voter's ideal point on the underlying dimension. We discuss two types of shifts, one in which voter ideal points are shifted toward the extremes and the other in which they are shifted toward the center and comment on the consequences of these shifts for two-candidate electoral competition. Finally, we relate our model to earlier work on the subject by Downs (1957) and Shepsle (1972).

The purpose of this paper is to demonstrate how voter uncertainty about the location of candidates on an underlying ideological dimension can affect standard results in the single dimensional theory of electoral competition. We will assume that voters think of candidates as occupying some position on the left/right or liberal/conservative dimension. This dimension is a shorthand device used by the voter to predict what each candidate will do in office, if he is elected. The way in which this position is determined is for each voter to reduce the positions that a candidate takes on the set of actual campaign issues to a single position on this underlying dimension.[1] However, because this is a complex task and because the informational requirements needed to carry out this task are great, the end result of this process is that the voter is unsure about precisely where the candidate is located on the underlying dimension. In other words, each candidate is perceived by the voters as a random variable on the underlying dimension. Further, it will be assumed that the variance of this random variable is a function of the spatial location of its mean value. Thus, we are positing a fundamental relationship between what is perceived as the mean value of a candidate on the underlying left/right dimension and the perceived degree of certainty associated with this mean value as a guide for predicting the future behavior of the candidate in office.

*Professor Hinich's research effort was supported by the National Science Foundation under grant SOC-7900034.

[1]For a fuller discussion of this process, see Hinich and Pollard (1979).

American Journal of Political Science, Vol. 25, No. 3, August 1981
 0092-5853/81/030483-11$01.30

Assuming also that voters possess quadratic utility functions and are expected utility maximizers, we arrive at some interesting results. What we find is that spatially dependent uncertainty results in a shift of each voter's ideal point on the underlying dimension. For example, suppose the center of the ideological dimension is the point of maximum variance, so that a candidate whose average position is perceived to be at this point creates the greatest amount of uncertainty in the minds of the voters as to what he will do in office. What happens as a result is that voter ideal points to the left of center are moved to the left and voter ideal points to the right of center are moved to the right. Conversely, if the center is the point of least variance, voter ideal points on either side are shifted toward the center.

After demonstrating how these results are obtained, we discuss their significance in light of previous work on voter uncertainty about candidate spatial location by Downs (1957) and Shepsle (1972). What we will point out is that our approach is based on a fundamentally different understanding of the source of voter uncertainty, which we see as only partially controllable by the candidates in a political campaign.

The Model

In this section, we will describe a revised form of the single-dimensional spatial model as conceived by Downs. We assume that all voters view the candidates in a campaign as being located somewhere on a left/right ideological dimension, but possess varying degrees of uncertainty about precisely where they are located. Let candidate Theta be perceived as the random variable $\widetilde{\theta} = \theta + \epsilon_\theta$, where $E(\epsilon_\theta) = 0$, so that θ is the perceived mean position of Theta on the ideological dimension, and ϵ_θ represents the random element of the voter's perception. The dependence of this random term on the mean perception of the candidate's position is an important feature of our model.[2] To simplify the exposition of the model and interpretation of its results, we will not assume idiosyncratic perception of θ or ϵ_θ. Were we to do so, the only effect this would have would be to individualize the results for each voter.

Let the utility of voter i for $\widetilde{\theta}$ be quadratic in form, i.e.,

$$u_i(\widetilde{\theta}) = c_i - (\widetilde{\theta} - x_i)^2 \tag{1}$$

where $c_i > 0$ is a constant representing the maximum utility obtainable by i, and x_i is his own most preferred point on the ideological dimension.[3] Recall

[2]The representation of a random variable as the sum of a fixed element (i.e., mean value) and random element is common practice in statistics, e.g., in the general linear model.

[3]Voter utility is defined only up to a positive linear transformation. Thus a symmetric quadratic utility function is either of the form given by equation (1), or it is of the form $1 - a_i(\widetilde{\theta} - x_i)^2$, where $a_i > 0$ is the salience of the left-right dimension for i. In equation (1), the a_i term is divided out.

that $\tilde{\theta}$ is a random variable, while x_i is a constant. What we wish to know is the value of θ that maximizes expected utility for i.[4]

To obtain the answer, we substitute $\theta + \epsilon_\theta$ for $\tilde{\theta}$ and take the expectation of $u_i(\tilde{\theta})$, giving us

$$E[u_i(\tilde{\theta})] = E[c_i - (\theta^2 + 2\epsilon_\theta\theta - 2x_i\theta + \epsilon_\theta^2 - 2\epsilon_\theta x_i + x_i^2)] \qquad (2)$$

But, since $E(\epsilon_\theta) = 0$, and $E(\epsilon_\theta^2) = \text{Var}(\epsilon_\theta) + [E(\epsilon_\theta)]^2 = \text{Var}(\epsilon_\theta)$, if we set $\text{Var}(\epsilon_\theta) = \sigma_\theta^2$, equation (2) reduces to

$$E[u_i(\tilde{\theta})] = c_i - (\theta - x_i)^2 - \sigma_\theta^2 \qquad (3)$$

A fuller explanation of how Equation 3 is derived from Equation 2 is contained in an appendix.

If we then differentiate Equation 3 with respect to θ, and set the result equal to zero, we obtain

$$\frac{\partial E[u_i(\tilde{\theta})]}{\partial\theta} = -2(\theta - x_i) - \frac{\partial\sigma_\theta^2}{\partial\theta} = 0$$

or

$$\theta_i^* = x_i - \frac{\partial\sigma_\theta^2}{2\partial\theta} \qquad (4)$$

Since the solution given on the right-hand side of Equation 4 is the mean candidate position that maximizes expected utility for i, given that the second-order condition $[(\partial^2\sigma_\theta^2/\partial\theta^2) > -2]$ is met, we label the left-hand side θ_i^* to note that this position is a revised ideal point for voter i under our conditions of uncertainty.

The Center as the Point of Maximum Uncertainty

Now, clearly, if σ_θ^2 is independent of θ, then $\theta_i^* = x_i$ satisfies both first- and second-order conditions required for a utility maximizing position. However, we are assuming that σ_θ^2 does depend upon θ. To be more specific, we shall assume two parabolic forms for σ_θ^2, the first of which is described by the graph in Figure 1. Without loss of generality, we will set μ, the point of maximum variance, equal to zero, which we will interpret to be the center of the ideological dimension. What we are describing in Figure 1, therefore, is the case in which the center is the point of maximum variance. In addition, it is necessary to set $\alpha, \beta > 0$ and to ensure that the variance is always nonnegative, $|\theta| \leq (\alpha/\beta)^{1/2}$.

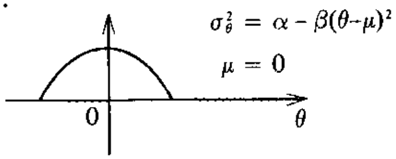

FIGURE 1

[4] For an analysis of voter preference and candidate strategy under individual decision rules other than expected utility maximization, see Weisberg and Fiorina (1980).

Now, substituting for $\partial\sigma_\theta^2/\partial\theta$ in Equation 4, we have

$$\theta_i^* = x_i + \beta\theta_i^* = \frac{1}{1-\beta}x_i \tag{5}$$

Checking for the second-order condition,

$$\frac{\partial^2 E[u_i(\widetilde{\theta})]}{\partial\theta^2} = -2 - \frac{\partial^2\sigma_\theta^2}{\partial\theta^2} = -2(1-\beta), \tag{6}$$

so, as long as $\beta < 1$, the solution given by Equation 5 is the candidate mean position that maximizes expected utility for voter *i*.

Let us now interpret this solution. Remembering that we have defined 0 as the center of the ideological dimension, if $0 < \beta < 1$, then $1/(1-\beta) > 1$ so that if $x_i > 0$, $\theta_i^* > x_i$ while if $x_i < 0$, $\theta_i^* < x_i$. In other words, under the conditions we have specified, voter ideal points to the right of center are shifted to the right, while voter ideal points to the left of center are shifted to the left. The result of this type of spatially dependent uncertainty, therefore, is to polarize the voters toward the two extremes of the ideological dimension—this polarization increasing in direct proportion to the size of β.

Let us now discuss the implications of our model for voter choice in two-candidate elections, given this form of σ_θ^2. Let there be two candidates, Theta and Psi, where Theta is perceived as the random variable $\widetilde{\theta} = \theta + \epsilon_\theta$ and Psi as the random variable $\widetilde{\psi} = \psi + \epsilon_\psi$. Further, let $E(\epsilon_\theta) = E(\epsilon_\psi) = 0$ and $\sigma_\psi^2 = \text{Var}(\epsilon_\psi)$. For convenience, let $\psi < \theta$. Now, voter *i* will prefer Theta to Psi if and only if $E[u_i(\widetilde{\theta})] > E[u_i(\widetilde{\psi})]$, which, upon substitution, reduces to

$$x_i > (1-\beta)\frac{\theta+\psi}{2} \tag{7}$$

Recalling that $0 < \beta < 1$ and that $\psi < \theta$, let us interpret the meaning of equation (7) for *i*'s candidate preference. Suppose $\theta, \psi > 0$, so that $\sigma_\theta^2 < \sigma_\psi^2$. Then, x_i need not be closer to θ than ψ for *i* to prefer Theta. Further, as β increases, $(1-\beta)(\theta+\psi)/2$ decreases to zero, so that as β approaches 1, *i* will prefer Theta to Psi whenever $x_i > 0$. For example, if $\beta = 1/4$, $\psi = 1/2$, and $\theta = 1$, then *i* prefers Theta to Psi if and only if $x_i > 9/16$. However, if $\beta = 3/4$, *i* prefers Theta to Psi if and only if $x_i > 3/16$, so that *i* will prefer Theta even if $x_i = \psi$!

On the other hand, if $\psi < 0$, σ_θ^2 may be equal to or greater than σ_ψ^2. If $\sigma_\theta^2 = \sigma_\psi^2$, then $\theta = -\psi$, in which case *i* will prefer Theta to Psi if and only if $x_i > 0$. Since the degree of uncertainty surrounding both candidates is the same, it is reasonable to expect that spatial distance should be the only factor governing *i*'s preference.

In the final case, $\sigma_\theta^2 > \sigma_\psi^2$ implies that $\theta < -\psi$ so that $(\theta+\psi)/2 < 0$. Again, as β approaches 1, *i* will prefer Theta to Psi if and only if $x_i > 0$.

Now, however, the point on the ideological dimension at which i is indifferent between Theta and Psi is shifted to the right of $(\theta + \psi)/2$. Thus, if $\beta = 3/4$, $\psi = -1$, and $\theta = -1/2$, then i prefers Theta if and only if $x_i > -3/16$.

We will now discuss the consequences of this type of spatially dependent uncertainty for two-candidate electoral competition. By the well-known median voter result, we should expect both candidates to locate themselves as close as possible to the ideological position most preferred by the median voter. However, this location is not the median x_i in our model, but is instead $1/(1-\beta)$ median x_i. Thus, if median $x_i < 0$, the two candidates will be drawn toward the left end of the ideological dimension, whereas if median $x_i > 0$, the two candidates will be drawn toward the right end of the dimension. Only if median $x_i = 0$ will this type of uncertainty have no effect on two-candidate electoral competition.

The Center as the Point of Minimum Uncertainty

On the other hand, suppose the relationship between σ_θ^2 and θ is given by the equation represented by the graph in Figure 2. Now the center (still at 0) is the point of least variance, so that for $\gamma > 0$, candidate mean positions closer to the extremes generate increasing levels of uncertainty for the voters. Once again, we can substitute for $\partial\sigma_\theta^2/\partial\theta$ in Equation 4, which yields

$$\theta_i^* = x_i - \gamma\theta_i^* = \frac{1}{1+\gamma} x_i \tag{8}$$

and, since $\partial^2 E[u_i(\tilde{\theta})]/\partial\theta^2 = -2\,(1+\gamma)$, our assumption that $\gamma > 0$ ensures that the second-order condition will be met.

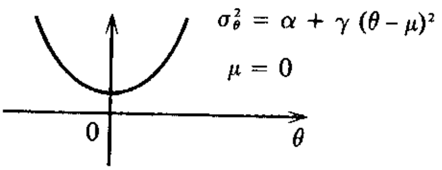

FIGURE 2

Under this type of uncertainty, since $[1/(1+\gamma)]\ x_i < x_i$, voter ideal points on either side of 0 are shifted toward the center—this shift being in direct proportion to the size of γ. Thus, in contrast to the first case, spatially dependent uncertainty can have a harmonizing effect on voter preferences, increasing the attractiveness of the center as opposed to the extremes.

Turning to the subject of voter choice, this type of spatially dependent uncertainty produces effects that are opposite to what occurs in the first case. Still assuming that $\psi < \theta$, voter i will prefer Theta to Psi if and only if

$$x_i > (1+\gamma)\frac{\theta+\psi}{2} \tag{9}$$

Recalling that $\gamma > 0$, if $\theta > -\psi$ the indifference point for i between Theta and Psi is shifted to the right of $(\theta+\psi)/2$. If $\theta = -\psi$, i's indifference point is at the midpoint between θ and ψ, while if $\theta < -\psi$, this point of indifference is shifted to the left of the midpoint.

Now, in contrast to the first case, the two candidates will be drawn toward the center. Wherever the median x_i happens to be located (except when median $x_i = 0$), $[1/(1+\gamma)]$ median x_i will be closer to 0 than the median x_i; so, given this type of spatially dependent uncertainty, the center will have a drawing power over the candidates in direct proportion to the size of γ.

Ideology and the Source of Voter Uncertainty

In this section, we will relate our approach to voter uncertainty to earlier work on the subject by Downs (1957) and Shepsle (1972). We will also suggest some of the implications of our results for electoral politics.

Both Downs and Shepsle are concerned with a more specific phenomenon than we are modeling in this paper. The phenomenon they consider is candidate (or party) *ambiguity*. We have intentionally avoided using this word until now, because, as we will argue, our understanding of voter uncertainty is somewhat different from Downs and Shepsle.

Candidate ambiguity, as used by Downs and Shepsle, is purposeful behavior. Candidates *choose* whether or not to be ambiguous, depending upon the incentives present in the electoral environment. Thus, Downs asserts that two-party systems will encourage both parties to "becloud their policies in a fog of ambiguity," while Shepsle demonstrates a connection between risk-acceptant voters and incentives for the challenger in a two-candidate race to adopt an ambiguous policy position. Both writers model elections, as we have done, in terms of a one-dimensional policy space.

Thus, what is central to both Downs' and Shepsle's models is that candidates control the level of voter uncertainty about the meaning of their policy proposals during the election period, which would seem to imply that candidates could adopt a point position on the policy scale and transmit this position clearly to the voters, if they wished to.[5]

Our view is somewhat different. Remembering that we are assuming that voters collapse candidate positions on the set of campaign issues into a

[5]In his "emphasis allocation theory" of candidate ambiguity, Page (1976) also views the candidates as the primary source of voter uncertainty.

position on an underlying ideological dimension, it seems more reasonable to suppose that the candidate is not the *only* source of uncertainty affecting this process. Instead, we would argue that this reduction process bears a heavy responsibility for the voter's inability to arrive at a point estimate of the candidate's underlying ideological position. Certainly, a candidate can create even greater confusion for the voter by uttering vague or ambiguous policy pronouncements. However, the very process of simplifying the candidate's positions (some of which may be imperfectly perceived) with a shorthand label such as "moderate-conservative" or "ultra-conservative" creates a residue of uncertainty in the voter's mind about what this underlying position is. Thus, we are saying that voter uncertainty about the ideological positions of the candidates is endemic to the process whereby voters attempt to reduce a wealth of issue information to some shorthand guide useful for predicting each candidate's future behavior.[6]

Our reason for assuming that voters engage in this reduction process is similar to Downs' explanation of why voters may be forced to compare parties on the basis of ideologies (1957, ch. 7). In a world of imperfect information, a world in which there are costs associated with gathering and evaluating new information, the voter, faced with a serious decision such as deciding which candidate would make a better president, is forced to utilize a shortcut method to arrive at his choice. This method is fraught with uncertainties, since the voter is typically faced with a candidate who does not easily lend himself to being reduced to a simple ideological label. Even so, it is interesting to note that this simplification process is practiced even by those who watch campaigns most closely—journalists—who certainly are much better informed than most voters about the complexity of candidates' statements and actions. The terms "conservative," "moderate," "liberal" and combinations thereof are heard repeatedly in the reports of broadcast and print journalists on American presidential candidates and seem an inescapable feature of our political system. For most voters to rely upon such labels would therefore seem quite reasonable.

The candidates, of course, may attempt to influence this labeling process. Thus, Ronald Reagan may have wished to modify his image as a hard-line conservative and appear more centrist, while Jimmy Carter may have tried to move away from the right and appear more liberal in preparation for the 1980 general election. Such images, however, may be very resistant to change.

[6]In Hinich and Pollard (1979), various means are discussed by which this reduction process may take place. However, in their model this reduction results in a point estimate of the candidate's position, while our model builds uncertainty into this process.

Before leaving this subject, we do not wish to leave the reader with the impression that we see all elections as occurring in a *single* underlying dimension. For the purposes of this paper, we have used the assumption of a single dimension in order to compare our results with those of the well-known Downsian spatial model.[7] All that we would argue is that what we have described as the voter reduction process is an endemic part of voter choice in complex elections. Empirical studies of American presidential elections have shown that no more than two underlying dimensions are necessary to represent voter attitudes toward the candidates. We see this evidence as supporting our position.

The last point which we wish to expand upon in this section is our assumption of spatially dependent voter uncertainty. This assumption, we feel, reflects another important aspect of real politics. While we have argued for the currency of ideological terms such as "center," "left," and "right" among voters, this is not to say that these terms are equally clear in the voters' minds. As Al-Adhadh and Hinich (1979) have documented, Arab political parties are generally perceived in the Arab world as occupying some location on a left/right ideological dimension. However, the ideological position of parties of the extreme left and right is much clearer to politically active citizens as a predictor of future actions than the ideological position of centrist parties, which try to offer an ideology that mixes traditional and revolutionary elements. Thus, the "center" in Arab politics is much less clear than the "left" or "right."

In American politics, we usually see the reverse phenomenon. Candidates of the center are usually more clearly understood by voters than candidates of the left or right. This is not to say that candidates of the left and right are generally ambiguous about their policy positions. As presidential candidates such as Barry Goldwater, Ronald Reagan, and George McGovern remind us, the opposite is more often the case. However, since public policies in this country tend to be the result of numerous compromises among politically interested groups, promises of sweeping reform that are frequently made by candidates of the left or right are apt to leave voters unsure about what the candidate will actually accomplish in office. Thus, an ideological position that is near the left or right end of the political spectrum is generally less useful for predicting the future actions of a candidate, if he is elected, than an ideological position near the center. In other words, it is generally clearer to the voters what a centrist candidate will do in

[7]Our results can be extended to the case of multiple underlying dimensions. However, the results do not lend themselves to as clear-cut an interpretation as in the single-dimensional case.

office than what a liberal or conservative candidate will do. However, in certain periods of our history, this may not have been true.

In this regard, our model may allow us to gain a better understanding of the advantages of incumbency.[8] Assume the ideological position of the incumbent creates the least amount of uncertainty in the minds of the voters. Then, voter ideal points will be shifted toward the incumbent's position, giving him an absolute advantage over his opponent. Further, if the median ideal point is to the right or left of the incumbent's position, a challenger who adopts the median ideal point in an election may still lose to the incumbent. In fact, even if public opinion shifts over time either to the left or right of the incumbent's position, the incumbent may still be able to beat a challenger who adopts a position closer to the new median of the ideal point distribution. Thus, the advantages of incumbency may be partially explainable in terms of the attractiveness to the voters of an incumbent who is at least a known (or well-understood) commodity as opposed to an alternative candidate whose position is not so well understood.

Conclusion

The main results of this paper concern the spatial shift of voter ideal points—either toward or away from the center of the underlying ideological dimension. These results have important implications for the outcomes of real elections. What we have shown is that unless the ideological position of the median voter is very close to the center, spatially dependent voter uncertainty can have a profound effect on electoral outcomes. When the center is the point of greatest variance, candidates will be drawn farther out toward the left or right end of the ideological dimension than the ideological position of the median voter. Thus, candidates will reinforce the ideological polarization occurring in the electorate. It seems plausible that the election of an ideologically extreme candidate will increase the uncertainty of the voters about centrist ideologies, in which case future elections will be won by candidates who are, ideologically, increasingly extreme. This pattern of increasing extremism is usually viewed by democratic theorists as portending the ultimate breakdown of the electoral system, so it would seem important to be able to identify those societies in which this type of uncertainty exists.

On the other hand, when voter perceptions of centrist candidates have least variance, the center has a powerful attraction for both candidates. Unless the ideological position of the median voter is at the center, both candidates will have a strong incentive to adopt an ideological position closer to

[8] We are grateful to an anonymous referee for bringing this point to our attention.

the center than that of the median voter. If we accept the idea that the election of a centrist candidate increases voter uncertainty about extremist ideologies, then this centripetal pattern reinforces itself. Thus, societies in which this type of uncertainty exists should experience an increasing degree of ideological stability from one election period to the next.

While our results are based on the assumption of a risk-averse voter, two points should be made. First, if some voters are risk-averse but others are not, the ideal point shift that we have described may still revise the location of the median ideal point. Thus, the effects we have described will exist, but to a lesser extent, if only some voters are risk-averse. Second, while electoral competition would not be affected by spatially dependent uncertainty if all voters were risk-neutral, it seems clear that there would be some effect on electoral competition if voters were risk-acceptant. The nature of this effect we leave as an open research question.

In closing, we wish to emphasize that the spatial theory of politics is an analogue of real-world processes. The theory does nothing more than attempt to formalize everyday notions about politics, such as the idea of a left/right dimension, in order to derive interesting consequences about real-world behavior. No matter how complex and intricate most political campaigns become, voters still think about candidates in terms of the left, center, and right. It would seem reasonable, therefore, for spatial theory to represent in some fashion this process of voter simplification. However, it is also important to recognize that the cost to the voter of thinking in these terms is a residual uncertainty about how well these terms predict the actual behavior of officeholders. It is precisely this uncertainty which we have attempted to model in this paper, keeping in mind that some of these ideological labels are better predictors than others.

Manuscript submitted 12 May 1980
Final manuscript received 30 October 1980

APPENDIX

$$\begin{aligned} E[u_i(\tilde{\theta})] &= E[c_i - (\theta^2 + 2\epsilon_\theta\theta - 2x_i\theta + \epsilon_\theta^2 - 2\epsilon_\theta x_i + x_i^2)] \\ &= c_i - E(\theta^2 + 2\epsilon_\theta\theta - 2x_i\theta + \epsilon_\theta^2 - 2\epsilon_\theta x_i + x_i^2) \end{aligned} \quad (2)$$

Now, θ is a constant, since it is the mean of the random variable $\tilde{\theta}$. Consequently, although the distribution of ϵ_θ depends upon θ, θ is not a random variable, and so $E(2\epsilon_\theta\theta) = 2\theta\, E(\epsilon_\theta) = 0$. Thus equation (2) reduces further to

$$c_i - \theta^2 + 2x_i\theta - E(\epsilon_\theta^2) - x_i^2 = c_i - (\theta - x_i)^2 - E(\epsilon_\theta^2)$$

But, since $\sigma_\theta^2 = \mathrm{Var}(\epsilon_\theta) = E(\epsilon_\theta^2) - [E(\epsilon_\theta)]^2$ and $E(\epsilon_\theta) = 0$, $\sigma_\theta^2 = E(\epsilon_\theta^2)$, and we have

$$E[u_i(\tilde{\theta})] = c_i - (\theta - x_i)^2 - \sigma_\theta^2 \quad (3)$$

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