Spatial Theory

> Assume that every voter's preferences are single-peaked and slope downward monotonically on either side of the peak (unless his peak lies at one extreme of the scale)....The best way [for each party] to gain more support is to move toward the other extreme, so as to get more voters outside of it—i.e., to come between them and its opponent. As the two parties move closer together, they become more moderate and less extreme in policy in an effort to win the crucial middle-of-the-road voters, i.e., those whose views place them between the two parties. This center area becomes smaller and smaller as both parties strive to capture moderate votes; finally the two parties become nearly identical in platforms and actions. (Downs, 1957, pp. 116-7).

One of the fundamental building blocks in the analysis of political phenomena is the representation of preferences. Without some means of capturing the essence of goals and trade-offs for individual choices, the mechanics of the public choice method are stalled. While there are many ways of representing preferences, the single most commonly used approach is the "spatial" model. The idea of conceiving preference in a kind of "space" is actually quite ancient, as the quote from Aristotle's Politics above shows. Furthermore, there are hints of several topics of modern spatial theory, including the power of the "middle," and the problem of instability in political processes.

Origins of Spatial Political Competition

It is important to recognize that the spatial model is not just an "as-if" form of reasoning about political phenomena. One can quickly find myriad references to "left" and "right" in political discourse, both in the media and in elite accounts. Although "space" is a metaphor, it is one that is used so widely that it must connect quite closely with human cognition about political representation.

The origin of the "left" and "right" metaphors, as is well known, is a reference to the physical positions occupied by different factions in the French National Assembly after 1789. The Girondins on the "right" of the huge meeting hall held power, and ran the government. The more radical Jacobin allies of Robespierre sat in the "mountain" on benches rising up the wall on the far "left." The Jacobins on the left were constantly agitating for change, while the Girondins on the right defended stability and the status quo. With only a very little adjustment for time and circumstance, these meaning still attach to "left" and "right" in political discourse today.

An alternative meaning, mapping an ideological left and right onto positions with respect to ownership of capital (right) and defense of labor (left), was created by Karl Marx, and is used today in a wide variety of surveys. This meaning, however, is at best misleading and is often simply incorrect. The former Soviet Union, particularly Russia, had a clearly defined left and right at the end of the 20th century. But "left" was understood to mean liberal reformers who favored markets and democracy, whereas the "right" was composed of former communists who demanded a return to central planning and a secure and stable, if unelected, government.

The Problem of Representation

In economics, the problem of representation of preferences has been refined to the point that is simply a mathematical problem. Suppose that there are many alternatives, and that for each pair of alternatives, I prefer one, or like them equally. Then it is possible (assuming transitivity) to construct an aggregate weak ordering that allows the individual to "rank" alternatives from best to worst, with each alternative either uniquely

or with a group of other alternatives associated with an ordinal level. If I like A better than B, we say, "A is preferred to B." If I like them equally well, then "I am indifferent between A and B."

"Representing" the preferences implied by this ranking requires the assignment of any mathematical function f that has the following properties: (1) If A is preferred to B, then $f(A) \geq f(B)$. (2) If A and B are equally preferred, then $f(A)=f(B)$. As should be obvious, if there exists at least one function that represents these preferences, there will be infinitely many (since, for example, f and $f'=(f/2)+37$ both have the same ordering of the index numbers associated with alternatives). In other words, any order-preserving transformation of a function that represents the preferences is equally good.

The work on preferences in economics has shown that relatively few assumptions are required to ensure representability. One common, and plausible, type of preferences that is not representable by a mathematical function is "lexicographic" preferences, but most preferences that obey simply convergence criteria are representable. Economic preferences, however, generally assume either non-satiety or free disposal. Can something like the same approach be used to "represent" *political* preferences, which may very well require interior ideal points?

To understand the problem, consider the difference between preferences for apples and preferences for education. We generally model preference for apples as nondecreasing, so that more is preferred to less. What about education, or more accurately education budget? If asked, most citizens will not say that they think that the education budget should be infinite. Instead, they will select some finite number of dollars they think is the "best" budget, and will argue that either a larger or smaller

budget is less preferred. This "interior" ideal point is illustrated in Figure 1. (The utility functions graphed in the figure are symmetric for the sake of simplicity, but there is nothing in spatial theory that requires symmetry).

One important research question in public choice is the relation of political preferences to economic preferences. More specifically, can political preferences with interior ideal points be derived from economic preferences, with the connection being the opportunity cost of taxes used to finance public programs? Interestingly, though the initial findings were hopeful (see, e.g., Barr and Davis, 1966), the answer turns out to be "no" (see Denzau and Parks, 1977; 1979; and Slutsky, 1977; for a review, see Hinich and Munger, 1994, Chapter 2).

Consequently, the basis of political preferences in "representation" is more tenuous than for economic preferences: the only way to justify interior ideal points rigorously is to connect preferences for the good with the recognition that the financing scheme requires that citizens also pay for the good. This means that preferences are being defined simultaneously over the underlying good and the price of the good, rather than just preferences for the good alone.

On the other hand, the mathematical underpinnings for political preference representation in a "space" are well defined and consistent, requiring only minimal assumptions about the convexity of the sets of alternatives enclosed by indifference curves (Schofield, 1984). To the extent that government decisions on property rights, security, and a currency system are logically antecedent to the problem of representing economic preferences, the problem may go the other way. That is, there is a failure of duality in the representation problem: it is perfectly easy to take preferences in either the

public or private sectors as primitive, and then use utility functions to represent them. As the literature cited in this section shows, once one starts with economic preferences, there is no consistent way to "induce" public sector preferences. However, it is equally true that if one takes political preferences as primitives, then it is the "induced" economic preferences that exhibit inconsistency. We will take spatial theory as a primitive, rather than induced or derived, means of representing public sector preferences.

The First Spatial Models

As Hinich and Munger (1994) point out, the first clear use of the spatial "model" appears in Aristotle's Politics, written down before 325 B.C.E., and perhaps amended and modified in several ways in the centuries that followed. Still, as the work comes down to us, it is clear that there is both a deep understanding of politics and stability, and a connection to the idea of a dimension, or simple space, that organizes political conflict.

> Now in all states there are three elements: one class is very rich, another very poor, and a third in a mean. It is admitted that moderation and the mean are best, and therefore it will clearly be best to possess the gifts of fortune in moderation; for in that condition of life men are most ready to follow rational principle...
> [T]hose states are likely to be well-administered, in which the middle class is large, and stronger if possible than both the other classes, or at any rate than either singly, for the addition of the middle class turns the scale, and prevents either of the extremes from being dominant...
> The legislator should always include the middle class in his government; if he makes his laws oligarchical, to the middle class let him look; if he makes them democratical, he should equally by his laws try to attach this class to the state. There only can the government ever be stable where the middle class exceeds one or both of the others, and in that case there will be no fear that the rich will unite with the poor against the rulers. (Aristotle, 1979, pp. 138-142).

As was discussed earlier, this understanding of politics seems to come naturally to human beings, with the clearest example deriving from the language used to describe the

conflict in the French Assembly. But what of models? How are we to think of the idea of a "space," or dimension of conflict, in a way that gives us testable propositions about political behavior and institutions?

The early literature in economics on "spatial" competition addressed what seem like similar considerations. Hotelling (1927), Lerner and Singer (1937), and Smithies (1941) all addressed the problem of location, in the sense that a set of firms selling zero cost, undifferentiated products might compete by choosing the physical setting for the business. The classic metaphor is the choice of two hot dog stands on a street or beach, with potential patrons distributed along the linear dimension of competition. The key assumption is that, since the products are undifferentiated (all hot dogs are of the same make), patrons will choose solely based on location. The equilibrium set of locations, as was shown by various means in this literature, was achieved when (in the case of two firms), the businesses converged to a "central place." With more than three competitors, the results are ambiguous (there are many possible equilibria), and with arbitrarily many firms very little can be said.

The interesting thing about the early spatial models in economics was the fact that the authors worked to develop normative implications. An important controversy was Hotelling's rather strong claim that capitalism was "wasteful," at least compared to a planned economy. The prediction of convergence implied that both (or all) firms ended up as close together as they could manage, increasing the average distance traveled by consumers. Hotelling concluded: "Our cities become uneconomically large and the business districts within them are too concentrated. Methodist and Presbyterian churches are too much alike; cider is too homogeneous." (p. 57).

Lerner and Singer (1937) disputed this claim, pointing out that it rested on tenuous assumptions (particularly about transport charges and the extreme inelasticity of demand assumed by Hotelling). Smithies (1941) pursued the matter further, showing that under some plausible assumptions there exist nonconvergent equilibria.

Unfortunately, the problems of spatial location for firms and spatial preference representation in politics are not isomorphic. The analogies in results are not very useful, and can be misleading. The idea that voters might choose the candidate "closer" to their own ideal seems plausible enough, but it is by no means clear what "close" means once the idea of simple Euclidean distance is dispensed with. Euclidean distance makes good sense in the hot dog stand competition, since it takes just as long to walk one hundred yards north as it does to walk one hundred yards south. But it is by no means clear that we would want to build in this extreme kind of symmetry in representing political spatial preferences.

The problem is worse if there are multiple dimensions. Euclidean distance makes two assumptions about preferences:

(1) separability -- My evaluation of issue i is not affected by the level of issue j I expect to result from the decision process.

(2) equal salience – Marginal changes in issue i have the same increment/decrement for my utility as marginal changes in issue j.

Neither of these is a problem for the spatial location problem, because my reaction to having to travel is based on distance, not whether the distance is in any one direction. But if we are to use a policy "space" to represent political preferences, the

assumptions of separability and equal salience are both empirically unrealistic and theoretically limiting.

The extension of this kind of reasoning to political problems, particularly of party competition, was accomplished by Downs (1957). It is clear, however, that Downs' analysis is of a piece with the earlier work; consider Smithies' first paragraph:

> The very fact that Professor Harold Hotelling's pioneer article explained so successfully the close similarity of the Republican and Democratic platforms in 1928 indicates that something more was needed in 1936. It was probably true to say in 1928 that by moving to the center of electoral opinion neither party risked losing its peripheral support. The situation at the present time requires no elaboration; suffice it to say that neither party feels itself free to compete with the other for the undecided votes at the center, in full confidence that it will retain its support from the extremes of political opinion.

This is a very sophisticated statement, recognizing that equilibria, if they exist, will depend on the reliability of turnout and support from those at the extremes. If, to use the economic analogy for the last time, the "elasticity of demand" of citizens is high, moving toward the center may actually reduce one's vote share, as the ardent supporters out in the wings lose interest. To be fair, Downs concentrated on the problems of turnout, and information, but Downs has come to be associated with the result that candidates converge to the middle, or median, in two-party elections. It has since been shown (for a review, see Berger, Potthoff, and Munger, 2000) that the convergence result is actually very fragile under the plausible set of "Downsian" assumption, and unlikely to be observed empirically.

A Rigorous Representation: Spatial Theory in the 1960s

The first rigorous statement of the spatial model as a representation of preferences, at a level of generality analogous to that of economics, was the result of the

collaboration of Otto Davis and Melvin Hinich. In three papers (Davis and Hinich 1966, 1967, 1968), they laid the groundwork for what is now thought of as spatial theory.

Using a generalized quadratic form for representing preferences, they were able to account for non-separability and differences in salience in an elegantly simple way. Further, in all three papers, but particularly in the 1968 piece, they addressed the normative problem of the "good" in the democratic choice problem: if we accept the idea of Aristotle's "mean" as the best choice for a democracy, there is a benchmark against which predicted outcomes in the spatial model can be compared. The most widely recognized paper in this collaboration, Davis, Hinich and Ordeshook (1970) is a general exposition of all the results in the series of papers, with some extensions, and is the generic original reference in the spatial theory literature.

Social Choice Theory

Social choice theory and spatial theory are related subjects, but there are many important differences. Social choice theory tends to focus on the consequences of aggregation of individual "lists", using different aggregation ("voting") rules. Very little restriction is placed on the form that these lists can take, other than each weak order is transitive. Some of the most important work, such as Arrow (1963), actually assumes explicitly that preferences are characterized by "universal domain," so that any ordering over elements of the choice set is possible. There is no requirement, in social choice theory, that the preferences are "representable." Instead, social choice theorists work directly with preference orderings themselves.

Spatial theory, on the other hand, focuses on preferences that are single-peaked, and which are amenable to mathematical representation. The simplest kind of spatial preferences, Euclidean preferences, make a very restrictive set of assumptions about the kind of function that can represent the underlying ordering over alternatives. The simplest way to think of the difference, then, is that social choice theory takes preferences as primitive, and unknown, with any ordering equally likely. Spatial theory uses the notion that "closer" alternatives are more preferred, though spatial theory can account for weighted Euclidean distance, so that the function representing preferences exhibits nonseparability and different salience for different issues.

There are some important overlaps between spatial theory and social choice theory. An early work, in many ways ahead of its time, was Black and Newing (1951). This book introduced something very close to the analytical tool now called "win sets," but at the time too little was known about the problems of aggregation to give a coherent account. Black recognized the limitations in the earlier work, and published his seminal Theory of Committees and Elections in 1958, though this book took more of a social choice than a spatial perspective.

Probably the best known example of the intersection of spatial and social choice theory is Plott's (1967) then revolutionary exposition of the problem of the nonexistence of equilibrium under most arbitrarily chosen configurations of voter ideal points (for an extension, see Enelow and Hinich, 1983). This paper led to a new research agenda, trying to identify some subset of the policy space that is likely to contain outcomes, if not unique equilibria, of majority rule voting processes. For example, Schofield (1978, 1984) offered a mathematically more general treatment than that of Plott, but works mainly

within the logic of the spatial representation of preferences. McKelvey (1976a, 1976b, 1979, 1986) generalized the concept of spatial equilibrium, and distilled some important solution concepts, including covering and dominance. The notion of the "uncovered set" in a spatial context derives from Miller (1980); for a review and some extensions, see Cox (1987).

Extensions

The spatial model has been extended in a number of useful ways, a review of which would extend beyond the scope of this short essay. Useful, though very different, reviews of the literature can be found in Coughlin (1992), Enelow and Hinich (1984; 1990), Hinich and Munger (1997), and Ordeshook (1986, 1997). But a brief list of extensions is worthwhile.

- One of the earliest, and most interesting, is the extension of the spatial model to account for the turnout decision, allowing for rational abstention. Hinich, Ledyard, and Ordeshook (1973) gather together many strands of literature, and raise some important questions about the notion of equilibrium in the spatial model.
- The idea of treating voter actions as outcomes of an idiosyncratic probability distribution function arises naturally from the Hinich, Ledyard, and Ordeshook work, and was taken up by Hinich (1977), and Enelow and Hinich (1989).
- The restriction of the "space" of conflict to only a few dimensions, based on the empirical phenomenon of clustering of issues, has resulted in two related, yet distinct, theoretical extensions of the spatial model. The idea that "ideologies" are

important for explaining mass behavior was developed by Hinich and Pollard (1981), extended by Enelow and Hinich (1984), and given a firmer theoretical foundation by Hinich and Munger (1994). The claim that "ideology" is an important empirical predictor of both the vote of members in Congress and of the structure of the space of competition itself can be found in Poole and Rosenthal (1996), which reviews Poole and Rosenthal's many previous contributions to the development of this idea.

- The spatial model has an important policy implication for agenda control, because it allows analysis of the role of the "setter." There have been many contributions on this point, but the most important is Romer and Rosenthal (1978). A review of the larger literature, and its importance, can be found in Rosenthal (1990).
- Finally, the spatial model has given rise to a number of tests using experimental methods and human subjects. A review of this literature can be found in McKelvey and Ordeshook (1990). The important thing about experimental work in the spatial model is that it can suggest patterns of outcomes empirically, since many of the theoretical results are simply negative, because of the absence of equilibria.

Conclusion

Spatial theory is the single most important analytical construct for representing citizen preferences over policies, public goods, and government actions. Though the mathematical generality of spatial models falls short of the standards of preference representation in economics, it is important to recognize three things. First, the problem

of representing political "preferences" is inherently more difficult than representing economic preferences. Thus, it is not clear whether our models are not very good, or the problem is just very hard. Second, spatial models perform very well in a wide variety of useful theoretical settings, and can be used to investigate the precise properties of different institutional arrangements, ranging from committee systems in legislatures to the assignments of ministry portfolios in parliamentary governments, and encompassing voting by the mass public on referenda or elections.

Finally, the spatial model is appealing because of its inherent verisimilitude. The notion of "left" and "right" as a description of the "location" of candidates or parties is nearly universal. The notion of "moving to the center" or "outflanking on the left/right" pervades media and elite discourse about politics. For all these reasons, knowledge of the basic results of spatial theory is one of the foundations of public choice theory.

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Figure 1: Utility Functions with Interior Ideal Points x_i^* for Education, Three Citizens