Episodic Nonlinearity and Nonstationarity in Alberta's Power and Natural Gas Markets^{*}

Danny Czamanski,[†] Paul Dormaar,[‡] Melvin J. Hinich,[§] and Apostolos Serletis[¶]

April 29, 2005

Abstract

This paper uses a new method of testing for linear and nonlinear lead/lag relationships between time series, introduced by Brooks and Hinich (1999), on Alberta's natural gas and power markets. The test, based on the concepts of cross-correlation and cross-bicorrelation, is used after pre-whitening of the data to test for the existence of residual nonlinearity as well as the episodic nature of the nonlinearity. Our evidence points to a relatively rare episodic nonlinearity within and across the two series, having important implications for forecasting these series.

Keywords: Nonlinearity; Bicorrelations; Cross-correlations *JEL* classification: C32, C53, Q43

^{*}Serletis gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada.

[†]Senior Vice President (Research), Canadian Energy Research Institute (CERI), #150, 3512-33 Street N.W., Calgary, Alberta, T2L 2A6.

[‡]Quantitative Specialist, Risk Management Department, TransCanada Corporation, 450 1st Street S.W., Calgary, Alberta, T2P 5H1.

[§]Applied Research Laboratories, The University of Texas at Austin, Austin, Texas, TX 78713-8029

[¶]Department of Economics, University of Calgary, Calgary, Alberta, T2N 1N4. Serletis is the corresponding author. Phone: (403) 220-4092; Fax: (403) 282-5262; E-mail: Serletis@ucalgary.ca; Web: http://econ.ucalgary.ca/serletis.htm.

1 Introduction

Recent leading-edge research has applied various innovative methods for modeling spot wholesale electricity prices — see, for example, Deng and Jiang (2004), León and Rubia (2004), Serletis and Andreadis (2004), and Hinich and Serletis (2006). These works are interesting and attractive, but have taken a univariate time series approach to the analysis of electricity prices. From an economic perspective, however, the interest in the price of electricity is in its relationship with the prices of various underlying primary fuel commodities. As Bunn (2004, p. 2) recently put it " \cdots take the case of gas, for example. This is now becoming the fuel of choice for electricity generation. The investment costs are lower than coal, or oil plant; it is cleaner and, depending upon location, the fuel costs are comparable. But with more and more of the gas resources being used for power generation, in some markets the issue of whether gas drives power prices, or *vice versa*, is not easily answered."

Our main objective in this paper is to study the relationship between Alberta's spot wholesale power and natural gas market. The Alberta power market is a local market with transportation congestion between neighboring markets and no ability to store local supply. In particular, suppliers offer power into a centralized authority (the Power Pool of Alberta) throughout the day for use at prices which are typically a function of their cost of production. In some cases, they offer prices of zero, either since power is a byproduct of thermal production and it is unreasonable to change production behavior as a function of the electricity market or it is too costly to turn the plant down if it is not required to supply incremental power. In other cases, producers have the ability to increase prices as demand increases up to a regulated price cap of \$1000.00 per megawatt-hours (MWh).

On the demand side, utilities draw power from the system as required and are pure price takers. As users take more or less electricity, power is made available as required, and the physical product is never in a surplus or deficit. Power is made available from local plants as required by the central authority. As demand increases, additional power is made available by increasingly more costly producers, starting from hydro, to coal, to natural gas, with limited imports. Coal production is sufficient to supply most of the offpeak (evening) demand, but incremental natural gas is required to meet peak (daytime) demand. The price paid to all producing suppliers is determined by the offer price of the most recently dispatched producer. As load increases during peak hours, more expensive generation must be dispatched to meet the additional capacity requirements which determines the current price. Even though the majority of demand is supplied by coal burning plants, during peak hours the price is determined by plants that use natural gas. Therefore, the price of power during peak hours is a function of the market price of natural gas.

In investigating the relationship between Alberta's spot wholesale power and natural gas market, we use data over the recent deregulated period from January 2, 1996 to March 15, 2005. In doing so, we follow Brooks and Hinich (1999) and draw two somewhat disparate areas of research into nonlinearity and multivariate time series analysis together, using a new test for nonlinearity, proposed by Brooks and Hinich (1999), which allows for cross-correlations and cross-bicorrelations between pairs of series. These tests can be viewed as natural multivariate extensions of Hinich's (1996) portmanteau bicorrelation and whiteness statistics which search for nonlinear cofeatures between time series.

The paper is organized as follows. In Section 2 we discuss the relationship between the Alberta natural gas and power markets. In section 3 we outline the testing methodology used. Section 4 describes the data and presents the empirical results. The final section provides concluding remarks.

2 Alberta's Natural Gas and Power Markets

Alberta commenced the restructuring of its electricity market in 1996 and allowed full retail access in 2001. The Alberta Electric System Operator (AESO) manages the spot and non-binding day-ahead markets for energy. In facilitating the energy market, the AESO accepts bids and offers submitted by the purchasers and suppliers of electricity in the market. The bids and offers are assembled and are a basis for a centralized dispatch schedule. Moreover, two independent third party power exchanges exist, operated by the Alberta Watt Exchange and the Natural Gas Exchange. Both exchanges allow for financial contracting, direct electricity purchases and transactions for physical delivery. Transactions conducted via these exchanges are outside the AESO's trading arrangements. However, any of the underlying purchases that require physical delivery must be nominated within the central dispatch schedule produced by the AESO.

Since the introduction of competition in Alberta's electricity market, investment in new generation has been forthcoming. Much of the additional capacity has been from natural gas fired co-generation plants related to oil sands and petrochemical activities. As a result, at 2,585 MW Alberta has the largest amount of co-generation capacity in Canada. The growth in installed wind power capacity has also been dramatic. But, it remains a very small portion of the total installed capacity in Alberta. Also, part of the deregulation called for the break-up of existing generation. The Government of Alberta facilitated this break-up by selling the generating facilities of the three formerly regulated utilities under Power Purchase Arrangements (PPAs). The purchasers of PPAs have the right to offer the electricity from the generating units into the market while at the same time compensating the original owners. To manage unsold PPAs, the Government of Alberta established an independent organisation called the 'Balancing Pool.'

The AESO facilitates the trade of energy in the real-time market and ancillary services to support its real-time system control operations. This is irrespective of whether the market participants are involved in direct sales agreements, forward sales of energy or whether they are directly transacting in the energy market, managed by the AESO, or through the Alberta Watt Exchange or the Natural Gas Exchange. While the settlement of the contracts is performed directly between the contracted parties, the AESO will settle the supply and demand variations experienced during the real-time operation of the electricity system. Thus, the AESO also manages a financial market in which market participants may secure contracts for differences to compensate for supply and demand variations experienced during the realtime operation of the electricity system. These variations are the deviations between scheduled and actual volumes that underlie the sales and purchases of suppliers and purchasers, respectively.

In the centralised dispatch process, bids and offers are assembled in a single economic 'merit order' (from lowest to highest). The outcome of this process is the dispatch schedule that permits the AESO to declare the market clearing price or system marginal price (SMP). As electricity demand shifts throughout the day, supply and demand are kept in balance by dispatching the next offers in the merit order. This ensures that Alberta's overall electricity needs are met by the lowest cost option. At the end of the hour, the time-weighted average of the 60 one-minute SMPs is calculated and published as the market price. Offers of energy imports are not allowed to submit prices with their offers while exporters must submit prices of \$999.99/MWh (i.e. they are not allowed to provide a price setting role in the market).

Restatements are another feature of the AESO's dispatch process. A

participant is allowed to change the offered volume of energy for an available asset on the trading day, as frequently as necessary. This includes increasing or decreasing the amount offered. However, participants are not permitted to supply more energy to the system than they have been dispatched, and they will only be dispatched if their offer is economically in-merit. The AESO also permits a locking restatement once per day per asset. While the day-ahead prices are binding, offered energy may be shifted from lower offer price blocks to higher offer price blocks or additional energy nominated to existing blocks. The submission of a locking restatement is allowed within the current trading period or 30 minutes prior to the start of that period only if an 'acceptable operational reason' exists, however prior to that period locking restatements are permitted for economic reasons.

The AESO accepts demand-side bids. Loads can submit a price at which they will decrease their consumption. Effectively, this sets the price at a lower level than it would have otherwise been if the next higher priced generating unit was dispatched to meet the demand. However, this mechanism is not frequently used. Instead, there are a number of loads in the province that decrease their consumption at certain price points, without submitting a bid. Through self-management, these consumers monitor the System Marginal Price and decrease their consumption accordingly.

It is also important to emphasize that the system operator dispatches supply on a 'as needed' basis and instantaneously. He needs to meet demand on a moment by moment basis. The system marginal price is calculated every minute. The 'pool' price used to settle contracts is an average of the 60 calculations made each hour. Thus, there is an inherent mismatch between the dispatch and settlement prices. Obviously, the resulting price signal is not that good and it causes participants to refuse their generating plants and to self-dispatch. Moreoever, suppliers have the flexibility to restate bids up to the time of dispatch in various forms. Some claim that these allowances cause the exercise of what can be called 'local' market power, a situation that affects the reported hourly prices. This situation is further complicated by a separation of capacity commitments and energy commitments.

Although the Alberta power market is a local market with transportation congestion between neighboring markets and no ability to store local supply, the Alberta natural gas market is a continental market with a strong ability to move gas between neighboring markets and with local storage capacity to meet changing supply and demand conditions. The short run price relationship between natural gas and power is weak due to local supply and demand shocks in power magnified by non-storability and transmission congestion, leading to noise that is not consistent with continental behavior of natural gas prices. For example, local shocks to the Alberta power market are typically not enough to affect the continental supply and demand conditions for natural gas. Therefore, spot price volatility in power is much greater than that of natural gas. However, since peak hour prices of power are a function of the market price of natural gas, a relationship between natural gas and power prices is expected to exist.

The fundamental premise of this paper is that the pattern of hourly electricity prices, as reported by the Alberta power pool, should be related to the natural gas prices in the relevant local market. This is a reasonable assumption, since natural gas is a fuel of choice during peak demand periods. It is possible, however, that for a variety of reasons base load and intermediate load generating plants may at times serve as peaking plants. For example, the dispatching merit order may be influenced by maintenance schedules and by environmental concerns, such as the availability of wind generating capacity. This would affect the resulting electricity prices and affect the relationship between natural gas and electricity prices. Nevertheless, a relationship in the two prices should be expected.

3 Testing Methodology

Let us consider a sample of length N of two jointly covariance stationary time series $\{x(t_k)\}$ and $\{y(t_k)\}$, which have been standardised to have a sample mean of zero and a sample variance of one, by subtracting the sample mean and dividing by the sample standard deviation in each case. Since we are working with small subsamples of the whole series, stationarity is not a stringent assumption. The null hypothesis for the test is that the two series are independent pure white noise processes, against an alternative that some cross-covariances between $\{x(t_k)\}$ and $\{y(t_k)\}$, denoted $C_{xy}(r)$ for $r \neq 0$,

$$C_{xy}(r) = E\left[x(t_k)y(t_k+r)\right],$$

or cross-bicovariances between $\{x(t_k)\}\$ and $\{y(t_k)\}\$, denoted $C_{xxy}(r, s)$,

$$C_{xxy}(r,s) = E\left[x(t_k)x(t_k+r)y(t_k+s)\right],$$

are nonzero. As a consequence of the invariance of $E[x(t_1) x(t_2) y(t_3)]$ to permutations of (t_1, t_2) , stationarity implies that the expected value is a function of two lags and that $C_{xxy}(-r,s) = C_{xxy}(r,s)$. If the maximum lag used is L < N, then the principal domain for the bicovariances is the rectangle $\{1 \le r \le L, -L \le s \le L\}$.

Under the null hypothesis that $\{x(t_k)\}\$ and $\{y(t_k)\}\$ are pure white noise, then $C_{xy}(r)$ and $C_{xxy}(r,s) = 0 \ \forall r, s$ except when r = s = 0. This is also true for the less restrictive case when the two processes are merely uncorrelated, but the theorem mentioned below to show that the test statistic is asymptotically normal requires independence between the two series. If there is second or third order lagged dependence between the two series, then $C_{xy}(r)$ or $C_{xxy}(r,s) \neq 0$ for at least one r value or one pair of r and s values, respectively. The following statistics give the r sample xy cross-correlation and the r, s sample xxy cross-bicorrelation, respectively

$$C_{xy}(r) = \frac{1}{N-r} \sum_{t=1}^{N-r} x(t_k) y(t_k+r), \qquad r \neq 0$$
(1)

and

$$C_{xxy}(r,s) = \frac{1}{N-m} \sum_{t=1}^{N-m} x(t_k) x(t_k+r) y(t_k+s)$$
(2)

where $m = \max(r, s)$.

The cross-bicorrelation in equation (2) can be viewed as a correlation between the current value of one series and the value of previous crossbicorrelations between the two series. Note that the summation in the second-order case (1) does not include contemporaneous terms, and is conducted on the residuals of an autoregressive fit to filter out the univariate autocorrelation structure so that contemporaneous correlations will not cause rejections. For the third-order test, we estimate the test on the residuals of a bivariate vector autoregressive model containing a contemporaneous term in one of the equations. The motivation for this prewhitening step is to remove any traces of linear correlation or cross-correlation so that any remaining dependence between the series must be of a nonlinear form. It can then be shown that

$$E [C_{xy} (r)] = 0,$$

$$E [C_{xxy} (r, s)] = 0,$$

$$E [C_{xy}^{2} (r)] = \frac{1}{N-r},$$

$$E [C_{xxy}^{2} (r, s)] = \frac{1}{N-m}$$

under the null hypothesis. Let $L = N^c$ where 0 < c < 0.5 — in this application we use c = 0.25, although the results and the null distribution of the test are not very sensitive to changes in this parameter. The test statistics for nonzero cross-correlations and cross-bicorrelations are given by

$$H_{xy}(N) = \sum_{r=1}^{L} (N-r) C_{xy}^{2}(r),$$

and

$$H_{xxy}(N) = \sum_{s=-L}^{L} \ ' \ \sum_{r=1}^{L} (N-m) C_{xxy}^2(r,s), \quad (\ '-s \neq -1, 1, 0),$$

respectively. These tests are joint or composite tests for cross-correlations and cross-bicorrelations (in a similar vein to the Ljung-Box Q^* test for autocorrelation), where the number of correlations tested for is L and the number of cross-bicorrelations tested for is L(2L-1). According to Hinich (1996, Theorem 1), H_{xy} and H_{xxy} are asymptotically χ^2 with L and L(2L-1)degrees of freedom, respectively, as $N \to \infty$.

4 The Data and Empirical Evidence

We study Alberta's spot wholesale power market, defined on hourly intervals (like most spot markets for electricity are), over the deregulated period after January 1, 1996 (to March 15, 2005). In doing so, we use hourly electricity prices (sourced from the Alberta Power Pool), denominated in megawatthours (MWh) and concentrate on Alberta's peak power market (in order to capture the relationship between natural gas and power), which is a 6 day per week and 16 hours per day market — Monday through Saturday from 8:00 a.m. to 11:00 p.m. Because the Alberta natural gas data is only available for weekdays and non-holidays, we aggregated the (load-weighted) power data for weekdays and non-holidays only. For natural gas, AECO is the most liquid intra-provincial index and daily spot prices were obtained from Bloomberg.

Figure 1 shows the spot prices of Alberta natural gas and power, whereas Figures 2 and 3 show the logged price changes for natural gas and power, respectively. The windowed test of Brooks and Hinich (1999) can be applied either to the logged first differences or to the residuals of an autoregressive fit of the data. Since our focus is to examine the stability of the underlying nonlinear dependency structures, we apply the test to the residuals of an AR(p) fit for each series, with p being chosen (optimally) using the Schwartz criterion; the fitted AR(p) model serves to remove linear dependencies from the data so that a rejection of the null of pure white noise at the specified threshold level is due to significant H statistics. It is found that an AR(14)model is sufficient to remove all the correlations from the logged first differenced power and natural gas price series.

The residuals of the AR(14) model are split into five sets of 115, 57, 38, 18, and 8 non-overlapping windows (or sub-samples) of 20, 40, 60, 125, and 250 observations in length (corresponding approximately to one-, two-, three-, six-, and twelve-month periods), respectively. The window length should be sufficiently long to provide adequate statistical power and yet short enough for the data generating process to have remained roughly constant. In splitting the original series into windows, the last window is not used if there are not enough data to fill that window. Using x to denote the Alberta natural gas price series and y the power price series, in Table 2 we report the number of significant frames at each of the five window sizes together with the percentage of the total number of frames where the null of pure white noise is rejected by the *H*-statistic. Table 2 also provides the dates when these episodic nonlinearities occurred, which is potentially useful for our future investigation into the causes of this detected episodic behavior.

For the natural gas correlations, there are two significant correlations in the 2-month window, three in the 3-month window, one in the half-year window, and two in the annual window. The power correlation tests show four significant correlations in both the 1-month and 2-month windows and one in each of the semi-annual and annual windows. The cross correlation tests show one significant cross correlation in each of the 1-month and 2-month windows. For the natural gas cross bicorrelations, there is one significant cross bicorrelation in the 1-month window and four in the 3-month window. In the power cross bicorrelations, there is one significant cross bicorrelation in both the 1-month and 12-month windows. Finally, in the natural gas bicorrelation tests, there are 5 significant bicorrelations in the 1-month window, ten in the 2-month window, and sixteen in the 3-month window, whereas in the power bicorrelations there are seven significant bicorrelations in the 1-month window, five in the 2-month window, three in each of the 3-month and 6-month windows, and two in the annual window.

These results demonstrate that the underlying nonlinear generating process for the Alberta power and natural gas price series is episodic in nature in which the nonlinear dependence appears only infrequently. Another pertinent feature is the transient nature of these dependencies, in which some correlations appear highly significant, but then quickly disappear, or become too weak to be detected in subsequent windows. This provides a plausible explanation for the failure of researchers to exploit the detected nonlinearity in making improved point forecasts. In particular, though the presence of nonlinearity implies the potential of predictability, the dependency structures do not seem to be persistent enough to benefit from it. That is, these dependencies show up at random intervals for a brief period of time but then disappear again before they can be exploited.

5 Conclusion

Researchers in economics and finance have been interested in testing for nonlinear dependence in time series for over twenty years now. Following relatively early work by Brock(1986), Hsieh (1989), and Scheinkman and LeBaron (1989), the number of applications has increased dramatically. There are at least two reasons for the popularity of this line of research. First, if evidence of nonlinearity is found in the residuals of a linear model, this must cast doubt on the adequacy of the linear model as an adequate representation of the data. Second, if the nonlinearity is present in the conditional first moment, it may be possible to devise a trading strategy based on nonlinear models which is able to yield higher returns than a buy-and-hold rule.

The most popular portmanteau tests for nonlinearity employed have been

the BDS test of Brock *et al.* (1987), now published as Brock *et al.* (1996), and the bispectrum test of Hinich (1982). The vast majority of researchers to use these tests have found strong evidence for nonlinearity, although the usefulness of nonlinear time series models for yielding superior predictions of asset returns is still undecided. There also exists a parallel literature which seeks to determine whether observed nonlinearities in financial time series are due to the existence of stochastic nonlinear relationships or fully deterministic chaotic dynamics. Over the years, a number of methods have been introduced for testing for chaos, but there is almost no evidence in favour of deterministic chaotic dynamics — see, however, Barnett and Serletis (2000) for some interesting ideas along these lines.

In this paper we have used a new method of testing for linear and nonlinear lead/lag relationships between time series, introduced by Brooks and Hinich (1999). The method provides a complement to Granger causality analysis, and is general enough to detect many types of nonlinear dependence between series in their conditional means. The test, based on the concepts of cross-correlation and cross-bicorrelation is used after pre-whitening the Alberta natural gas and electricity price series to test for the existence of residual nonlinearity as well as the episodic nature of the nonlinearity, if any exists. Our results indicate that there exists statistically significant episodic nonlinearity both in the natural gas and electricity prices and between the two price series. The nonlinearity appears to occur in one month (20 days) but in different months for different bicorrelation statistics. Thus the evidence points to a relatively rare episodic nonlinearity within and across the two series.

The evidence of episodic nonlinearity in the power and natural gas price series has important implications for forecasting these series. We know how to forecast linear dynamical systems with constant coefficients. We may differ about the best way to deal with trends in the series but the dynamics is modeled by a vector autoregressive model and the model is fit by one of the standard least squares methods. The trend plus dynamical model is then used to produce forecasts. However, although the linear modeling and fitting approach may yield useful forecasts of a nonlinear process, there is no way to know when the linear forecasts are very wrong.

Moreover, the usefulness of a linear approach to forecasting an episodic nonlinear process is even more questionable than the use of a linear approach to forecasting a stationary nonlinear process. If we can learn how to detect when the energy market series become nonlinear then we can use linear methods for making short term forecasting during the linear regimes. There is no known method for forecasting nonlinear processes with non zero bicorrelations and cross bicorrelations. Forecasting of such nonlinear processes is an important and difficult mathematical and statistical problem that should attract more attention than it has received in the time series field.

References

- [1] Barnett, William A. and Apostolos Serletis. "Martingales, Nonlinearity, and Chaos." *Journal of Economic Dynamics and Control* 24 (2000), 703-724.
- [2] Brock, W. A. "Distinguishing Random and Deterministic Systems: Abridged Version." Journal of Economic Theory 40 (1986), 168-195..
- [3] Brock, W. A., Dechert, W. D., and Scheinkman, J. A. "A Test for Independence Based on the Correlation Dimension." Mimeo. Department of Economics, University of Wisconsin at Madison (1987).
- [4] Brock, W. A., Hseih, D. A. and LeBaron, B. Nonlinear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence. M.I.T. Press, Reading, Mass (1991).
- [5] Brock, W. A., Dechert, W. D., Scheinkman, J. A. and LeBaron, B. "A Test for Independence Based on the Correlation Dimension." *Econometric Reviews* 15 (1996), 197-235.
- [6] Brooks, C. and Hinich, M. J. "Cross-Correlations and Cross-Bicorrelations in Sterling Exchange Rates." *Journal of Empirical Finance* 20 (1999), 385-404.
- [7] Bunn, Derek W. "Structural and Behavioural Foundations of Competitive Electricity Prices." In Derek W. Bunn (ed.), *Modelling Prices in Competitive Electricity Markets* (Wiley Series in Financial Economics) 2004, pp. 1-17.
- [8] Deng, Shi-Jie and Wenjiang Jiang. "Quantile-Based Probabilistic Models for Electricity Prices." In Derek W. Bunn (ed.), *Modelling Prices in Competitive Electricity Markets* (Wiley Series in Financial Economics) 2004, pp. 161-176.
- [9] Hinich, Melvin J. "Testing for Gaussianity and Linearity of a Stationary Time Series." *Journal of Time Series Analysis* 3 (1982), 169-176.
- [10] Hinich, Melvin J. "Testing for Dependence in the Input to a Linear Time Series Model." Journal of Nonparametric Statistics 6 (1996), 205-221.

- [11] Hinich, Melvin J.and Apostolos Serletis. "Randomly Modulated Periodic Signals in Alberta's Electricity Market." *Studies in Nonlinear Dynamics* and Econometrics (2006, forthcoming).
- [12] Hsieh, D. A. "Testing For Nonlinear Dependence in Daily Foreign Exchange Rates." Journal of Business 62 (1989), 339-368.
- [13] León, Angel and Antonio Rubia. "Testing for Weekly Seasonal Unit Roots in the Spanish Power Pool." In Derek W. Bunn (ed.), *Modelling Prices in Competitive Electricity Markets* (Wiley Series in Financial Economics) 2004, pp. 131-145.
- [14] Scheinkman, J. A. and B. LeBaron. "Nonlinear Dynamics and Stock Returns." Journal of Business 62 (1989), 311-337.
- [15] Serletis, Apostolos and Ioannis Andreadis. "Nonlinear Time Series Analysis of Alberta's Deregulated Electricity Market." In Derek W. Bunn (ed.), *Modelling Prices in Competitive Electricity Markets* (Wiley Series in Financial Economics) 2004, pp. 147-159.

Figure 1. Alberta Natural Gas and Power, Jan 2/96 to March 15/05



Figure 2. Alberta Natural Gas Logged Price Changes, Jan 2/96 to March 15/05





Figure 3. Alberta Power Logged Price Changes, Jan 2/96 to March 15/05

TABLE	1
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	Window size									
	20		40		60		125		250	
C_x	0	(0.0%)	2	(3.5%)	3	(7.9%)	1	(5.6%)	2	(22.2%)
C_y	4	(3.5%)	4	(7.0%)	0	(0.0%)	1	(5.6%)	1	(11.1%)
C_{xy}	1	(9.0%)	1	(1.8%)	0	(0.0%)	0	(0.0%)	0	(0.0%
C_{xxy}	1	(9.0%)	0	(0.0%)	4	(10.5%)	0	(0.0%)	0	(0.0%
C_{uux}	1	(9.0%)	0	(0.0%)	0	(0.0%)	0	(0.0%)	1	(11.1%
C_{xxx}	5	(4.3%)	10	(17.5%)	16	(42.1%)	0	(0.0%)	0	(0.0%)
C_{uuu}	7	(6.1%)	5	(8.8%)	3	(7.9%)	3	(16.7%)	2	(22.2%)

NUMBER OF SIGNIFICANT FRAMES AT VARIOUS WINDOW SIZES

Note: Numbers in parantheses are percentages of significant frames.

Window Size	C_{xxy}	C_{yyx}	C_{xxx}	C_{yyy}
20	Dec $04/96$ to Jan $02/97$	Dec $04/96$ to Jan $02/97$	Dec $02/98$ to Dec $30/98$	Apr 29/97 to May 27/97
	- / /		Apr 24/01 to May 21/01	Dec $02/98$ to Dec $30/98$
			Jun 05/02 to Jul 02/02	May 26/99 to Jun 23/99
			Jan 03/03 to Jan 31/03	Jun 24/99 to Jul 22/99
			Jun 23/04 to Jul 20/04	Sep 20/99 to Oct 15/99
			, , ,	Nov 15/99 to Dec 13/99
				Feb $04/04$ to Mar $02/04$
				, , ,
40			Jan $23/96$ to Mar $19/96$	Apr 29/97 to Jun 24/97
			Nov $05/96$ to Jan $02/97$	Dec $02/98$ to Jan $29/99$
			Apr 15/98 to Jun 10/98	May 26/99 to Jul 22/99
			Dec $02/98$ to Jan $29/99$	Jan $07/04$ to Mar $02/04$
			Oct 26/00 to Dec 22/00	Mar $03/04$ to Apr $27/04$
			Apr 24/01 to Jun 19/01	, _ ,
			Oct 12/01 to Dec 06/01	
			Jun 05/02 to Aug 01/02	
			Feb 03/03 to Mar 31/03	
			Mar 03/04 to Apr 27/04	
60	Sep 19/97 to Dec $12/97$		Jan 23/96 to Apr $17/96$	Dec $02/98$ to Dec $02/98$
	Sep $04/98$ to Dec $01/98$		Jul 12/96 to Oct $07/96$	Aug $20/99$ to Nov $12/99$
	Jan 26/01 to Apr $23/01$		Oct $08/96$ to Jan $02/97$	Apr $09/02$ to Jul $02/02$
	Apr $09/02$ to Jul $02/02$		Jun $25/97$ to Sep $18/97$	
			Sep 19/97 to Dec $12/97$	
			Mar 16/98 to Jun 10/98 $$	
			Jun 11/98 to Sep $03/98$	
			Sep 04/98 to Dec 01/98	
			Dec $02/98$ to Mar $01/99$	
			Oct 26/00 to Jan 25/01 $$	
			Jul 19/01 to Oct $11/01$	
			Oct $12/01$ to Jan $09/02$	
			Apr $09/02$ to Jul $02/02$	
			Jan $03/03$ to Mar $31/03$	
			May $26/04$ to Aug $17/04$	
			Nov 10/04 to Feb $01/05$	
125				Jan 15/99 to Jul 15/99
				Jul 16/99 to Jan 11/00
				Jan 10/02 to Jul 11/02
050		$I_{00} = \frac{10}{09} + c = \frac{16}{09} + c$		$I_{00} = 15/00 + 0$ $I_{00} = 11/00$
200		Jan 10/02 to Jan 10/03		$J_{an} = \frac{10}{99} t0 J_{an} = \frac{16}{02}$
				Jan 10/02 to Jan 10/03

TABLE 2. DATES OF SIGNIFICANT FRAMES