STRUCTURAL CHANGE IN MACROECONOMIC TIME SERIES: A COMPLEX SYSTEMS PERSPECTIVE

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ABSTRACT

We demonstrate that the process of generating smooth transitions can be viewed as a natural result of the filtering operations implied in the generation of discrete time series observations from the sampling of data from an underlying continuous time process that has undergone a process of structural change. In order to focus discussion, we utilize the problem of estimating the location of abrupt shifts in some simple time series models. This approach will permit us to address salient issues relating to distortions induced by the inherent aggregation associated with discrete-time sampling of continuous time processes experiencing structural change. We also address the issue of how time irreversible structures may be generated within the smooth transition processes.

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INTRODUCTION

Macroeconomists have always found it difficult to deal with structural change in their modeling strategies (see Foster (2004)). For example Hendry (2000) explores the issue of modeling structural change in more depth than usual and he concludes that it is difficult to detect structural change in the data generating process except when there is a 'mean shift' in what is presumed to be the 'long run equilibrium' part of a vector equilibrium correction model (VECM). However, there is little discussion of the theoretical aspects of such structural change processes and little or no concern with some of the statistical problems that arise when highly aggregated, low frequency data are employed in macroeconomic modeling.

Because of the high level of aggregation, it is difficult to associate macroeconomic models with microeconomic (or even 'mesoeconomic') behaviour. Thus, there has been a preference for uniform agent assumptions that simply remove the problem. However, in so doing, the fixed representation of economic behaviour adopted, by definition, precludes any real analysis of structural change. Although this is a difficult problem that, more than likely, will require some integration of macroeconomics and modern evolutionary economics, there is no real excuse for ignoring the statistical issues that are presented when using macroeconomic data.

In the absence of any coherent theorizing about structural change, some modelers have preferred to use nonlinear econometric methods that allow for the effects of institutional changes and shifts in behaviour. In particular, structural change and transition have been the subject of an extensive literature relating to both the statistical investigation of the properties and 'regime-shift' based explanations of the business cycle[1]. However, a range of statistical questions that have a significant bearing upon the nature of structural transitions have remained unanswered in this literature. It is this gap that we seek to fill in this article, if not to provide definitive answers to what are often very difficult questions, at least to provide a coherent research agenda. This has been made possible because of methodological advances in the area of time series analysis that have permitted the construction of appropriate test statistics concerning the nature and extent of structural change. Also, the emergence of the view that the economy and its constitutent parts are complex systems that exhibit nonlinear features in their data has helped to advance this kind of research (Foster (2005)).

A feature of complex adaptive systems is the fact that they are capable of 'self-organization'. Such phases in economic evolution are characterized by a growth process that is logistic in form across the relevant time series data. However,

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[1] Regime shift was a central part of the famous Lucas (1976) critique. Hamilton (1989, 1990) delivered a new impetus for the study of regime shifts and "regime-switching" in the 1990s. Further references include e.g. Tsay (1989, 1998), Terasvirta and Anderson (1992), Terasvirta (1994), Potter (1995), Ramsey and Rothman (1996), and Hinich and Rothman (1998).

the observation of the logistic curve is not new: it has been observed in a very large number of studies of innovation and it was Griliches (1957) and Mansfield (1968) who pioneered an econometric modeling approach to innovation diffusion. This approach was extended to deal more broadly with economic self-organisation in Foster and Wild (1996, 1999a, 1999b). Wild (2002) offers a more extended treatment of the statistical properties of this class of nonlinear models derived from complex systems theory and synergetics.

The various complex systems explanations focus on the role of endogenously determined structural change within the economic system as a naturally occurring phenomena. Generally, processes of structural change in statistical data defined at high levels of aggregation are determined by underlying processes that involve the interactions between many smaller sub-systems in an inherently nonlinear manner. These interactions can produce unanticipated emergent behavior and associated structural instability in a model, as well as a significant degree of time irreversibility that can lead to asymmetries in the probabilistic structure of macroeconomic data (Forrest and Jones (1994)).

In fact the "symmetry breaking" nature of emergent behavior in complex adaptive systems, resulting in a loss of uniqueness in system trajectories, heuristically underpins the formulation of recent time series tests for time reversibility, notably, the test proposed in Hinich and Rothman (1998).[2] This

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[2] Also see Lawrance (1991) and Ramsey and Rothman (1996).

particular test rigorously activates a proposal originally advocated in Brillinger and Rosenblatt (1967). Specifically, the bispectrum test for time reversibility rests on the null hypothesis that the imaginary part of the bispectral representation of a time reversible process is not significantly different from zero. This implies that the bispectral representation will be real and symmetric, thereby admitting a unique spectral representation under the null hypothesis. Time irreversible processes have a bispectral representation that has a statistically significant imaginary part. Thus the spectral representation of a time irreversible process will be *complex*. Brillinger and Rosenblatt (1967) demonstrated that such representations do not admit a unique spectral representation, thereby providing a link between time irreversibility and loss of uniqueness.

In the time series econometrics literature, models encompassing regime shifting behavior seem to most closely approximate the types of behavior commonly associated with complex economic systems. Specifically if one is willing to view different regimes as separate subsystems, then the commonly cited (smooth) transition paths between regimes might simply reflect the nonlinear linkages between sub-systems that are so crucial in the generation of time irreversible probabilistic structure in data concerned with the behaviour of complex economic systems. From the perspective of regression analysis, once the key structural parameters determining regime identification and selection have been determined, the types of structural change defining the different regimes (sub-systems) can typically be interpreted as encompassing two

particular forms of structural change. The first type refers to changes in intercepts and/or slopes of estimated relationships. The second type refers to the possible emergence of latent structures associated with particular sub-system specifications.

The broad problem of dealing with the first type of structural change – that is, intercept and slope shifts, is not a new problem in mathematical statistics. The statistical theory of detecting and estimating structural change has evolved over at least sixty years beginning with work on detecting departures from tolerance limits in the theory of statistical quality control (Shewhart (1931, Ch 19-20)) and with the seminal contributions by Page (1954, 1955, and 1957), Chernoff and Zacks (1964) and Hinich and Farley (1966). These papers, in turn, motivated further generalizations resulting in the emergence of related but distinct problems.

First, issues relating to estimation and inference about the change in the mean of a stationary random process were examined in Farley and Hinich (1970a, 1970b), Hinkley (1970), Hawkins (1977), Hsu (1979), Talwar (1983), Worsley (1986), Ritov (1990), Bai (1994) and Lavielle and Moulines (2000). Second, another related but distinct problem involved estimation and inference about regime shifts associated with intersecting or "broken line" regressions were examined in Hudson (1966), Hinkley (1969, 1971) and Feder (1975a, 1975b). The major distinguishing feature of this particular problem is that the regression function is constrained to be continuous at the change point. This means that the change point phenomenon can be characterized as involving a

jump in the first derivative of the log-likelihood function of the regression problem, implying a change in the slope instead of the position of the regression function at the change point. This is in contrast with the first problem where the change point phenomenon involves a jump in the mean or position of the regression function, causing a discontinuity at the change point in the log-likelihood function of the regression problem.

Another issue in the literature on inference about mean and slope shifts discussed above concerns the peculiar nature of the hypothesis testing problem associated with tests of the significance of the shift itself at the change-point. This problem is one where inference must be undertaken in the presence of a nuisance parameter (for example, the new value of the mean at the change point) that is typically not identified under the null hypothesis (of no mean shift), e.g. see Davies (1977, 1987), Worsley (1986), Andrews (1993), Andrews and Ploberger (1994) and Hansen (1996). This problem also extends to inference about transition in the 'newer' time series econometric models, such as Threshold Autoregression (TAR) models (Potter (1995, p. 111) and Tsay (1998, p. 1189)).

A third related problem is the determination of whether a change in the intercept and/or slope of a linear statistical model has occurred. The seminal papers by Chow (1960) and Quandt (1960) were followed by Brown, Durbin and Evans (1975), Farley, Hinich and McGuire (1975), Feder (1975a, 1975b), James, James and Siegmund (1987), Kim and Siegmund (1989), Andrews (1993), Kim (1994), Bai (1996), and Yashchin (1995, 1997).

In this article, we demonstrate that the process of generating smooth transitions can be viewed as a natural artifact of the filtering operations implied in the generation of discrete time series observations that result from sampling data that flow from an underlying continuous time process that has undergone a process of structural change. We also address the circumstances in which the phenomenon of time irreversibility can occur. To accomplish this task, we focus on the problem of estimating the location of abrupt shifts in some simple time series models. These simple models are chosen in order to allow us to focus attention on some salient issues relating to distortions induced by the inherent aggregation associated with discrete-time sampling of continuous time processes that generates the phenomenon of smooth transition. We also address the issue of the generation of time irreversible structures within the smooth transition processes.

This question is also important because it focuses attention on the location of the structural change itself – that is, when the structural change or regime shift occurred. This knowledge allows for the possibility of re-initialization of estimation about the actual transition if the underlying process approximates a piecewise linear process. Our focus, however, differs somewhat from much of the research in the time series econometric literature in that we are not directly addressing the mechanics of the smooth transition process itself, which typically addresses the process of regime selection or identification. For example, we are not addressing whether the transition process is of a discrete character (i.e. Threshold Autoregressive (TAR) model), a more gradual process

with a regime indicator function based on a continuous function (for example, the logistic cumulative distribution function associated with Smooth Transition Autoregressive (STAR) model), or based on a Markov Switching process (see Brooks (2002, p.561)). Instead, we interpret the smooth transition process associated with regime change broadly, as a phase of structural change with priority given to identifying the temporal location of this change if we think it has occurred.

The above-mentioned problems all share one common property - the starting point is a discrete time series model. It was recognized early in the literature that the assumption of discrete time is one of the key determinants of the discontinuity problem associated with the log-likelihood function in conventional change point problems (Quandt (1960, p. 876 and footnote 7), Hinkley (1970, p. 6) and Feder (1975a, p. 52-53)). It is common in time series analysis to begin with a discrete-time sample of the time series. Whereas, in engineering and science applications of statistical signal processing methods, it is understood that any discrete-time series is the result of filtering a continuous time data flow (often termed a signal) and then decimating the filtered output (discrete-time sampling) to obtain the discrete-time sample. The filtering operation is designed to remove high frequency components of the continuous time data flow, whose frequencies are higher than twice the sampling rate used, in order to avoid aliasing.

Even if the continuous time data flow is not deliberately filtered to remove high frequency components, any measurement system has finite bandwidth

and there is a natural high frequency cutoff in an observed signal. This finite bandwidth property can be viewed, in a fundamental sense, as being a consequence of the constraints imposed by the second law of thermodynamics and its analogue in information transmission systems. In such systems there is no thing such as a "free lunch" or unlimited growth potential.

The science-based approach to obtaining discrete-time observations of an underlying continuous time data flow process is not a standard topic in most statistical time series texts. The nth observation $t_n = n\tau$ for the sampling frequency $f_s = 1/\tau$ is either implicitly or explicitly assumed to be the true value at time t_n in the standard statistical time series literature, eg. Grenander and Rosenblatt (1957, p.57), Box and Jenkins (1970, p.399-400), Hannan (1970, Sect. II3), Brillinger (1975, p. 178), Fuller (1995, Sect. 1.3) and Shumway and Stoffer (2000, Sect. 1.3). This assumption is wrong. Any discrete-time observation made is the result of some smoothing of the underlying data flow process and is the average of the continuous time process in a time slice around t_n.

In contrast, we formulate the problem as a continuous time process which experiences one type of structural change during the observation period yielding, heuristically, two regimes either side of the point of structural change. We then assume an ideal situation where we either low-pass filter the signal or society low-pass filters it and then samples it at the Nyquist rate. This filtering produces a discrete time problem where structural change has been smoothed

by a filtering operation, thereby producing a smooth transition path between the two regimes.

In economic applications, macroeconomists use data that is typically supplied by national statistical agencies. This data is usually aliased because it is under sampled. For example, surveys from which quarterly data are generated are often conducted over a week's duration either once a month or once a quarter. The results of these surveys contain information that is benchmarked with additional information obtained from annual surveys (or censuses) which are then used to generate both quarterly and annual national accounts statistics. This data processing is a complicated, albeit imperfect, filtering process that smoothes out any abrupt changes in the economy. For example any abrupt change appearing within a quarter will show up as a change across consecutive quarters. But the information content in the resultant quarterly time series data will not be sufficient to locate and model the change as it appeared within the quarter in which it occurred. Clearly, there are technical issues concerning sampling and filtering that have to be addressed before it is possible to assess whether structural change has occurred in such time series data. It is to such issues that we now turn.

2   SAMPLING A CONTINUOUS TIME PROCESS

Most time series can be conceived as a continuous flow of data through time that is measured by some procedure. The measurement procedure used in engineering and science applications is deliberative but in the social sciences

the filtering and sampling methods that are used to generate discrete-time samples are rarely discussed explicitly. Each discrete-time observation $\tilde{x}(t_n)$ is treated as if it was the true value at time t_n rather than an average value around t_n.

An engineered time series sampling method is as follows. A continuous time process $x(t)$ is filtered to remove all frequency components above a cutoff frequency f_o and then the filtered process is sampled at a rate greater than or equal to $2f_o$ in order to avoid aliasing the sampled data (Priestley (1981, Sect. 7.1.1)). Recall that the nth observation of the discrete-time process is $t_n = n\tau$ where $\tau = (2f_o)^{-1}$ denotes the sampling interval if the process is sampled at the Nyquist sampling rate $2f_o$. The sampled value $\tilde{x}(t_n)$ is an aggregated value of the process $x(t)$ around time t_n.

Let us turn to the basic technicalities of linear filtering. Suppose that the filter is *linear* and *causal*. Then the filter is characterized by its *impulse response* function, denoted $h(s)$ where $h(s) = 0$ for $s < 0$. The output of the filter is the convolution

(2.1) $$\tilde{x}(t) = \int_{s=0}^{\infty} h(s)x(t-s)ds .$$

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The impulse response is the output of the system to a unit impulse occurring at time t = 0. The output of the filter at time t depends only on past (observed) values of filter input.[3]

 When the sampling method is engineered, the impulse response used is part of the design. Thus it is a known function. In social sciences the sampling methodology is usually inexact. The impulse response is an unknown function that is ignored in the quest for a statistical model of the stochastic process.

 The response of the filter is equivalently represented by the filter's transfer function, which is the Fourier transform of the filter function $h(s)$, namely

$$(2.2) \qquad \Gamma(f) = \int_{-\infty}^{\infty} h(s) e^{-i2\pi f s} ds$$

This Fourier transform exists if $h(s)$ is absolutely integrable (Priestley (1981, pp. 264-266)). The transfer function is complex valued and can be expressed heuristically as

$$(2.3) \qquad \Gamma(f) = \gamma(f) \exp(i\varphi(f))$$

where $\gamma(f) \geq 0$ is called the *gain* and $\varphi(f)$ is called the *phase response* of the filter. The gain function is always symmetric and the phase function is anti-symmetric about zero. When the input is passed through the filter, the

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[3] Such filters are clearly non-anticipative because they do not depend on future (non-observed) values of filter input (Priestley (1981, p.265)).

amplitude of the input will be multiplied by the filter gain $\gamma(f)$ and the phase

will be shifted by $\varphi(f)$ (see Priestley (1981, pp. 270-271)).

To illustrate the way the gain and phase of a linear filter operate on a cycle we

use the causal filter whose impulse response is $h(t_n) = \sqrt{\frac{2\kappa}{\pi}} \exp\left(-\frac{\kappa}{2} t_n^2\right)$. This

filter is a half gaussian density function scaled to have unit area. It is a type of

low pass filter since its gain function is gaussian. Thus, high frequency terms

are reduced in amplitude relative to the low frequency terms. The cycle we use

in our calculations is

$$(2.4) \qquad s(t_n) = \sum_{k=1}^{K} \left(1 + \frac{k}{K}\right) \left[\sin\left(2\pi \frac{k}{P} t_n\right) + \cos\left(2\pi \frac{k}{P} t_n\right)\right]$$

where P is the fundamental period of the cycle with K harmonics and N is the

sample size used in the simulation. The fundamental frequency of the cycle

is $f = 1/P$. The output of the filter is shown in <u>Figure 1</u> for the bandwidth

coefficient κ = 1.

Figure 1 about here.

The phase response is *zero* if, and only if, the impulse response is symmetric

about zero, which will only arise in the case of a very special type of non-causal

filter called a *symmetric* filter. Thus, all causal and all but the symmetric filter

is a *time irreversible* operation. The key implications of this outcome will be

examined in more detail in Section 4.

The filter operation in (2.1) smoothes the input since the filter removed frequency components of the input for $f \geq f_o$. Let the impulse response have unit area to simplify notation. Then, if the continuous time process experiences structural change – for example an abrupt shift in the mean from μ to $\mu + \delta$ at an *unknown* time t_0 - the shift in the output of the process is given by $\delta H(t + t_0)$ where $H(t) = \int_{s=0}^{t} h(s) ds$. If the filtered signal is sampled at the Nyquist rate then the mean shift is $H(t_n + t_0) = \sum_{m=0}^{n+n_0} h(t_m)$ and $t_0 = n_0 \tau$. Since the filtered output is a continuously differentiable function of the shift time parameter t_0, a maximum likelihood estimate of this parameter exists. The properties of the MLE estimate are given in Hinich and Wild (2003).

If the form of the structural change involves an abrupt shift in the variance from σ^2 to $\sigma^2 + \delta$ at an unknown time t_0, the shift in the output of the process is given by

(2.5) $$H(t + t_0) = \sigma^2 \int_{s=0}^{\infty} h^2(s) ds + \delta \int_{0}^{t - t_0} h^2(s) ds.$$

The first term on the right hand side of (2.5) is the output of the filter, assuming no structural change in the variance of the input. The second term is the additional output of the filter assuming the structural change in the variance occurred at time t_0.

The view expressed above, that the underlying data flow process is a continuous time phenomenon can be readily extended to economic and social

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systems. In the context of economic systems, for example, the time continuity of the underlying process is related to the notion of continuity of exchange. Specifically, the structure of both domestic and international trade and finance, together with the availability of spot and forward markets, engenders a continuous flow of economic transactions. However, some sort of filtering operation is still latent in social science applications, even if the investigator believes that each value of $x(t_n)$ is the true value of the process at time $t_n = n\tau$.

The fundamental uncertainty principle accepted in the natural sciences applies to all measurements made in the social sciences. In particular, it should be recognized that the consequences of inherent limitations to coding, transmitting and analyzing information on institutional and other forms of organizational behavior, in the face of complex real-world situations, constitutes an important source of finite bandwidth in social systems. As a consequence it is impossible to obtain a precise measurement of a process at a precise time. However in this article we are not going to address the inherent error in the time of measurement. We treat t_n as the true time of measurement but $x(t_n)$ is really $\tilde{x}(t_n)$ from equation (2.1) for some filtering operation with a usually unknown impulse response.

It should be noted that the assumption of finite bandwidth is also at odds with the conventional requirement of infinite bandwidth $(-\infty, \infty)$ underpinning the conception of continuous white noise typically employed in continuous time econometric and time series problems (see, for example, Bergstrom, (1976) and

Priestley, (1981, pp. 234-235)). The assumption of infinite bandwidth also ensures that the sampling interval approaches zero, a condition that seems to be necessary and implied in applications of standard stochastic techniques such as Brownian motion.

It is our contention that the latter situation cannot be viewed as an appropriate approximation in any sense to any meaningful real world problem, irrespective of whether the problem is in the realm of the natural or social sciences. This is because, even in an ideal setting, both nature and society will ensure that f_0 will have an upper bound that, while possibly being very large in magnitude, is definitively finite. This value of f_0, "bequeathed" to us by nature or society in turn determines the Nyquist sampling rate $2f_o$ and associated sampling interval $\tau = \left(2f_o\right)^{-1}$. This means that the appropriate continuous white noise concept is *bandlimited white noise*, whose spectral density is constant over the finite pass band range $\left(-f_o, f_o\right)$ and zero outside this range.

3   *FILTERED TIME SHIFTS*

We next turn to the issue of the impact of filtering operations on mean shifts and slope changes in continuous time processes. Our main objective is to demonstrate the distortions that the filtering operation can exert on the discrete-time output of a filter process in terms of its ability to track the true changes occurring in the continuous time process.

Suppose that the slope parameter β of the simple linear time trend model $y(t) = \beta x(t) + e(t)$ shifts at time t_0. Assume that $x(t)$ is observed with no error and that the joint distribution of the noise process $e(t)$ is independent of $x(t)$. Suppose further that the independent variable $x(t)$ and dependent variable $y(t)$ are smoothed by a bandlimited causal filter whose impulse response is $h(t)$ and then sampled at or above the Nyquist rate. Then the discrete-time (sampled) observations are of the form $\tilde{y}(t_n) = \beta \tilde{x}(t_n) + \tilde{e}(t_n)$, where the tildes indicate that the process is the output of the causal filter as per equation (2.1) as long as β is constant. If β shifts to $\beta + \delta$ at time $t_0 = n_0 \tau$, that is

(3.1)
$$\beta(t) = \begin{cases} \beta & \text{if } n < n_0 \\ \beta + \delta & \text{if } n \geq n_0 \end{cases}$$

then the output of the discrete-time process is now given by

(3.2)
$$\tilde{y}(t) = \beta \sum_{m=n-n_0}^{\infty} h(t_m) x(t_{n-m}) + (\beta + \delta) \sum_{m=0}^{n-n_0} h(t_m) x(t_{n-m})$$

$$= \beta \sum_{m=0}^{\infty} h(t_m) x(t_{n-m}) + \delta \sum_{m=0}^{n-n_0} h(t_m) x(t_{n-m})$$

$$= \beta \tilde{x}(t_n) + \delta \sum_{m=0}^{n-n_0} h(t_m) x(t_{n-m}).$$

The first term in (3.2) gives the output of the filtering operation, assuming no structural change. The second term gives the additional output of the filtering

operation, assuming a process of structural change associated with the slope change from β to $\beta + \delta$ at time t_0.

To illustrate how the measurement filter can distort the input when there is a slope shift we simulate a particular low-pass filter operating on a curvilinear trend plus a cycle. Computation is necessary to show how the filtering operation distorts a slope shift in a curvilinear trend with cycles since the output cannot be expressed in any meaningful closed form. We use the same half gaussian causal filter model as was used to generate Figure 1.

To illustrate the effect of such a filter on a linear trend with a slope shift, this impulse response was convolved with a trend plus a cycle of the following form

$$(3.3) \qquad\qquad x(t_n) = \beta(t_n)t_n + c\left(t_n^2 - \frac{T}{2}\right) + a\, s(t_n)$$

where $s(t_n)$ is the cycle defined in (2.4).

The cycle's amplitude a is zero for the first three examples of a trend with a slope shift. Figure 2 shows the true trend and the filtered trend where c = 0 and the slope is β = 0.5 with a shift of δ = 5 in the middle. The filtered trend stays below the true trend after the shift.

Figure 2 about here.

Figure 3 shows the true and filtered trend with a linear slope of β = 1.5, a quadratic coefficient of c = -0.04 and a linear slope shift of δ = -2.5.

Figure 3 about here.

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Once again the filtered trend is biased away from the true trend after the shift. The same is also true in Figure 4 where β = 3.5.

Figure 4 about here.

Figure 5 shows the effect of the filtering operation for a true trend plus a cycle with seven harmonics and an amplitude of a = 30. In this simulation, β = 1.5, the quadratic coefficient c = -0.04 and a linear slope shift of δ = -2.5 is also adopted. The filtered trend is biased down from the true trend once again and the cycle is time shifted due to the phase shifts on the causal filter's complex frequency response.

Figure 5 about here.

All these examples show that the effect of the filter's operation is to *smooth* the response of the system to the abrupt types of structural change envisaged in each respective simulation by attenuating the high frequency components relative to the low frequency components. In the cases involving linear trend simulations documented in Figures 2, [3 and 4], this effect shows up in the form of a downward [upward] bias in the sampled output (the estimated trend) when compared with the true trend. In all three cases, the true extent to the structural change is understated by the filtering operation.

In the case of the simulation involving trend plus cycle (Figure 5), the principal effect of the filter once again is to *smooth* (or understate) the extent of periodic behavior of the sampled output (the estimated trend) when compared to the true process. The phase shift arising from the complex frequency

response of the filter operation is also apparent from inspection of Figure 5 in terms of the horizontal displacement of the peaks and troughs of the estimated trend when compared with the peaks and troughs of the true process. Finally, the inherent aggregation implied by the filter operation is also apparent from Figure 5 in terms of the extent of vertical displacement of the estimated trend, which is diminished when compared with the true process. This reflects the filter's attenuation of the high frequency components relative to the low frequency components.

A curvilinear trend was used to simplify our exposition. Trends in economic data vary over time and we advocate transforming the levels to growth rates. A time series of growth rates will have a much flatter spectrum than the spectrum of levels and the growth rate process will appear to be more stochastic.

4 FILTER OPERATIONS AND GENERATION OF TIME IRREVERSIBLE PROBABLISTIC STRUCTURE

Let $\{x(t_n)\}$ denote the discrete-time input to a linear filter whose output is $\{\tilde{x}(t_n)\}$. Assume that the density of the $x(t_n)$ has finite support and thus the cumulants of $x(t_n)$ are bounded. Recall from Section 2 that $\Gamma(f) = \gamma(f)\exp(i\varphi(f))$ is the transfer function of the filter. Then the bispectrum of the filter output $\{\tilde{x}(t_n)\}$ is

$$\begin{aligned}
(4.1) \quad B_{\tilde{x}}\left(f_1, f_2\right) &= \Gamma\left(f_1\right)\Gamma\left(f_2\right)\Gamma\left(-f_1-f_2\right)B_x\left(f_1, f_2\right) \\
&= \gamma\left(f_1\right)\gamma\left(f_2\right)\gamma\left(f_1+f_2\right)\exp\left[i\left(\varphi\left(f_1\right)+\varphi\left(f_2\right)-\varphi\left(f_1+f_2\right)\right)\right]B_x\left(f_1, f_2\right)
\end{aligned}$$

where $B_x\left(f_1, f_2\right)$ is the bispectrum of the input process which is assumed to be non-zero.[4] Within the structure denoted by (4.1), the property of *time irreversibility* of the filter output $\{\tilde{x}(t_n)\}$ can arise, in principle, in the following two ways:

(a) if the filter is causal (or non-causal but non-symmetric), the imaginary part of the bispectrum of the filter output $(B_{\tilde{x}}\left(f_1, f_2\right))$ will be non-zero and the output of the filter will be *time irreversible* because of the phase function of the filter, irrespective of whether the input is time reversible or irreversible; and

(b) if the filter is non-causal and symmetric (i.e. a symmetric filter), then the output of the filter will be time reversible *(time irreversible)* if the input is time reversible *(time irreversible)* – that is, if the bispectrum of the input $(B_x\left(f_1, f_2\right))$ has a zero *(non-zero)* imaginary part. If the input is time irreversible, this outcome will be linked to the phase function of the input bispectrum, and the bispectrum of the filter output $(B_{\tilde{x}}\left(f_1, f_2\right))$ will also consequently have a non-zero imaginary part, thereby confirming the time irreversibility of the filter output $\{\tilde{x}(t_n)\}$.

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[4] This will require that the input is non-gaussian and non-symmetric.

In social science applications the measurement filter will be causal but it's impulse response will be unknown. If the input is white noise and the filter is invertible then the filter's impulse response can be estimated from the covariance function of the output or, equivalently, its spectrum. If the input is pure white noise, that is the variates are independently and identically distributed, then the estimated bispectrum can be used to estimate the transfer function of a linear filter and thus obtain an estimate of its inverse Fourier transform, the impulse response, e.g., see Lii and Rosenblatt, 1982. However, if the input is not pure white noise, then the gain and phase of the input cannot be disentangled from the gain and phase of the filter and the estimated bispectrum cannot be used to obtain an estimate of the impulse response of the linear filter. Most economic processes are unlikely to be white noise even in growth rates. Thus the measurement filter is an unknown yet important aspect of the transformation of the continuous time input to a time series.

Finally, suppose that the output of the measurement process (i.e. the discrete-time series data obtained from the measurement filter) is used as source time series in econometric investigations. Suppose further that this discrete-time data is time irreversible, by construction, from the discrete-time sampling process. Then the output from econometric modeling exercises using this data as regressor variables will also retain the property of time irreversibility irrespective of the properties of the econometric model that is used to model the process. If the source time series is used as a dependent

variable, attempts to model this data must then also be able to account for the time irreversibility in its probabilistic structure. If the input entering this modeling process is assumed to be time reversible (such as pure white noise), then the econometric model itself must be able to account for the observed time irreversibility in the probabilistic structure of the dependent variable series.

5   *CONCLUDING REMARKS*

We raise some fundamental statistical issues that macroeconomic researchers must confront in relation to structural change in the economic systems and associated economic processes that time series data reflect. The discrete data points that are typically used in macroeconomic modeling are samples drawn from continuous-time processes which generate continuous flows of data. This must be taken into account in any investigation of structural change. The next issue that we deal with is the impact of filtering operations on mean shifts and slope changes in continuous time processes. We demonstrate that the distortions that such operations exert on discrete-time output can affect our ability to track the actual changes occurring in the continuous time process. Finally, we discuss the link between time irreversibility and loss of uniqueness in the spectral representation of a data generation process, i.e. what we might expect in the presence of a complex system. Time irreversible processes have a bispectral representation that has a statistically significant imaginary part and thus it is important to test whether this is the case before conventional methods are applied. For example suppose that the sampling process

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generating the discrete source time series data used in econometric investigations produces data that is time irreversible, by construction. Then if one assumes, in common with econometric theory, that the input process is time reversible (such as a pure white noise process), then the model used must be able to account for any observed time irreversibility in the source time series data.

Economic systems are complex adaptive systems. When we are using discrete time series data to understand the processes that these systems undergo, we must recognize the limitations of methods and techniques that are constructed under the presumption that we are dealing with simplistic representations of complex systems. This is clearly a problem that is most acute in studies employing macroeconomic time series data, yet it is mostly ignored in the applied macroeconomics literature where it is unusual for even the most rudimentary examination of the spectral properties of data series to be undertaken. This problem is quite a general one but it is likely to be most marked in cases where it is acknowledged that structural change has occurred.

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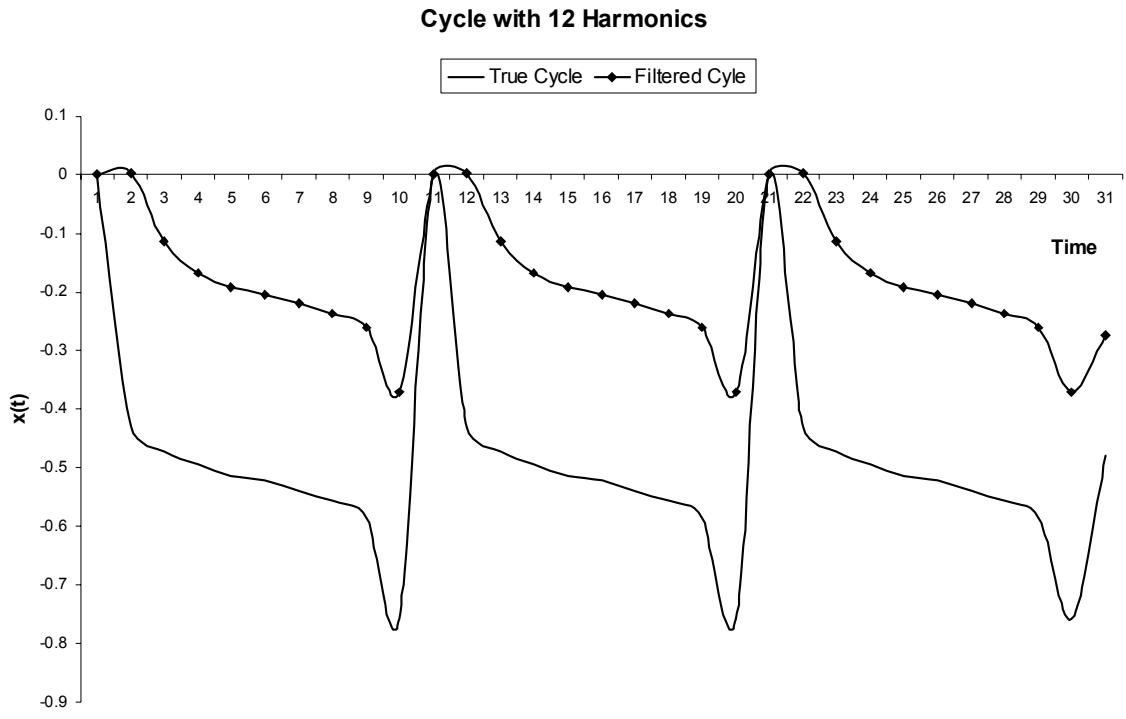
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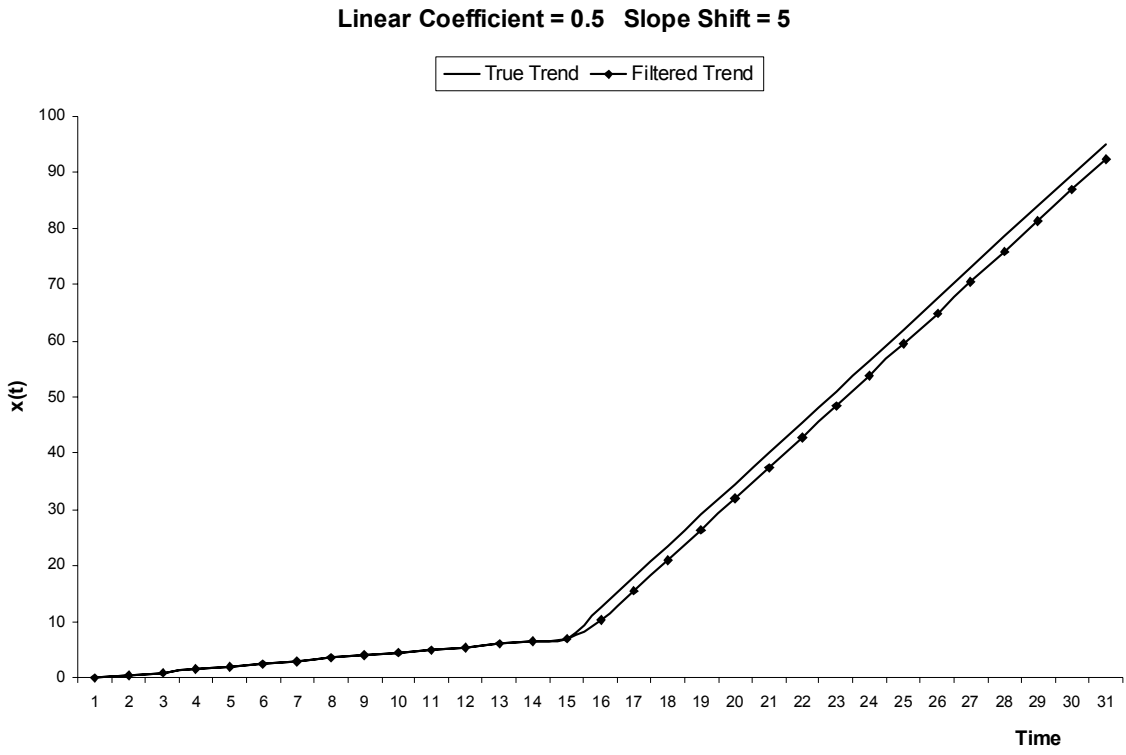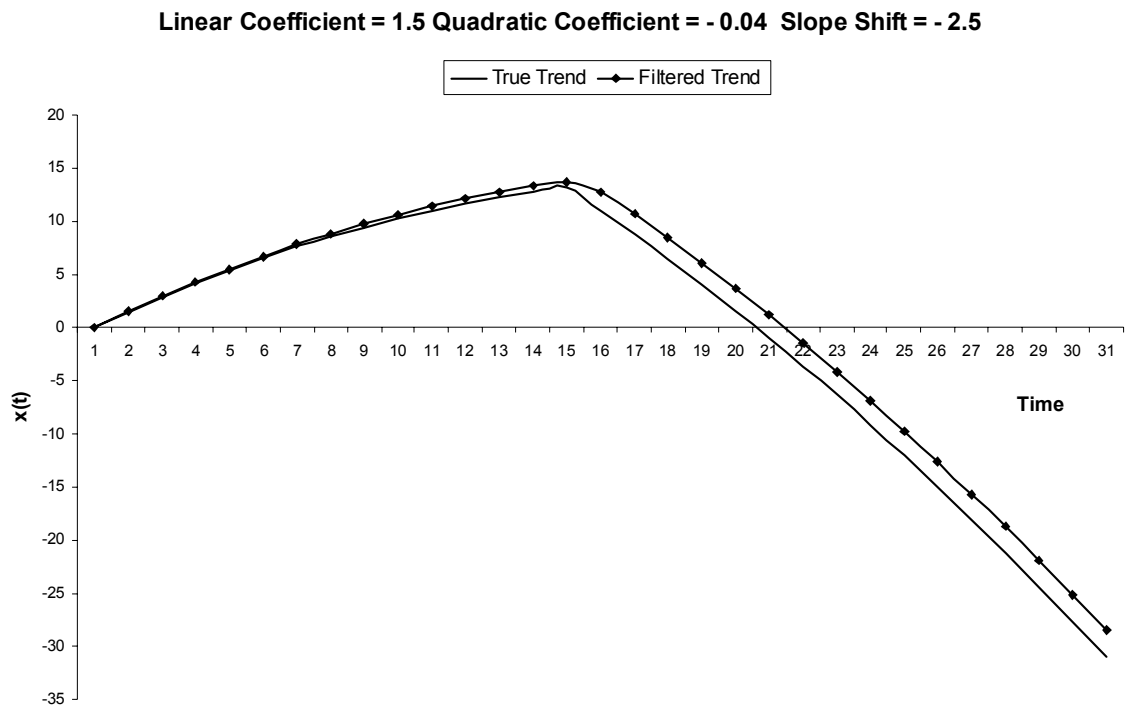
Figure 1.

Cycle with 12 Harmonics
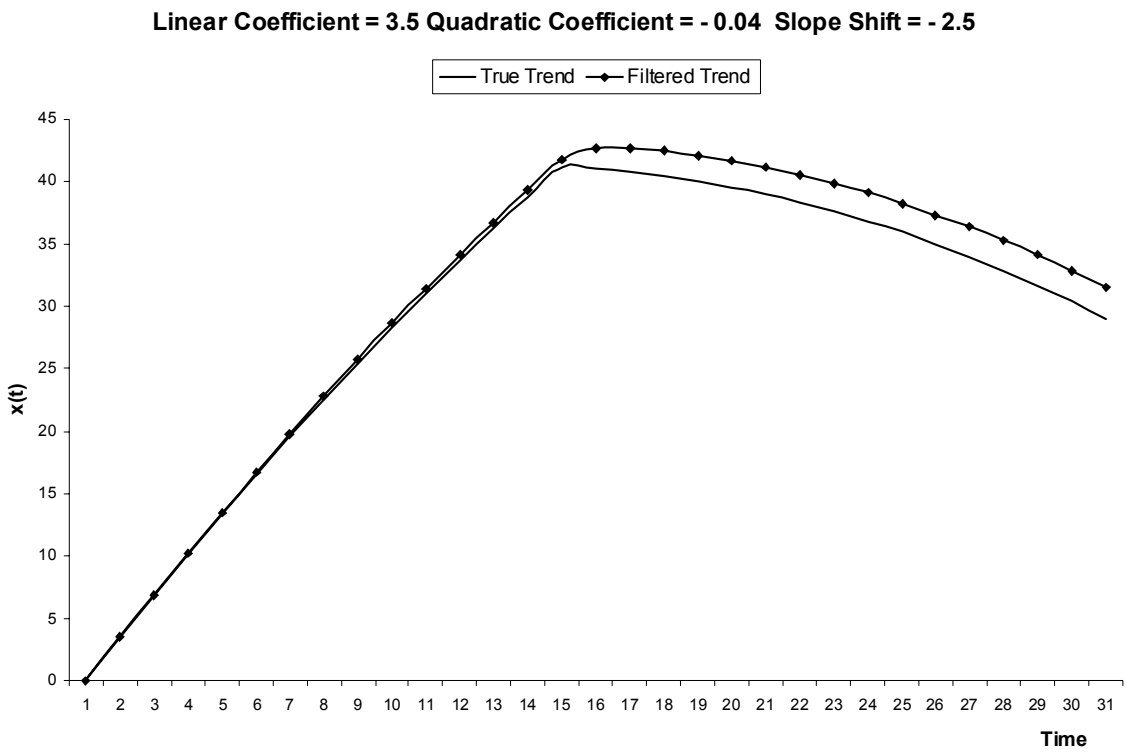
Figure 2.

Linear Coefficient = 0.5   Slope Shift = 5

Figure 3.

Linear Coefficient = 1.5 Quadratic Coefficient = - 0.04 Slope Shift = - 2.5

Figure 4.

Linear Coefficient = 3.5 Quadratic Coefficient = - 0.04 Slope Shift = - 2.5

Figure 5.

Linear Coefficient = 1.5 Quadratic Coefficient = -0.04 Slope Shift = -2.5
Cycle Amplitude = 30 Harmonics = 7