Leading Global Currencies: On the Role of Neglected Nonlinearity*

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Abstract

We perform numerous univariate tests for non-linearity and chaotic structure using daily data for leading currencies that include the Australian dollar, British pound, Brazilian real, Canadian dollar, euro, Japanese yen, Mexican peso, and the Swiss franc to resolve whether these currencies are driven by fundamentals or exogenous shocks to the global economy.

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1 Introduction

For many centuries, national economies have been linked to one another financially primarily because of trade. The importing nation received goods and paid in some pre-agreed currency. Currency trading predates both bond and stock trading as a financial innovation. However, there is very little doubt that during the past 50 years globalization grew at a remarkable pace and with it, currency trading. Today, the daily volume of currency transactions in currency futures, forwards, swaps and options dominates all other types of trading volumes. This volume is driven by globalization that includes both trade and foreign direct investments, by portfolio diversification and by hedging and speculation, among other factors.

After a rapid bibliographical review of the importance of global currencies in the next 3 sections we use daily data to perform state-of-the-art statistical tests to identify economic and statistical characteristics of several key currencies.

The paper is organized as follows. In Section 2 we offer a rapid historical development of the energy industry to illustrate its economic dynamics. This is followed by an assessment of the impact of the energy sector on the U.S. economy to simply reemphasize the often-cited fact of the currently diminished role of energy in contrast to its elevated significance three decades ago. In Section 6 we present the data and investigate the univariate time series properties of the crude oil, gasoline, heating oil, propane, and natural gas time series. In section 7 we discuss a number of tests for nonlinear structure, apply each of these tests to each of the five energy price series, and present and discuss the empirical results. The final section provides a brief summary and conclusion.

2 Globalization

Globalization is an extension of labor specialization beyond national borders and it is important to understand the international economic developments of the last fifty years. With a deepening of specialization, a growing population and improved attitudes toward taking risks over a widening area, production has become increasingly international. The technological advances of recent decades have increased the effects of globalization on economic growth and during the last decades policymakers have been trying to better understand global and technological changes that seemed to have changed world economic development.

Changes in information and communication technologies, for example, have accelerated the processing and transmission of data and ideas to a level far beyond our capabilities of a decade or two ago. Real-time information, by eliminating much human intervention, has significantly reduced errors in all forms of recordkeeping and lead times on purchases. These changes have had positive effects on the economic well-being for most of the economic participants but if globalization is to sustain the necessary public support we need also to consider an equitable distribution of global benefits among participating nations. Fortunately, global trade has long been viewed ans a positive sum game that benefits all participants. Although such benefits are clearly positive they need not be equal.

The dynamics of globalization include the lowering of tariffs and various trade barriers, deregulation, increased innovation and competition, the emergence of multinational firms, increased global trade and direct foreign investments and a faster pace in global GDP than in earlier decades. As a result, domestic economies are increasingly exposed to international competition.

Production of traded goods has increased in economies with large, low-wage labor forces and as a consequence, significant additions to world production and trade have put downward pressure on domestic and global prices. This trend of declining prices all over the world has been an important factor in the decrease of world economic volatility. To many people, the combination of increasing globalization and monetary policy has become increasingly effective in achieving the goal of price stability.

This view called the Great Moderation dominated economic thinking during 1990-2006 but the global financial crisis of 2007-2009 has challenged this paradigm. The current thinking suggests that globalization contributed to stable and low prices that encouraged various central banks to maintain low interest rates since there were no obvious risks of inflation and such an easy monetary policy on a global scale contributed to the emergence of financial bubbles both in global stock markets and housing prices.

Considering that so much of our recent experience with globalization has little precedent, we cannot fully determine how long the current globalization dynamic will last. We have little evidence that economic forces that are fostering international specialization, and hence cross-border trade and increasing dispersion of current account balances, are as yet diminishing.

The increasing globalization of the post-war world was supported at its beginnings by the judgment that burgeoning prewar protectionism was among the primary causes of the depth of the Great Depression of the 1930s. As a consequence, trade barriers began to fall after the war. Globalization was enhanced further when the inflation-ridden 1970s provoked rethinking of the philosophy of economic policy, the roots of which were still planted in the Depression era.

Globalization has expanded markedly in recent decades. Not only has the ratio of international trade in goods and services to world GDP risen inexorably over the past half-century, but a related measure, the extend to which savers reach beyond their national borders to invest in foreign assets, has also risen.

During some time after the World War II, countries were used to invest most of their domestic savings in domestic capital assets without considering the potential for superior risk-adjusted returns if they were to invest abroad. In the beginning of the 1990s, this bias to invest in its domestic capital assets started do decrease and countries started to invest their current account balances in different places. Thus, this expanding globalization enabled the USA to finance and, hence, incur so a large current account deficit. As a result of these capital flows, the ratio of foreign net claims against U.S residents to our annual GDP has risen to approximately one-fourth. While some other countries are far more in debt to foreigners, at least relative to their GDPs, they do not face the scale of international financing that we require.

We may not be able to usefully determine at what point foreign accumulation of net claims on the US will slow or even reverse, but up until recently it postulated that the greater the degree of international flexibility, the less the risk of a crisis. Obviously we now know that globalization during the last three years has caused serious financial imbalances, in particular in terms of emerging nations such as China financing along with advanced economies such as the EU, Japan and England the US twin deficits of the federal budget and the balance of payments.

In a world economy that is flexible, as debt projections rise, product and equity prices, interest rates, and exchange rates presumably would change to reestablish global balance. However, the penchant of humans for quirky, often irrational, behaviors gets in the way of this conclusion as recent history has demonstrated. A discontinuity in valuation judgments, often the cause building a bubble may quickly reverse and contribute to bursting such bubbles. Such developments can occasionally destabilize even the most liquid and flexible of markets as was witnessed during the current global financial crisis, in particular during the month of September 2008.

3 Portfolio Diversification

Increased financial globalization has offered important opportunities for portfolio diversification. There is a wide spectrum of financial risks that include firm specific risks, industry wide risks and country risks. Several macroeconomic policies such as monetary policy, fiscal policy and level of regulation are included in country risks, among several other factors. The growth in the size and complexity of international financial markets has been one of the most striking aspects of the world economy over the last decade. Lane and Milesi-Ferretti (2001, 2006) document the increase in gross holdings of cross-country bond and equities for a large group of countries. They describe this as a process of financial globalization. Economists and policy makers have speculated on the implications of financial globalization for the design of monetary policy. Most central banks now either explicitly or implicitly follow a policy of inflation targeting. Under this policy, price stability, appropriately defined, is the principal goal of monetary policy.

Devereux and Sutherland (2007) results' imply that financial globalization does not affect the fundamental aims of monetary policy. Although their model produces an international financial structure where countries are holding large offsetting gross nominal asset positions, so that exchange rate movements can generate substantial 'valuation effects', the presence of these effects does not directly change the optimal monetary rule. Because portfolios are chosen optimally, the wealth redistribution arising from exchange-rate-induced valuation effects represent the workings of an efficient international financial structure.

However, Devereux and Sutherland (2007) argue that the effects of monetary policy on other variables may be very different in a model with endogenous portfolio choice than in the standard analysis. Because the monetary rule leads to changes in the structure of international portfolios, the effects of monetary policy may be the opposite of what traditional reasoning would imply. For instance, a policy putting more weight on price stability may increase rather than reduce exchange rate volatility and the volatility of international capital flows. Because the exchange rate represents the excess return on nominal bonds, this means that an optimal monetary policy may increase rather than reduce asset price volatility.

Over the period 1975 to 2005, the US dollar and the euro and Swiss franc have moved against world equity markets. Thus, these currencies should be attractive to risk-minimizing global equity investors despite their low average returns. The risk-minimizing currency strategy for a global bond investor is close to a full currency hedge, with a modest long position in the US dollar.

Many investors hold indirect positions in foreign currency when they buy foreign equities or bonds without hedging the currency exposure implied by the foreign asset holding. Such investors receive the foreign-currency excess return on their foreign assets, plus the return on foreign currency.

Following Glen and Jorion, Campbell et al. (2009) considered an equity investor who chooses fixed currency weights to minimize the unconditional variance of her portfolio. Such an investor wishes to hold currencies that are negatively correlated with equities. Their first novel result is that at one extreme, the Australian dollar and the Canadian dollar are positively correlated with local-currency returns on equity markets around the world, including their own domestic markets. At the other extreme, the euro and the Swiss franc are negatively correlated with world stock returns and their own domestic stock returns. The Japanese yen, the British pound, and the US dollar fall in the middle,

with the yen and the pound more similar to the Australian and Canadian dollars, and the US dollar more similar to the euro and the Swiss franc.

4 Hedging and Speculation

When considering currencies in pairs, Campbell et al. found that risk-minimizing equity investors should short those currencies that are more positively correlated with equity returns and should hold long positions in those currencies that are more negatively correlated with returns. When considering all seven currencies as a group, they found that optimal currency positions tend to be long the US dollar, the Swiss franc, and the euro, and short the other currencies. A long position in the US-Canadian exchange rate is a particularly effective hedge against equity risk.

Campbell et al. achieved the second novel result when they considered the risk-minimization problem of global bond investors rather than global equity investors. They found that most currency returns are almost uncorrelated with bond returns and thus risk-minimizing bond investors should fully currency-hedge their international bond positions. The US dollar is an exception to the general pattern in that it tends to appreciate when bond prices fall, that is when interest rates rise, around the world. This generates a modest demand for US dollars by risk-minimizing bond investors.

The third novel result was obtained after Campbell et al. analyzed the historical average returns on currency pairs. They found that high-beta pairs have delivered higher average returns. However the historical reward for taking equity beta risk in currencies

has been quite modest, and much smaller than the historical average excess return

on a global stock index.

The fourth novel result is that increases in interest rates have only modest effects on currency-equity covariances. Over the full sample period, and particularly the first half of the sample, increases in interest differentials are, if anything, associated with decreases in these covariances. This implies that risk-minimizing equity investors should tilt their portfolios towards currencies that have temporarily high interest rates, amplifying the speculative "carry trade" demands for such currencies rather than offsetting them.

5 Hypotheses

The above rapid discussion leads us to the conclusion that certain global currencies have received great significance the past few decades. When we also consider the creation of the euro this significance becomes even greater since several important national currencies such as the German marc, the French franc, the Italian lira and several others were replaced by the euro. This global significance translates into a search for the pricing of these currencies. The challenge becomes even greater since currencies are priced one in terms of another. One may view the issue of pricing currencies as a comparison of all economic and financial fundamentals between two nations.

When pricing equities, economists have observed that returns have exhibited strong, short, and medium term serial correlation. The phenomenon of persistent performance over time has challenged the traditional random walk assumption for stock returns. The basic idea is that a single normal distribution is insufficient to describe the observed stock returns, evidenced by fat tails, skewness and excess kurtosis.

The classical intertemporal CAPM model of Merton (1980) indicates that risk premiums are positively related to market volatility. However, other researches have found different results. A large stock price drop is usually associated with a concurrent increase in volatility, which makes equity returns and volatility negatively correlated. This market phenomenon has led to two academics explanations. One is the leverage effect and the other one is the volatility feedback effect.

The leverage effect argues that a stock price drop increases the debt to equity ratio, which makes the stock riskier and raises the equity risk premium, that is, the leverage effects relates how realized stock returns are to future returns volatility.

The volatility feedback effect, which reverses the direction of causality, assumes that volatility is incorporated in stock prices and carried over to future returns, a positive volatility shock increases the future required return on equity and, therefore, stock prices are expected to fall simultaneously. Conclusions drawn from both of these academic thoughts are not satisfactory due to insignificant relations implied in their empirical work and researches using the same model were able to find competing results.

Instead of pricing currencies in this paper we study the time series behavior of leading currency relative prices. Our main hypothesis is that perfect randomness in price changes for all currencies does not hold. If price changes are not random but follow nonlinear deterministic patterns such information may allow economists to better evaluate the overall impact of leading currencies in the global economy. This paper performs state-of-the-art univariate tests to uncover the structure of currency prices for several leading currencies.

Mandelbrot and Hudson (2004) give a detailed description and of the presence of nonlinear determinism in financial markets. Empirical evidence of chaotic dynamics in financial data such as stock market indexes, foreign currencies, macroeconomic time series and several others have been performed by various researchers Kyrtsou and Vorlow (2007) recently and in much more detail earlier by Brock, Scheinkman and LeBaron, (1989) and Brock and Malliaris (1989). However, there is very little empirical work done to study nonlinear chaotic determinism in currency markets.

6 The Data

We use daily exchange rates (per United States dollar), provided by www.barchart.com, on the Australian dollar, British pound, Brazilian real, Canadian dollar, euro, Japanese yen, Mexican peso, and the Swiss franc. The sample period is from January 3, 2000 to June 2, 2009 (a total of 2,492 observations). Figure 1 plots U.S. dollar prices of each of the eight currencies. A rise in these plots indicates a strengthening of the currency (a weakening of the U.S. dollar). In Figure 2, we plot first logarithmic differences of the nominal exchange rates and in Figure 3 we show the frequency distributions of the first logged differences in the form of histograms. Summary statistics (not reported here) indicate that the skewness parameters are close to zero for all series, but that the kurtosis parameters are all greater than 3, suggesting peaked (leptokurtic) distributions relative to the normal distribution, consistent with the evidence in Figure 3. In fact, the Jarque and Bera (1980) test statistic, distributed as a $x^2(2)$ distribution under the null hypothesis of normality, leads to the rejection of the null hypothesis of a normal distribution with $p < .0001$ for each exchange rate series.

The first step in conducting nonlinear analysis is to test for stochastic trends (unit roots) in the autoregressive representation of each individual time series. In doing so, we use four alternative testing procedures to deal with anomalies that arise when the data are not very informative about whether or not there is a unit root. In the first three columns of Table 1, we report p-values for the augmented Weighted Symmetric (WS) unit root test [see Pantula *et al.* (1994)], the augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1981)], and the nonparametric, $Z(t_{\widehat{\alpha}})$, test of Phillips (1987) and Phillips and Perron (1988). These p-values (calculated using TSP 4.5) are based on the response surface estimates given by MacKinnon (1994). As discussed in Pantula *et al.* (1994), the WS test dominates the ADF test in terms of power. Also, the $Z(t_{\widehat{\alpha}})$ test is robust to a wide variety of serial correlation and time-dependent heteroskedasticity. For the WS and ADF tests, the optimal lag length was taken to be the order selected by the Akaike information criterion (AIC) plus 2 — see Pantula *et al.* (1994) for details regarding the advantages of this rule for choosing the number of augmenting lags. The $Z(t_{\widehat{\alpha}})$ test is done with the same Dickey-Fuller regression variables, using no augmenting lags. Based on the p-values for the WS, ADF, and $Z(t_{\widehat{\alpha}})$ test statistics reported in Table 1, the null hypothesis of a unit root in the first logged differences can be rejected for each exchange rate series.

Given that unit root tests have low power against relevant (trend stationary) alternatives, we also follow Kwiatkowski *et al.* (1992) and test for level and trend stationarity to distinguish between series that appear to be stationary, series that appear to be integrated, and series that are not very informative about whether or not they are stationary or have a unit root. KPSS tests for level and trend stationarity are presented in columns 4 and 5 of panel A of Table 1. As can be seen, the t-statistic $\widehat{\eta}_\mu$ that tests the null hypothesis of level stationarity is large relative to the 5% critical value of .463 given in Kwiatkowski *et al.* (1992). Also, the t-statistic $\widehat{\eta}_\tau$ that tests the null hypothesis of trend stationarity exceeds the 5% critical value of .146 [also given in Kwiatkowski *et al.* (1992)]. Hence, combining the results of our tests of the stationarity hypothesis with the results of our tests of the unit root hypothesis, we conclude that all the first logged differenced exchange rate series are stationary.

7 Nonlinearity Tests

7.1 Bispectral Tests

Hinich (1982) developed a statistical test for determining whether a sampled stationary time series $\{x(t)\}$ is linear. This is a direct test for linearity and also a test for Gaussianity; it is possible that $\{x(t)\}$ is linear without being Gaussian, but all of the stationary Gaussian time series are linear. The Hinich (1982) test involves estimating the bispectrum of a stationary time series — see also Hinich and Patterson (1989) for more details. If the process generating the data is linear then the skewness of the bispectrum will be constant. If the test rejects constant skewness then a non-linear process is implied. It is to ne noted, however, that as Barnett *et al.* (1997) show, the Hinich bispectrum test did poorly in a competition among tests for nonlinearity, while a test by Kaplan (1994) did far better.

Let's present a brief description of the Hinich (1982) bispectrum-based linearity and Gaussianity tests. Consider a third order stationary time series $\{x(t)\}$, where the time unit t is an integer. The third-order cumulant function of $\{x(t)\}$ is defined to be

$$C_{xxx}(r,s) = E\left[x(t+s)x(t+r)x(t)\right],$$

for each (r,s) when $E\left[x(t)\right] = 0$, in which $s \leq r$ and $r = 0, 1, 2, \cdots$. Because third-order cumulants are hard to interpret, the bispectrum, which is the double Fourier transform of the third-order cumulant function, $C_{xxx}(r,s)$, is calculated. The bispectrum at frequency pairs (f_1, f_2) is defined as

$$B_x(f_1,f_2) = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} C_{xxx}(r,s) \exp\left[-i2\pi(f_1 r + f_2 s)\right],$$

assuming that $|C_{xxx}(r,s)|$ is summable.

The symmetries of $C_{xxx}(r,s)$ translate into symmetries of $B_x(f_1,f_2)$ that yield a principal domain for $B_x(f_1,f_2)$ given by $\Omega = \{0 < f_1 < 0.5, f_2 < f_1, 2f_1 + f_2 < 1\}$. Since the (ordinary power) spectrum of $x(t)$ at frequency f, $S_x(f)$, is given by

$$S_x(f) = \sigma^2 \left|A(f)\right|^2,$$

the skewness function of $\{x(t)\}$, $\psi(f_1,f_2)$, is defined by

$$\psi^2(f_1,f_2) = \frac{|B_x(f_1,f_2)|^2}{S_x(f_1)S_x(f_2)S_x(f_1+f_2)},$$

for all f_1 and f_2 in Ω and $A(f) = \sum_{s=0}^{\infty} \alpha(s) \exp(-i2\pi f s)$.

Linearity and Gaussianity of $\{x(t)\}$ can be tested using a sample estimator of the skewness function. In particular, linearity of $\{x(t)\}$ is tested through the null hypothesis that the skewness function, $\psi(f_1, f_2)$, is constant over all frequencies. Gaussianity of $\{x(t)\}$ is tested through the null hypothesis that $\psi(f_1, f_2)$ is zero over all frequencies.

Columns 1 and 2 of Table 5 present p-values for Hinich's (1982) bispectrum-based Gaussianity and linearity tests. The results reject the null hypothesis of Gaussianity in all eight exchange rate series (see column 1). Although Gaussianity and linearity tests are linked, a rejection of Gaussianity does not necessarity rule out linearity. However, the p-values in column 2 reject the null hypothesis of a linear generating mechanism in all eight exchange rate series, suggesting the existence of nonlinear dependencies within the daily changes.

7.2 Bicorrelation Tests

Hinich (1996) proposed a modified version of the Box and Pierce (1970) portmanteau Q-statistic for autocorrelation and a third order portmanteau statistic, which can in a sense be viewed as a time domain analogue of the bispectrum test. A full theoretical derivation of the test statistics and a number of Monte Carlo simulations to assess their size and power are given in Hinich (1996) and Hinich and Patterson (1985).

Let $\{x(t)\}$ denote the sampled data process, where the time unit t is an integer. In this paper the time series will be daily energy returns. The method is to break the observed series into equal length frames and apply a number of statistics to each frame, generating a multivariate time series of frame statistics which are then used to test for linear and nonlinear serial dependencies. In particular, if n is the window length, then the kth window is $\{x(t_k), x(t_k+1), \cdots, x(t_k+n-1)\}$. The next window is $\{x(t_{k+1}), x(t_{k+1}+1), \cdots, x(t_{k+1}+n-1)\}$, where $t_{k+1} = t_k + n$. We define $z(t_k)$ as the standardized observations (created by subtracting the sample mean of the window, and dividing by its standard deviation) at time $t = k$, that is,

$$z(t_k) = \frac{x(t_k) - \mu_x}{\sigma_x},$$

where μ_x and σ_x^2 are the sample mean and sample standard deviation of the window. The null hypothesis for each window is that $x(t)$ are realizations of a statioanary pure noise process that has zero bicorrelation. The alternative hypothesis is that the process in the window is random with some non-zero correlations or non-zero bicorrelations.

The C (or correlation) statistic, which has been developed for the detection of linear serial dependencies, is defined as

$$C = \sum_{r=1}^{L} \left[C^2(r)/(T-r-1) \right] \sim \chi^2(L), \tag{1}$$

where

$$C(r) = \sum_{k=1}^{T-s} z(t_k)z(t_{k+r})$$

is the sample correlation.

The H statistic, which has been developed for the detection of nonlinear serial dependencies, tests for certain forms of nonlinearity using third-order correlations. It is defined as

$$H = \sum_{s=2}^{L}\sum_{r=1}^{s-1} \left[G^2\left(r,s\right)/(T-s)\right] \sim \chi^2(L\left(L-1\right)/2), \tag{2}$$

where

$$G(r,s) = \sum_{k=1}^{T-s} z(t_k)z(t_{k+r})z(t_{k+s})$$

is the (r,s) sample bicorrelation.

In (1) and (2), the number of lags L is specified as $L = T^c$ with $0 < c < 0.5$, where c is a parameter under the choice of the analyst. Based on results from Monte Carlo simulations [see Hinich and Patterson (1995)], the use of $c = 0.4$ is recommended in order to maximize the power of the tests whilst ensuring a valid approximation to the asymptotic theory even when T is small.

The C statistic in (1) is asymptotically distributed, under the null of pure white noise, as a chi-square with L degrees of freedom for large T if $L = T^c$ with $0 < c < 0.5$. It is closely related to the Box-Pierce portmanteau test statistic which detects correlated (non white) noise — see Box and Pierce (1970). Usually, the Box and Pierce Q-statistic for autocorrelation is applied to the residuals of a fitted ARMA model, but the C statistic is applied to the standardized observations, $z(t_k)$. Moreover, the Box and Pierce test does not specify the number of lags L to be used; that decision is left to the analyst. The C statistic specifies $L = T^c$ with $0 < c < 0.5$.

The H statistic in (1) is asymptotically distributed, under the null that the observed process is pure white noise (i.i.d.), as a chi-square with $L\left(L-1\right)/2$ degrees of freedom for large T if $L = T^c$ with $0 < c < 0.5$. It tests for certain forms of nonlinearity using third-order correlations and is considered as a generalization of the Box and Pierce portmanteau test. In particular, the test is of a null of pure white noise against an alternative that the process has m non-zero correlations or bicorrelations in the set $0 < r < s \leq L$, i.e. that there exists second or third order dependence in the data generating process, and relies on the property of pure noise that it has zero bicovariance. The test is particularly useful in detecting nonlinear dependencies, since it has much better small-sample properties, and does not have such stiff data requirements as many of its competitors, such as the BDS test [Brock *et al.* (1996)] for a useful survey.

Columns 3 and 4 of Table 5 present p-values for the correlations (C) and bicorrelations (H) test statistics. The results show that the null of pure noise is strongly rejected by both the C and H statistics in all eight exchange rate series.

8 Modeling Heteroscedasticity

In conventional econometric models, stochastic variables are assumed to have a constant variance (and are called homoskedastic, as opposed to heteroskedastic). Many macroeconomic and financial variables, however, exhibit clusters of volatility and tranquility (i.e., serial dependence in the higher conditional moments). In such circumstances, the homoskedasticity assumption is inappropriate.

Having concluded that the logged first differences of the exchange rates are stationary, we estimate the best fitted autoregressive model for each series according to equation (2):

$$\Delta \ln z_t = \sum_{i=1}^{r} \phi_i \Delta \ln z_{t-i} + \sum_{k=1}^{5} d_k D_{kt} + \varepsilon_t. \tag{3}$$

In equation (3), D_{kt} are day of the week dummy variables, r is the order of the autoregression, and ϕ and d are unknown parameters to be estimated.

We first select a minimum autoregression order $r_{\min}$ for each series such that the models show no autocorrelation according to the $Q(36)$ test statistic. Then we use both the Schwartz Information Criterion (SIC) and the Akaike Information Criterion (AIC) to optimally determine the value of r in equation (3), by estimating several models with $r = r_{\min}$ to $r = 25$. However, as the AIC tends to overparameterize the model while the SIC tends to select the true model as the sample size increases (and if the true model is included in the choices), we follow the SIC in selecting the optimal lag length of the autoregression, r. The results are reported in Table 1.

Both visual inspection and the use of the $Q(36)$ statistic for residual serial correlation (as seen in the last two columns of Table 2) suggest that the residuals of the autoregressive model with the order of the autoregression, r, chosen as above are not serially correlated. However, the $Q^2(36)$ statistic, which represents the Q-statistic for the squared residuals and is designed to pick nonlinearities and the presence of heteroskedasticity, is highly significant providing evidence for the presence of conditional heteroskedasticity in the error term. For this reason in order to capture the heteroskedasticity in the error term we estimate the autoregressive AR(r) model (3) for each series assuming that ε_t is $IN(0, \sigma_t^2)$ with σ_t^2 following a GARCH(p, q) process as follows,

$$\sigma_t^2 = w_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{k=2}^{5} d_k D_{kt} \tag{4}$$

or an EGARCH(p,q) process as follows

$$\log\left(\sigma_t^2\right) = w_0 + \sum_{i=1}^{q}\left(\alpha_i\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| + \gamma_i\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) + \sum_{j=1}^{p}\beta_j\log(\sigma_{t-j}^2) + \sum_{k=2}^{5}d_kD_{kt} \tag{5}$$

see, for example, Bollerslev (1986) and Nelson (1991), respectively, for more details.

In both equations (4) and (5) above, $p, q \in [1,2]$ such that eight different conditional heteroskedasticity specifications are estimated for each series. The lagged values of the error term, ε_{t-i}, $i = 1, \cdots, q$, in equations (4) and (5) represent news in the market about volatility in the previous period, while the lagged values of the conditional variance, σ_{t-j}^2, $j = 1, \cdots, q$, are lagged forecasted variances. Thus, this period's variance prediction is formed as a weighted average of a long term average (the constant, w_0), the forecasted variance from previous periods, and information about volatility observed in earlier periods. This variance modeling is consistent with the volatility clustering observed in the returns of the eight series (see Figure 2).

In Table 3 we report the AIC and SIC for the alternative specifications of the conditional variance for the eight exchange rate series and in Table 4 the model selected by the two criteria. For reasons discussed earlier we again use the SIC to select the best model for each exchange rate series and according to this for all series a GARCH$(1,1)$ is the best specification of the conditional variance with the exception of the Brazilian real where the EGARCH$(1,1)$ is selected.

The models estimated and selected so far use the normal distribution as the density function for the error term. Now we explore different error distributions in an attempt to improve the fit of the models. In particular, in addition to the normal distribution we use the Student's t distribution, used by Bollerslev (1987), and the generalized error distribution (GED), used by Nelson (1988) for each of the eight exchange rate series.

The Student's t distribution is given by

$$f(z) = \theta^{-.5}\pi^{-.5}\Gamma\left(\frac{\theta+1}{2}\right)\Gamma\left(\frac{\theta}{2}\right)^{-1}\left(1+\frac{z^2}{\theta-2}\right)^{-.5(\theta+1)}$$

where $\theta > 2$ is the degrees of freedom (controling the tail behavior) and $\Gamma(\cdot)$ is the gamma function. This distribution is normalized to have unit variance and becomes the standard normal distribution when $\theta \to \infty$. Under a Student's t distribution for the errors, the log likelihood function takes the form

$$L = -\frac{T}{2}\log\left(\theta^{-.5}\pi^{-.5}\Gamma\left(\frac{\theta+1}{2}\right)\Gamma\left(\frac{\theta}{2}\right)^{-1}\right) - \frac{1}{2}\sum_{t=1}^{T}\log\sigma_t^2 - \frac{\theta+1}{2}\sum_{t=1}^{T}\log\left[1+\frac{\varepsilon_t^2}{\sigma_t^2(\theta-2)}\right].$$

Moreover, the generalized error distribution (GED), used by Nelson (1988), is used. The density of a GED random variable normalized to have a mean of zero and a variance of one is given by

$$f(z) = \frac{\nu \exp\left[-\frac{1}{2}\left|z/\lambda\right|^{\nu}\right]}{\lambda 2^{(1+1/\nu)}\Gamma\left(1/\nu\right)},$$

where $-\infty < z < \infty$, $0 < \nu \leq \infty$, $\Gamma(\cdot)$ is the gamma function, and

$$\lambda \equiv \left[\frac{2^{(-2/\nu)}\Gamma\left(1/\nu\right)}{\Gamma\left(3/\nu\right)}\right]^{1/2}.$$

Above, ν is a tail-thickness parameter, $\nu > 0$. When $\nu = 2$, z has a standard normal distribution. For $\nu < 2$, the distribution of z has thicker tails than the normal (for example, when $\nu = 1$, z has a double exponential distribution). For $\nu > 2$, the distribution of z has thinner tails than the normal (for example, for $\nu = \infty$, z is uniformly distributed on the interval $\left[-3^{1/2}, 3^{1/2}\right]$. Under a GED distribution for the errors, the log likelihood function takes the form

$$L = -\frac{T}{2}\log\left(\frac{\Gamma\left(1/\nu\right)^{3}}{\Gamma\left(3/\nu\right)\left(\nu/2\right)^{2}}\right) - \frac{1}{2}\sum_{t=1}^{T}\log\sigma_t^2 - \sum_{t=1}^{T}\left[\frac{\Gamma\left(3/\nu\right)\varepsilon_t^2}{\sigma_t^2\Gamma\left(1/\nu\right)}\right]^{\nu/2}.$$

In Table 5, we produce the AIC and SIC for the estimated models according to the three different distributional assumptions and we use the SIC to determine the best overall model as it is reported in Table 6 for the eight exchange rate series. With the exception of the Australian Dollar and the Japanese Yen where the Student's-t distribution is selected, for the rest of the exchange rates the GED provides the best fit. According to the $Q(36)$ and $Q^2(36)$ statistics that are reported in the last two columns of Table 6, for the selected models we cannot reject the null of no autocorrelation and no non-linearities in the residuals.

8.1 Chaos Tests

Finally, we test for chaos by applying the recently developed methods by Whang and Linton (1999), Linton and Shintani (2003), and Shintani and Linton (2004) and construct the standard error for the Nychka *et al.* (1992) dominant Lyapunov exponent — see Serletis and Shintani (2003) for a detailed discussion of the methodology and an application to the U.S. stock market or Serletis and Shintani (2006) for an application to U.S. monetary aggregates.

Lyapunov exponent point estimates, along with p-values for the null hypothesis $H_0 : \lambda \geq 0$, are reported in Table 6, for the logarithmic first differences of the series. The results are presented for dimensions 1 through 6, with the optimal value of the number of hidden units in the neural net being chosen by minimizing the BIC criterion — see, for example, Serletis and Shintani (2006) for more details.

As can be seen, the reported Lyapunov exponent point estimates are negative and in every case we reject the null hypothesis of chaotic behavior. Of course, the failure to detect low-dimensional chaos does not preclude the possiblity of high-dimensional chaos in these series

— see, for example, Barnett and Serletis (2000). The presence, however, of dynamic noise makes it difficult and perhaps impossible to distinguish between (noisy) high-dimensional chaos and pure randomness. Thus, as Granger (1991, p. 268) put it, "it will be a sound, pragmatic strategy to continue to use stochastic models and statistical inference."

9 Conclusion

We have discussed a number of (widely used) univariate tests from dynamical systems theory to distinguish between deterministic and stochastic origin for time series.

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