THE EPISODIC BEHAVIOR OF DEPENDENCIES IN HIGH FREQUENCY STOCK RETURNS

Melvin J. Hinich
University of Texas at Austin
e-mail: hinich@mail.la.utexas.edu

Douglas M. Patterson
Virginia Polytechnic Institute
and State University
(540)231-5737
e-mail: amex@vt.edu

April 2001

Preliminary Draft

THE EPISODIC BEHAVIOR OF DEPENDENCIES IN HIGH FREQUENCY STOCK RETURNS

Melvin J Hinich
University of Texas at Austin

Douglas M. Patterson
Virginia Polytechnic Institute
and State University

ABSTRACT

The paper investigates the following conjecture: the inability of investigators to make meaningful point forecasts of stock returns despite strong evidence of nonlinear dependence is caused by the episodic, or transient, nature of the dependencies. A new methodology for detecting transient dependence in a pure white noise process is applied to intra-day returns of a sample of stocks who are members of the DJIA. The methodology makes use of two portmanteau test statistics. The first, or C statistic, is similar to the Box-Pierce correlation test statistic. The second statistic, called H, detects third-order correlation, and is therefore a test for certain types of nonlinearity. In addition, Engle's LM test for ARCH/GARCH, another form of nonlinearity, is used to search for transient episodes of conditional heteroskedastic volatility in daily stock returns.

The potential for making point predictions of stock returns generated considerable excitement in econometric circles with the discovery, over a decade ago, of nonlinear dependence in financial time series. The idea of complex stock market dynamics with some degree of predictability has theoretical appeal to many economists. A prodigious amount of empirical evidence supporting the existence of nonlinear dependence in economic and financial time series has accumulated over the past decade. Much of the evidence is related to the phenomenon of conditional heteroskedasticity, a form of nonlinear dependence which can be generated by a martingale difference process. More generally, nonlinear dependence can be classified into two broad groups: those processes which are also martingale differences, and those which are not. The former do not admit forecasts of the conditional mean, a significant distinction in finance. In its totality, the scientific evidence suggests that both classes of dependence are present in stock returns.

The first report of nonlinear dependence in stock market returns can be found in Hinich and Patterson (1985). That paper applied the Hinich (1982) bispectral linearity test to a sample of daily common stock returns for fifteen different stocks selected at random from the daily CRSP file. The Hinich test is designed to detect third-order moment dependence, a complicated form of correlation.

The best known example of a nonlinear martingale model in the finance field is the Autoregressive Conditional Heteroskedastic (ARCH) model of Engle (1982), and its close cousin the Generalized Autoregressive Conditional Heteroskedastic (GARCH) model of Bollerslev (1986a). These models have had wide appeal to finance researchers because they are capable of modeling time varying stock market volatility. ARCH and GARCH are specific models whose parameters can be estimated from data given there is suitable evidence that this type of conditional volatility is present in the data.

In his 1982 paper, Engle presents a Lagrange multiplier test which is capable of detecting ARCH/GARCH effects in time series. Bollerslev (1986b) is the first published paper that documents the estimation of an autoregressive conditionally heteroskedastic model using stock market data. Asymptotically, the Hinich bispectrum test can not distinguish between an i.i.d. process and ARCH/GARCH because the Hinich test[1] is based upon third-order moments, whereas the unconditional moments of ARCH and GARCH are of the fourth-order. Hsieh (1991), and Nelson (1991), present evidence that suggests ARCH-type models do not explain all of the nonlinear dependence in stock returns. Indeed, Diebold and Nason (1990) comment that "It is not clear, however, that the ARCH effects are structural, i.e. that they are a characteristic of the true data-generating process (DGP). Instead, ARCH may indicate misspecification, serving as a proxy for neglected nonlinearities in the conditional mean."

A third widely-used test for detecting dependence has been developed by Brock, Dechert, and Scheinkman (1986). The BDS test makes use of the correlation integral in the test statistic, which gives it an interesting connection with chaos. The ideas behind the BDS test are used to study nonlinear dependence in stock returns in Scheinkman and LeBaron (1986); also see Chapter 3 in Brock, Hsieh and LeBaron (1991), and Hsieh (1991) for specific examples of applying the BDS test statistic to stock market returns. The BDS test is sensitive to all forms of dependence, including serial correlation, and ARCH/GARCH volatility effects.

---

[1]A partial list of the papers which apply the bispectrum test to stock market returns are Hinich and Patterson (1985a), Hinich and Patterson (1985b), Ashley and Patterson (1989), and Hinich and Patterson (1993).

The ARCH and GARCH models are not the only mathematical specification that produces an apparent non stationarity in variance. Consider a quadratic nonlinear moving average (MA) process of the forum:

$$x(t) = e(t) + c_1 e(t-1) \sum_{j=2}^{L} (c_1)^{j-2} e(t-j), \tag{1.1}$$

Where e(t)~NIID(0,1). Figure -1 is a plot of the e(t) process which is the innovation series for (1.1). A plot of a fifteen term quadratic MA driven by the e(t) innovation series is shown in Figure 0; it is not a martingale difference process nor is it a Gaussian normal process. In figures -1 and 0 we have added horizontal dashed lines to indicate the limits of a 95% confidence interval - i.e. ± two estimated standard deviations – around the observations. In Figure 0 the relatively large number of observations falling outside these limits is indicative of the non-Gaussian nature of this process. A histogram (not shown) of the data plotted in Figure 0 indicates that this data is characterized by "fat tails" compared to a Gaussian process.

The idea that nonlinear structure in stock returns would lead to superior predictions of the mean has not born fruit. To date, no successful prediction method has been reported in literature, although there have been serious attempts by various researchers. One problem with developing a prediction method is that the form of the nonlinearity is unknown, thereby forcing researchers to employ approximation techniques. See, for example, Diebold and Nason (1990), and Hsieh (1991), who have applied the nearest neighbor regression technique to the prediction of financial time series. Although these authors conclude that the evident nonlinearities do not appear to be exploitable in the sense of making improved point forecasts, they offer no opinion on the apparent contradiction between the existence of nonlinearity which is not a martingale difference, and their inability to improve upon naive forecasting techniques such as a random walk. Taking a different tack, White (1988) reports that a neural network set up did no better than a simple random walk in predicting IBM stock returns.

It is our conjecture that the failure to exploit nonlinearity in order to make improved point forecasts is a reflection of the episodic nature of the nonlinear phenomenon. We believe that the high power of tests such as the bispectral linearity test and the BDS test masks the episodic appearance and disappearance of nonlinear dependence in stock returns.

The paper applies a new methodology for detecting epoches of transient dependence in pure white noise to the study of intra-day stock returns. Both linear and nonlinear dependence are considered. The method views the candidate stochastic process as a *pure-noise* series[2], that from time to time, due to unknown factors, switches to a dependent stochastic process for some unknown length of time, and then switches back to *pure-noise*. The structure of the dependence follows some unknown linear, or nonlinear, operation on lagged values of the process. The probability model of the dependence is unknown, and may vary from epoch to epoch. The epoch occurrence times are sparse, and their distribution is unknown. The *pure-noise* process need not follow the normal law. The single assumption made about the *pure-noise* process is that it is stationary and has finite moments up to order twelve. Thus the specification of the problem is very loose. Because little is known about the probability structure, standard approaches for estimating parameters of a switching model are not appropriate here.

I. TESTING FOR DEPENDENCE

---

[2]A stationary time series is called *pure-noise* if $x(n_1),...,n_N$. A white-noise time series, by contrast is one for which the autocovariance function is zero for all lags. Whiteness does not imply that x(n) and x(m) are independent for $m \neq n$ unless the series is Gaussian (normal).

A. Overview

The sample design is to pass the data through a non-overlapped window. The data consists of eight time series of stock returns sampled every 10 minutes over a six and one-half year period. The eight series correspond to eight of the thirty DJIA stocks. The length of the window corresponds to one trading day. We look at individual windows because we want to detect episodic events if present.

Each window is tested for dependence under a null hypothesis of *pure-noise*. Three different portmanteau tests are performed: linear - similar to the Box-Pierce correlation test, nonlinear - considers third-order correlations, cross nonlinear - cross third-order correlation between return and volume.

We also test for conditional heteroskedasticity using Engle's Lagrange multiplier test. In keeping with the spirit of ARCH/GARCH, we apply this test at the window level; that is we look for evidence of conditional heteroskedasticity which evolves across windows.

B. Test Statistics

Below we introduce and define each test statistic. The stated distribution of each statistic holds asymptotically. However, the portmanteau tests are run on small samples, 36 observations per window, which makes the asymptotics questionable. Therefore, in Appendix A, we report the size and power of the statistics for a sample size of 36.

Let the sequence $\{x(t_n)\}$ denote the observed sampled data process. Set the time unit such that the sampling interval $t_{n+1}-t_n=1$, and use the standard convention that t is an integer. As stated at the top of this section, the sampling design employs a non-overlapped data window. If n is the window length, the kth window is $\{x(t(k)),x(t(k)+1),...,x(t(k)+n-1)\}$. The next non-overlapped window is {x(t(k+1)),...,x(t(k+1)+n-1)} where t(k+1)=t(k)+n. From now on let t(k) = 1. The statistics used in this paper are computed for each window. The null hypothesis for each window is that the transformed data {x(t)} are realizations of a stationary pure-noise process with finite moments up to order twelve. The alternative hypothesis is that the process in the window is random with some non-zero correlations $cx_2(r)=E[x(t)x(t+r)]$ or non zero bicorrelations $c_{x3}=E[x(t)x(t+r)x(t+s)]$.

B.1. C Statistic

Our C, or correlation, statistic is closely related to the widely used Box-Pierce portmanteau test statistic which detects correlated (non white) noise (Box and Pierce, 1970). The Box-Pierce statistic is usually applied to the residuals of a fitted ARMA model, but here it is a function of the standardized observations. First, define z(t) as:

$$z(t)=[x(t)-m_x]/s_x \tag{1.1}$$

for each t=1,..., n where m_x and s_x are the sample mean and sample standard deviation of the window. Next, define

$$C = \sum_{r=1}^{\ell} (c_{z2}(r))^2 \tag{1.2}$$

where $$c_{z2}(r) = (n-r)^{-1/2} \sum_{t=1}^{n-r} z(t)z(t+r).$$

The Box-Pierce test does not specify the number of lags ℓ to be used; that decision is left to the user. Here, we specify $\ell=n^b$ with $0<b<0.5$. In Box and Pierce (1970) it is proven that C is asymptotically approximately chi-square with ℓ degrees-of-freedom.

B.2. H Statistic

The H statistic tests for certain forms of nonlinearity using third-order correlations. The (r,s) sample bicorrelation is

$$c_{z3}(r,s) = (n-s)^{-1} \sum_{t=1}^{n-s} z(t)z(t+r)z(t+s) \qquad 0 \leq r \leq s. \tag{1.3}$$

Let $G(r,s) = (n-s)^{1/2}\, c_{z3}(r,s)$ and define H as,

$$H = \sum_{s=2}^{\ell} \sum_{r=1}^{s-1} G^2(r,s). \tag{1.4}$$

H is asymptotically distributed under the null as a chi-square with $(\ell-1)/\ell/2$ degrees-of-freedom when $\ell=n^b$, $0<b<0.5$ as above. (See Hinich (1995) for the proof).

The H statistic, which detects third-order correlations, can be considered a generalization of the Box-Pierce portmanteau test. It is our experience that the value b=0.4 for the exponent on n is a good compromise between: 1) using the asymptotic result as a valid approximation for the sampling properties of H for moderate sample sizes, and, 2) having enough sample bicorrelations in the test statistic to have reasonable power against non-independent variates.

B.4. E Statistic

This is a lagrange multiplier test attributed to Engle (1982). Consider the OLS regression of the autocorrelated {x(t)} on its past:

$$x(t) = W_t \beta + e(t) \tag{1.7}$$

where w_t = (1, x(t-1), ..., x(t-m)), and $\beta = (\beta_0, \beta_1, ..., \beta_M)^T$, with M some predetermined lag integer, and T denoting the transpose. Suppose (1.7) is adequate for whitening the x(t)'s, that is, the residuals $\{\hat{e}(t)\}$ from (1.7) can be regarded as white-noise, but not necessarily *pure-noise*.

Construct the sequence of squared residuals $\{\hat{e}^2(t)\}$, and denote it $\{a(t)\}$. Regress a(t), using OLS, on $\{1, a_{t-1}, ..., a_{t-p}\}$:

$$a(t) = U_t \phi + \mu(t) \qquad (1.8)$$

where $\mu(t) = \{1, a_{t-1}, ..., a_{t-p}\}$, and $\phi = (\phi_0, \phi_1 \phi, ..., \phi_p)^T$. Let R^2 be the R-squared of the auxiliary regression (1.8). Then the E test statistic is

$$E = nR^2 \qquad (1.9)$$

with n the number of observations used in (1.8). Note that E has a limit null distribution of chi-square with p degrees-of-freedom.

II. The Data

We analyze 15 of the 30 DJIA stocks for the period January 2, 1980, through August 31, 1985.[3] To illustrate the difference between our window by window methodology and standard time series methodology for making inferences, we revisit the stock data we used in our earlier papers. The included stocks are:

Alcoa, Inc.	International Business Machines
American Express	International Paper
Bethlehem Steel	McDonald's Corp.
Chevron Corp.	Merck & Co.
Coca-Cola Co.	Minnesota Mining and Manufacturing Co.
Eastman Kodak	
General Motors	Proctor & Gamble o.
Goodyear Tire and Rubber	United Technologies Corp.

The source data is in the form of trade-by-trade prices provided by Fitch Investor Services. Each price series was first adjusted for dividends and stock splits. Next, each series was smoothed using

---

[3]The exception is McDonald's, where the period is January 2, 1980, through December 31, 1985.

a Tukey filter. This removes low-frequency or trend components. Last, each series was sampled every 10 minutes, and these sampled prices converted to returns by taking the first-difference of log prices. A 10 minute sample interval produces 36 returns per trading day. The data window is 36 observations long, and covers exactly a single trading day, with no overlap unless the stock failed to open at 10:10 a.m. or earlier. The first sample return for each day is at 10:10 a.m. and the last is at 4:00 p.m. Each stock series contains approximately 1,435 daily windows. From equation (1.4), b is set to 0.4 which yields ℓ=4 correlations and six bicorrelations for each day. The mean, standard deviation (sig), skewness, kurtosis (k4), maximum, and minimum were computed for each window along with the two test statistics H and C. The standard deviation, skewness, kurtosis, and range (max-min) were normalized by dividing each value by their sample values computed from the whole sample period. The H and C statistics are transformed by the cdf of the standard normal N(0,1) so that they have a uniform U(0,1) distribution if the process is pure noise and when asymptotic normality applies for their distributions. The descriptive statistics over the whole period for these eight stocks are shown in Table 1. All the returns series are highly leptokurtic.

Low-pass filtering the returns is necessary in order to mitigate the influence of aliasing on the time series statistics of the data. In other words, if a time series is not independent increments, then proper sampling technique requires that the data be band limited before sampling; see Hinich and Patterson (1989) for a discussion sampling high-frequency stock data.[4]

---

[4]In Hinich and Patterson (1989) we use tick data to study the same stocks as included here. The time period covered in our 1989 paper was September 1978 through August 31, 1981, and we employed a 15 minute sampler rather than a 10 minute sampler. This paper applies more statistical tests than our earlier paper, and, among other differences, here we examine individual days rather than the entire record as a single series. Further, we didn't consider nonlinear cross coupling between volume and returns in our 1989 paper.

Given that the data consists of the sequence of traded prices, some readers may be concerned that bid-ask bounce and the possible sequential execution of limit orders is going to confound our statistical tests, especially the correlation test. However, these effects are mitigated for three reasons: (1) low-pass filtering (bid-ask bounce is a high frequency phenomenon), (2) skip sampling of prices, and (3) this sample of stocks (DJIA members) is frequently traded. In Section III we will return to this issue, and present some direct evidence that bid-ask bounce is not influencing our statistical results in any significant sense.

III. Intra Window Results

In part A of this section we present the results from applying the C and H tests to the univariate return series using a one-day data window. These results are summarized in Table 2. The evidence to be presented in subsection A supports the assertion that the stock generating process is non stationary in return dependence. We will see that the rejection of the i.i.d. null using either the C or H statistic is episodic in nature, with the null being accepted most of the time. In part B we consider nonlinear cross-coupling between trading volume and returns through the HXY statistic. Part C deals with various econometric issues, including bid-ask bounce.

A. Returns

The C and H statistics are calculated for each data window. A window covers a trading day, and contains 36 non overlapping observations. Four lags were used in calculating the C and H statistics. A 1% significance level is used in reporting the results.

A.1. C Statistic

The third column in Table 2 displays the number of windows where the null of i.i.d. returns is rejected by the C statistic. In parenthesis is the percentage of the total number of windows (days) where rejection occurs. For example, in the case of Coke, the null was rejected on 48 days, which is equivalent to 3.3% of the days tested. By random chance, we would be expected to reject 1% of the days at the 1% level, so in fact our rejection rate is more than three times greater than expected under the null. IBM and GM, and possibly McDonald's, are stocks where evidence against the null is weak. For the remaining four stocks. the evidence against the null is reasonably strong.

The portmanteau correlation statistic, C, is calculated from estimated correlations up to and including lag 4 within a window. The computer software that implements the tests prints out the correlation estimates for each window where C was significant. Typically, the largest correlations were of the order 0.10 to 0.50. Likewise, the smallest correlations fell in the range -0.10 to -0.50. Occasionally, magnitudes as high as 0.70 were observed.

The occurrence of days on which the C statistic rejects tends to be isolated with infrequent clusters of two or three significant days in sequence. Figure 1(a) is typical of the patterns we observed. It is a plot of one minus the prob-value level of the C statistic for Coca-Cola during 1983. We report the P-values this way for plotting purposes so that a very significant outcome is plotted as a value near 1.0. For clarity, we only show those days when one minus the P-value is $\geq$ .095. Note that a few of the plotted values are very close to 1.00.

A.2. Correlation Across Windows

In this subsection we consider the following question: Is it possible that there exist patterns in the returns which extend across windows which exhibit a significant H statistic? One approach to pattern detection is as follows. Suppose there is a significant positive correlation between the returns at lag 9 and 22 in a particular (significant) window. It would be interesting to know if lags

9 and 22 are also correlated in other significant C or H windows. As an example calculate the correlation measure

$$\hat{\rho}(9,22) = \frac{1}{M} \sum_{k=0}^{M} r(36k+9)\, r(36k+22) \qquad (3.1)$$

where M is the number of significant H windows, and the significant windows are grouped sequentially. More generally, estimate the cross-window correlations for all possible combinations of i, j ($1 \leq i, j \leq 36$). Table 3 shows the 10 largest (positive) and 10 smallest (negative) correlations for each stock in the study (all correlations are nominally significant at the 1% level). The magnitude of the correlations is surprisingly high, although not out of line with the intra-window correlations at lags 1, 2, 3, and 4. Of course, most of the windows included in the calculations of Table 3 did not have a significant C statistic. Turning to the negative correlations, notice that many are located at adjacent lags, which indicates strong reversals over the two 10-minute intervals.

The reader is advised to use caution in the interpretation of the evidence of correlation presented in Table 3. Because the correlation measure is non standard, and because all possible lag pairs were considered, the sampling properties of the estimator are not clear. As a rough check on how many significant correlations we might expect by chance, the exercise was repeated using only insignificant windows. Note: the number of significant windows was always less than 100, whereas the number of insignificant windows was around 1,400. With only the insignificant windows, the magnitudes of the correlations are smaller than those shown in Table 3. The positive correlations are generally smaller than 0.10, and the negative correlations fall in the range -0.10 to -0.15. Again, some of the negative correlations are located at adjacent lags. Therefore, it can be concluded that there exist patterns of dependence across windows. However, exactly what these patterns mean is not clear. Whether the observed reversals are linked to bid-ask bounce, despite the filtering procedure, requires additional investigation, which is reported in Section C.1 below. For now it can be said that it is unlikely that bid-ask bounce can explain the reversals. Remember that the reversals occur at the same time on different days where the particular days in questions have in common the fact that the H statistic is significant. Although some reversals can be observed on other days, the strength of the correlation is lower.

A.3. H Statistic

The H statistic is used as a nonlinearity test derived from the sample bicovariance function. One can think of the test as measuring the cross-correlation between the level and the correlations. Refer to Table 2 for a summary of the results. At the 1% level, we reject the null in 1.8% of the windows for IBM and up to 4.3% of the windows for Proctor and Gamble, and the nonlinearity test rejects more frequently than the correlation test. When linearity is rejected for a window, the rejection tends to occur at a very high probability level despite the small sample size (36 observations per window). This is brought out in Figure 1(b) where we plot the probability levels for Coca-Cola during 1983.

In Figure 2, graphical evidence is presented that supports the idea that returns behave differently on days when the H statistic rejects, than on days when the null is not rejected. The figure is analogous to using a scatter plot of a variable versus its lagged value to spot correlation. Here the plot consists of lagged cross-products versus the contemporaneous value of the return, for example, x(k-1)x(k-2) versus x(k). Because the H statistic considers lags 1 through 4, all possible cross-products of returns involving lags 1-4 are considered. The process involves calculating the absolute value of the products, summing these six cross products together, and plotting the sum against the contemporaneous return. In figure 2, the returns are drawn from three consecutive days when the H statistic rejected for McDonald's Corporation. Compare this with Figure 3, where we have repeated the experiment over three consecutive days in the same month when the H statistic did not reject. It can be seen in Figure 3 that the points cluster around the origin, in sharp contrast to Figure 2.

In summary, the evidence seen in Subsection A shows that that intraday security returns exhibit episodes of significant dependence followed by periods where returns appear to be identically and independently distributed.

B. Bid-Ask Bounce and Other Issues

In this final subsection of Section III, the question of bid-ask bounce is again considered, as well as issues such as the correlation between test statistics, and the possible impact of poorly behaved higher-order moments on the reported results.

B.1. C and H Tests Under Bid Prices

As is well known, there are certain situations where the price of a stock may sequentially bounce between the specialist's "bid" price and the specialist's "ask" price. Clearly, such behavior will cause negative correlation in sequential returns. If left unchecked, such negative correlation would confound the C statistic, and possibly the H statistic. Time series statisticians regard these frequent reversals as a source of high frequency noise, which in turn, can be filtered from the data using a low-pass digital filter. A filtering approach to the issue is taken here. In particular, a Tukey filter with a base width of 15 minutes is applied to the data. As a check on the adequacy of this method, we ran the C and H tests using bid prices for McDonald's during 1983.[5] Table 5 compares the C and H test statistics when using traded prices versus bid prices (because of missing days in the data base containing the bid prices, that sample size is slightly smaller). As shown in the table, the rejection frequencies using the C statistic are virtually identical. Surprisingly, the rejection frequency for the H statistic is slightly higher using bid prices. Why this is the case is not known, but it can be concluded that bid-ask bounce does not appear to be responsible for intraday rejections by either the C statistic or the H statistic.

B.2. Other Econometric Issues

Readers familiar with the statistics of stock returns know that they are characterized by heavy tails vis-à-via the normal distribution. Eugene Fama (1968) carefully documented this behavior in the 30 DJIA stocks 30 years ago and explained that such a characteristic is referred to as leptokurtosis. Fama argued that a Stable Paretian distribution with a characteristic exponent less than 2.0 fit the data much better than a normal distribution. One immediate consequence of this proposition is that the second moment does not exist. Fama explained that non existence would manifest itself in the form of a poorly behaved variance estimator, and, indeed, provided some evidence that for his sample of stocks the variance did appear to be poorly behaved. We provide this background not because we want to enter the argument, still being carried on today, over the existence of the second moment in stock returns, but rather to focus on the more narrow issue of

---

[5]Bid prices came from the ISSM tapes whereas the data employed is from Fitch Investor services. The data available to us with ISSM tapes does not cover the entire period included in this study. We also found a number of missing days for those stocks and years where bid prices were available. For these reasons it was decided to only replicate the tests for McDonald's Corp. in 1983.

whether or not the variance and higher order moments are sufficiently ill-behaved in our data such that they are causing false rejections by the H statistics.

Evidence on this question is presented in Table 6. For those windows where the H statistic is significant at 1% the table shows correlation of H and C, the correlation of H and standard deviation, the correlation of H and kurtosis, and finally, the correlation of H with range. The correlation of H and C is included because we are interested in whether or not correlation and nonlinearity tend to occur in sympathy (asymptotically, the two test statistics are independent under the null). The two standard error interval in this table is approximately 0.3. Correlation between H and C appears to be the case for Coke, IBM, and GM, to some degree, but not the case for the other stocks. Next, if moment behavior is a problem, one would expect to see positive correlation between H and various measures of dispersion such as standard deviation, kurtosis, and range. But from Table 6 it can be seen that this is not the case. Regarding sigma, Coke has a moderate correlation, but of the wrong sign. The correlation of H with kurtosis tend to be low, but again of the wrong sign, and the sample correlations of H and range are mostly small, and with one exception of the wrong sign. Given the above evidence it is difficult to see how poorly behaved moments could be driving our nonlinearity results in any meaningful way.

IV. Inter Window Results

A. The E Statistic

The Autoregressive Conditional Heteroskedastic (ARCH), and the Generalized Autoregressive Condition Heteroskedastic (GARCH) are two widely studied nonlinear time series models. These models have often been fitted to financial time series, including common stock returns, and have a natural appeal to financial economists because they are capable of explaining (time varying) conditional volatility; see, for example, Schwert and Sequin (1990).

Good econometric practice requires identifying the order(s) of ARCH/GARCH processes before model estimation. Engle's lagrange multiplier test is a popular method for identification. Here his test is used to detect the likely presence of ARCH/GARCH in the data. The spirit of these models is to capture the relatively slow (longer than a day) evolution of volatility changes. Hence, one is interested in testing for the presence of ARCH/GARCH in daily returns, rather than the 10 minute returns. The variate under test is the standardized sample variance of the returns in each window (a little more than 1400 windows for each stock).

For each stock the E (Engle) test statistic is calculated over the entire sample, and then over 7 subgroups of about 214 days each. Table 7 displays the result of the experiment (the test statistic is chi-square with 11 d.f.). Regarding the entire sample, all series wildly reject at the 1% level, with the exception of United Technologies. The subgroups, however, tell a different story. Here we again see evidence of episodic nonlinearity, as was the case with the H statistic. IBM and General Motors reject the null in five of the seven subgroups, and the other stocks reject less often. It should also be noted that there is no pattern with regard to which subgroups reject. Although United Technologies did <u>not</u> reject for the entire sample, subgroups 3 and 6 did reject at the 1% level.

B. Is ARCH/GARCH Responsible for Rejection of the Null by the H Statistic?

Because the presented evidence suggests the presence of third-order nonlinearity (H statistic) and fourth-order nonlinearity (E statistic) it is natural to speculate as to whether or not fourth-order nonlinearity (i.e. ARCH/GARCH) is causing false alarms in the H test. Two methods are employed to investigate this question. First, a computer simulation of a GARCH model is carried out, and the size of the H statistic reported. Second, the simulated GARCH data is transformed to a binary series (0,1). Because ARCH/GARCH is a martingale difference process, the transformed data should be a bernoulli sequence if the original distribution is symmetric.

All the ARCH/GARCH specifications have the following structure: x(t) = h(x,e)e(t), where {e(t)} is a zero mean stationary pure-noise process, and h(x,e) is a positive functional of {x(s), e(s); s=t-1,t-2,...}. The process is ARCH if the functional only depends on e(s), and is GARCH if it depends on x(s) and e(s) for s<t. Thus {x(t)} is a stationary martingale difference process since the conditional expectation of x(t) given x(s) for s<t is zero. For example, a GARCH(1,1) model is x(t)=h(t)e(t), where $h(t) = [b_0+b_e e^2(t)+b_h h(t-1)]^{1/2}$ where b_0, b_e, and b_h are positive parameters. In most applications it is assumed that the e's are gaussian.

In order to get a handle on the validity of the large sample approximation for the H statistic, a GARCH(1,1) model with parameters b_0=0.0108, b_e=0.1244, and b_h=0.8516 was simulated. These parameters are typical of a GARCH(1,1) fit to financial time series. The same three noise processes used in the simulations of Appendix B are employed: gaussian, exponential, and uniform. For 1000 repetitions of the GARCH(1,1) with 51,622 observations (1433 windows) per repetition, the H statistic rejected for 1.4% of the windows. This is within the two standard error band for a 1% test, so we conclude that the H statistic has the appropriate size in the face of a GARCH(1,1).

The second way of looking at the false alarm question is with the binary transformation. Define the following transformation of {x(t)} to {y(t): y(t) = 1 if x(t) $\geq$ 0 and y(t) = -1 if x(t) < 0}. Thus if {x(t)} is a ARCH or GARCH process with a symmetric distribution, the {y(t)} is a stationary binary pure noise process. Although a GARCH(1,1) is a martingale difference, and thus white noise, it is not pure-white noise. The maintained hypothesis of this paper is therefore not true for

GARCH(1,1). However the binary transformation turns an ARCH/GARCH into a pure noise process whose cumulants are very well behaved with respect to the asymptotic theory presented in Appendix A. Therefore, if the C or H statistics are significant for a sample, and if their large sample distribution is valid, then the presence of ARCH/GARCH effects cannot be used to explain the rejections.

The GARCH(1,1) model described above was used to simulate the source process. The x(t)'s so generated were then transformed to binary variates as defined above. The estimates of the size for six thousand replications of length n=36, and length n=50, are given in Table 8. The sizes are conservative.

V. Binary Stock Data

The binary transformation of Section IV.B. can be easily applied to the 10 minute stock returns. Doing so will provide another check on the validity of the results reported earlier for intra day returns. The exercise is of independent interest because the C test can be interpreted as a "runs" test. That is to say, in a runs test of a stock series a plus is assigned if the stock price increased over an interval, and a minus if it declines. A run is defined as a sequence of pluses, or a sequence of minuses. The number of runs observed is compared to the number expected under a sequence of bernoulli trials. If an observed binary sequence passes a runs test, it will also not cause the C test to reject. Next, we report the results of the C and H tests when the stock market data is transformed to binary sequences.

The C and H statistics were computed for a sequence of n=36 daily windows using the binary return data. The days were the same as used for the real returns. The number of significant windows for the binary data is given in Table 9. Please note that the number of rejections is smaller than the number reported for the real returns but nevertheless much larger than the nominal level of 1.0%.

The pattern of significant windows is different for the binary data as compared with the patterns for the real returns. A larger fraction of the rejections is attributed to the C statistic in the binary data. Figure 4(a) shows a comparison with the pattern for the C's shown in Figure 1 for Proctor and Gamble. Only a minority of the windows have statistically significant C's for both the real and the binary data. The same holds for the H's (Figure 4(b)). Yet the results for the binary data suggest that these data series are not statistically independent, that is, they are not generated by a random walk.

The maintained hypothesis after the binary transformation of the data remains iid. Therefore, it is useful to examine the correlation pattern of the binary data. The results for Coca-Cola are typical of what was found. The mean of the sample $C_{z2}(1)$ (see equation (1.2)) for the 49 significant windows is $<C_{z2}(1)>=-0.30$, with a standard deviation of 0.30. This mean is slightly biased towards zero since some of the rejections were due to the H rather than the C statistic. The mean of $C_{z2}(1)$ for the 1380 non significant windows is $<C_{z2}>=-0.12$ with a standard deviation of 0.167. Thus the standard errors for the means are 0.039 and 0.005 respectively, and the difference between the means is clearly statistically significant.

The results for the other three correlations are as follows: $<C_{z2}(2)>=0.25$ for the significant windows and $<C_{z2}(2)>=0.00$ for the rest (standard dev's of 0.19 and 0.166 respectively), $<C_{z2}(3)>=-0.14$ for the significant windows and $<C_{z2}(3)>=-0.05$ for the rest (standard dev's of 0.31 and 0.165), and $<C_{z2}(4)>=0.10$ for the significant and $<C_{z2}(4)>=-0.05$ for the rest (standard dev's of 0.29 and 0.171). Only four lags are used for a window size of 36. Windows of size n=72 and 108 were also considered and it was found that the correlations for significant windows have at most a lag of seven periods, which is 70 minutes. These results indicate that the signs of price changes in the 10 minute returns cannot be regarded as bernoulli sequences, although the dependence may die out quickly.

5. CONCLUSION

The portmanteau bicorrelation test of Hinich (1995) and a modified Box-Pierce test have been used to detect epoches of transient dependence in eight high frequency time series of stock returns. The results provide evidence that there is some form of nonstationarity in the underlying generating process. The results can be regarded as consistent with a switching process. There appear to be epoches of dependence in the ten-minute returns followed by long periods where no evidence of second or third-order dependence is observed in the data. Significant short term correlations and bicorrelations appear for certain days in the binary sign sequences.

The lagrange multiplier test of Engle (1982) provided additional evidence of nonlinearity in stock returns at daily intervals. This nonlinearity is consistent with the presence of ARCH/GARCH effects in the return generating process. There appears to be no reason to believe that ARCH or GARCH is responsible for the rejections by the bicorrelation portmanteau tests.

REFERENCES

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. **Journal of Econometrics** 31, 307-327.

Engle, R.F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation, **Econometrica** 50, 987-1007.

Box, G.E.P. and Pierce, D.A. (1970) Distribution of residual autocorrelations in autoregressive-integrated moving average time series models. **J. Amer. Statist. Assoc.** 65, 1509-1526.

Harvey, A.C. (1989) **Forecasting, Structural Time Series Models and the Kalman Filter**. Cambridge: Cambridge University Press.

Hinich, M.J. and Patterson, D.M. (1989) Evidence of linearity in the trade-by-trade stock market return generating process. Chapter 16, **Economics Complexity: Chaos, Sunspots, Bubbles, and Nonlinearity**, ed. by W.A. Barnett, J. Geweke, and K. Shell. New York: Cambridge University Press, 383-409.

Hinich, M.J. and Patterson, D.M. (1993) Intra-day nonlinear behavior of stock prices. ed. by R. Day, and P. Chen, **Nonlinear Dynamics and Evolutionary Economics**. Oxford: Oxford University Press.

Hinich, M.J. (1995) Testing for dependence in the input to a linear time series model, forthcoming, **J. Nonparametric Statistics**.

__________ (1999) Sampling Dynamical Systems Macro Economic Dynamics, 3, 602-609.

Schwert, G.W. and Seguin,P.J. (1990) Heteroskedasticity in stock returns, **Journal of Finance**, 45 1129-1155.

APPENDIX

The purpose of this appendix is to provide evidence concerning the size and power of the C and H tests. Assume that {x(t)} has non-zero bicorrelations. Then under the alternative hypothesis, $G(r,s)=(n-s)^{1/2}c_{z3}(r,s)+O(1)$ by applying the central limit theorem for dependent variates to equation (1.4) in the text. Thus $G^2(r,s)=(n-s)(c_{z3}(r,s))^2+O(n^{1/2})$, and consequently $H=(n/\ell)B^2[1+O(1/\ell\, n^{1/2})]$ where

$$B^2 = \sum_{s=2}^{\ell}(1-s/n)\sum_{r=1}^{s-1}(c_{z3}(r,s))^2.$$

The power of the H test will be near one if the $n^{(1-c)}B^2$ is large, and clearly the test is consistent as $n\to\infty$ for any alternative with at least one non-zero bicorrelation. A similar results holds for C as long as there are some non-zero correlations for the alternative. Asymptotic theory does not give much insight into whether the asymptotic normal distribution for H applies for a given sample size. In order to provide such insight, we turn to artificial data analysis.

The sizes (Type I errors) of the H and C tests were estimated for three sample sizes (n=50, 200, 1000) and for three types of distributions for pure noise $\epsilon(k)$'s - normal N(0,1), one tailed exponential, and uniform U(0,1) - using six thousand (6000) replications. The $\epsilon(k)$ variates were generated using the IMSL pseudo-random number generators GGNML, GEXP, and GGUB. Table 1 shows the estimated sizes of the H and C statistics for the levels of the 1% and 5% nominal sizes using the N(0,1) approximation. The sizes are well approximated by the tests even for a sample size of 50.

In order to obtain some idea of the power of the H and C tests, pseudo-random x(k) variates were generated by the following nonlinear autoregressive model which has non-zero bicorrelations as well as significant skewness and kurtosis. Let $\{\epsilon(k)\}$ be a pure noise process where t_k-k for all integers. Then the nonlinear AR model is $x(k)=ax(k-2)x(k-1)+\epsilon(k)$ if $|ax(k-2)|<1$ or $x(k)=\epsilon(k)$ otherwise. For a=0.50 this model has a correlation at lag three of about 0.42.

Table 2 shows the estimated power of the H statistic for the 1% and 5% levels using the N(0,1) approximation. As with the size, the $\epsilon(k)$'s were computed using the normal, exponential, and uniform pseudo-random generators and n=50, 200, and 1000. The scales used were a=0.10, 0.25, and 0.50. The H test correctly rejected more than 50% of the runs at the 5% level for n=50 and a=0.50. It correctly rejected about 50% of the runs for n=200 at the 5% level for all three values of a.

Although the model generates few non-zero correlations, the C test correctly rejected the null hypothesis of pure noise at a 50% rate for n=50 and a=0.50. The power of the C test increases with increasing n, as is indicated by the theoretical calculation for the power.

Table 1. Summary Statistics of 10 Minute Returns for the Fifteen DJIA Stocks

Name	Mean %	Stand Dev %	Skew	Kurtosis	Max %	Min %	Sample Size-Days
Alcoa	7.15E-04	0.276	0.36	10.2	4.07	-3.86	51516-1431
American Express	1.71E-03	0.288	-0.050	12.0	3.50	-5.46	51516-1431
Bethlehem Steel	1.15E-04	0.299	0.267	14.9	5.86	-6.34	51552-1432
Chevron	9.29E-04	0.264	1.17	40.6	8.82	-4.34	51516-1431
Coke	1.42E-03	0.217	-0.174	14.2	2.64	-4.72	51622-1432
GM	8.90E-04	0.191	0.097	17.3	2.72	-4.70	51622-1432
Goodyear	1.56E-03	0.297	-0.019	10.2	3.94	-6.61	51552-1432
IBM	1.26E-03	0.169	0.467	12.3	2.91	-2.34	51585-1431
Int. Paper	8.26E-04	0.225	0.507	21.1	4.82	-4.18	51552-1432
Kodak	8.36E-04	0.191	0.10	14.2	2.91	-3.72	51552-1432
McDonalds	1.95E-03	0.205	1.06	36.6	5.74	-3.00	54540-1515
Merck	8.64E-04	0.194	0.303	9.8	2.63	-2.22	51622-1432
MMM	9.32E-04	0.196	0.350	9.4	3.21	-2.19	51622-1432
Proctor & Gamble	9.57E-04	0.188	0.514	14.5	4.78	-2.02	51586-1431
United Tech	1.19E-03	0.241	1.76	76.9	10.10	-4.52	51549-1430

Table 2. C and H Statistics Using a Threshold of 0.10%

Name	Total Number Windows (Days)	Significant[1] C Windows	Significant[1] H Windows
Alcoa	1431	11 (0.77%)	26 (1.82%)
American Express	1431	3 (0.21%)	6 (0.42%)
Bethlehem Steel	1432	13 (0.19%)	35 (2.44%)
Chevron	1431	10 (0.70%)	14 (0.98%)
Coke	1432	13 (0.91%)	16 (1.12%)
GM	1432	4 (0.28%)	26 (1.82%)
Goodyear	1432	21 (1.47%)	24 (1.68%)
IBM	1431	6 (0.42%)	3 (0.21%)
Int. Paper	1432	6 (0.42%)	31 (2.16%)
Kodak	1432	5 (0.35%)	7 (0.49%)
McDonalds	1515	4 (0.26%)	23 (1.52%)
Merck	1432	9 (0.63%)	16 (1.12%)
MMM	1432	11 (0.77%)	18 (1.26%)
Proctor & Gamble	1431	7 (0.49%)	23 (1.61%)
United Tech	1430	7 (0.49%)	23 (1.61%)

[1]Four lags used in calculating C and H portmanteau statistics.

Table 3. Test for patterns across H windows which are significant at 5.0%. Shown are the largest statistically significant correlations (10 positive and 10 negative), when the ith return, r(i), in a significant window is correlated with return r(j) in all other significant windows.

Alcoa	(i,j)	=	(30,31)	(1,15)	(21,24)	(31,35)	(14,17)	(30,36)	(22,25)	(3,23)	(2,23)	(3,24)
+	$\hat{\rho}$	=	0.36	0.27	0.27	0.26	0.26	0.26	0.25	0.24	0.22	0.22
	(i,j)	=	(12,13)	(6, 7)	(33,34)	(18,19)	(17,18)	(26,29)	(2, 4)	(16,17)	(3, 4)	(24,30)
-	$\hat{\rho}$	=	-0.57	-0.43	-0.38	-0.35	-0.33	-0.32	-0.30	-0.29	-0.29	-0.28
American Express	(i,j)	=	(18,34)	(28,21)	(9,12)	(2, 3)	(3,13)	(6,36)	(26,29)	(1,10)	(6,13)	(2,13)
+	$\hat{\rho}$	=	0.46	0.37	0.32	0.32	0.32	0.31	0.31	0.30	0.29	0.29
	(i,j)	=	(29,30)	(6, 7)	(18,20)	(1, 4)	(20,34)	(4,10)	(5, 6)	(23,27)	(13,14)	(32,36)
-	$\hat{\rho}$	=	-0.49	-0.48	-0.48	-0.48	-0.41	-0.35	-0.35	-0.34	-0.34	-0.33
Bethlehem Steel	(i,j)	=	(10,13)	(1, 2)	(13,23)	(2,18)	(25,28)	(14,20)	(28,31)	(10,22)	(7,12)	(17,28)
+	$\hat{\rho}$	=	0.32	0.28	0.27	0.24	0.22	0.22	0.21	0.21	0.20	0.20
	(i,j)	=	(5, 6)	(10,11)	(35,36)	(31,32)	(18,20)	(26,27)	(16,17)	(29,30)	(22,24)	(19,20)
-	$\hat{\rho}$	=	-0.52	-0.52	-0.46	-0.42	-0.42	-0.41	-0.39	-0.35	-0.34	-0.34
Chevron	(i,j)	=	(12,13)	(7, 8)	(17,30)	(13,16)	(31,36)	(31,34)	(13,30)	(5,28)	(12,28)	(15,30)
+	$\hat{\rho}$	=	0.63	0.53	0.47	0.40	0.38	0.38	0.37	0.31	0.31	0.30
	(i,j)	=	(13,14)	(28,29)	(12,14)	(7, 9)	(33,34)	(22,23)	(19,20)	(20,21)	(17,18)	(12,22)
-	$\hat{\rho}$	=	-0.57	-0.43	-0.43	-0.41	-0.40	-0.39	-0.39	-0.37	-0.35	-0.34

Table 3. Test for patterns across H windows which are significant at 5.0%. Shown are the largest statistically significant correlations (10 positive and 10 negative), when the ith return, r(i), in a significant window is correlated with return r(j) in all other significant windows.

Coke	(i,j) =	(20,29)	(5, 6)	(19,22)	(6,20)	(9,32)	(15,22)	(19,26)	(11,34)	(28,34)	(5,25)
+	$\hat{\rho}$ =	0.32	0.31	0.30	0.28	0.26	0.25	0.25	0.23	0.23	0.22
	(i,j) =	(20,21)	(6, 7)	(10,11)	(24,26)	(22,23)	(7, 8)	(32,33)	(11,36)	(21,22)	(15,16)
-	$\hat{\rho}$ =	-0.46	-0.46	-0.41	-0.39	-0.39	-0.34	-0.34	-0.31	-0.30	-0.30
GM	(i,j) =	(17,28)	(22,23)	(1, 2)	(27,28)	(24,29)	(19,25)	(9,15)	(3,23)	(25,27)	(14,18)
+	$\hat{\rho}$ =	0.39	0.37	0.37	0.29	0.29	0.27	0.27	0.25	0.25	0.25
	(i,j) =	(20,24)	(8,12)	(22,25)	(24,27)	(6,10)	(5, 6)	(1,23)	(23,28)	(1, 3)	(23,27)
-	$\hat{\rho}$ =	-0.40	-0.38	-0.35	-0.34	-0.31	-0.31	-0.30	-0.30	-0.28	-0.28
Goodyear	(i,j) =	(19,22)	(8,11)	(28,34)	(28,31)	(12,28)	(29,32)	(8,26)	(20,25)	(25,29)	(11,36)
+	$\hat{\rho}$ =	0.33	0.33	0.32	0.30	0.29	0.25	0.25	0.24	0.23	0.23
	(i,j) =	(19,20)	(5, 6)	(22,23)	(35,36)	(28,29)	(4, 5)	(20,22)	(11,12)	(17,18)	(13,14)
-	$\hat{\rho}$ =	-0.47	-0.47	-0.46	-0.45	-0.45	-0.40	-0.40	-0.40	-0.39	-0.39
IBM	(i,j) =	(35,36)	(34,35)	(34,36)	(21,36)	(32,35)	(16,12)	(11,13)	(30,31)	(21,35)	(20,36)
+	$\hat{\rho}$ =	0.71	0.68	0.60	0.48	0.42	0.41	0.40	0.39	0.37	0.35
	(i,j) =	(12,13)	(10,11)	(10,13)	(11,12)	(16,17)	(8, 9)	(31,32)	(23,36)	(25,35)	(5, 9)
-	$\hat{\rho}$ =	-0.64	-0.57	-0.52	-0.50	-0.38	-0.38	-0.38	-0.36	-0.35	-0.35

Table 3. Test for patterns across H windows which are significant at 5.0%. Shown are the largest statistically significant correlations (10 positive and 10 negative), when the ith return, r(i), in a significant window is correlated with return r(j) in all other significant windows.

Int.Paper	(i,j)	=	(20,21)	(31,35)	(31,33)	(1, 2)	(25,34)	(20,25)	(2,15)	(21,25)	(13,17)	(33,35)
+	ρ̂	=	0.69	0.48	0.43	0.29	0.28	0.28	0.27	0.27	0.26	0.26
	(i,j)	=	(21,22)	(20,22)	(18,19)	(8,11)	(35,36)	(31,36)	(2, 6)	(15,17)	(16,18)	(19,27)
-	ρ̂	=	-0.58	-0.47	-0.40	-0.37	-0.29	-0.29	-0.29	-0.27	-0.27	-0.27
Kodak	(i,j)	=	(29,35)	(30,33)	(32,33)	(27,36)	(18,23)	(8,14)	(16,20)	(27,33)	(24,29)	(2,17)
+	ρ̂	=	0.57	0.42	0.41	0.38	0.32	0.31	0.30	0.30	0.30	0.30
	(i,j)	=	(27,35)	(22,24)	(24,27)	(23,29)	(33,34)	(14,15)	(33,35)	(22,35)	(27,29)	(2, 3)
-	ρ̂	=	-0.49	-0.49	-0.44	-0.43	-0.43	-0.40	-0.37	-0.37	-0.33	-0.33
Mc-Donalds	(i,j)	=	(11,28)	(21,33)	(13,19)	(9,36)	(9,17)	(23,29)	(20,29)	(9,12)	(9,10)	(6,17)
+	ρ̂	=	0.36	0.32	0.27	0.27	0.27	0.27	0.26	0.25	0.25	0.24
	(i,j)	=	(33,34)	(12,14)	(31,34)	(19,23)	(7, 8)	(32,34)	(13,14)	(27,29)	(28,30)	(10,14)
-	ρ̂	=	-0.37	-0.35	-0.31	-0.31	-0.30	-0.30	-0.30	-0.29	-0.29	-0.27
Merck	(i,j)	=	(12,14)	(14,15)	(19,22)	(31,33)	(2,18)	(31,36)	(10,14)	(31,32)	(4,26)	(22,31)
+	ρ̂	=	0.45	0.34	0.33	0.28	0.28	0.27	0.25	0.25	0.24	0.24
	(i,j)	=	(14,16)	(15,16)	(20,21)	(12,16)	(21,22)	(1, 5)	(17,18)	(6, 7)	(4, 5)	(35,36)
-	ρ̂	=	-0.64	-0.56	-0.46	-0.43	-0.40	-0.38	-0.33	-0.32	-0.30	-0.28

Table 3. Test for patterns across H windows which are significant at 5.0%. Shown are the largest statistically significant correlations (10 positive and 10 negative), when the ith return, r(i), in a significant window is correlated with return r(j) in all other significant windows.

MMM	(i,j)	=	(13,15)	(4, 5)	(1, 2)	(15,17)	(30,34)	(28,33)	(8,17)	(4,25)	(2,34)	(17,32)
+	$\hat{\rho}$	=	0.36	0.36	0.31	0.31	0.29	0.28	0.26	0.25	0.24	0.24
	(i,j)	=	(18,19)	(5, 6)	(9,10)	(15,19)	(7, 8)	(30,35)	(4, 6)	(10,11)	(10,18)	(21,23)
-	$\hat{\rho}$	=	-0.58	-0.50	-0.45	-0.37	-0.36	-0.35	-0.33	-0.32	-0.31	-0.31
Proctor & Gamble	(i,j)	=	(5, 6)	(23,26)	(5,18)	(33,36)	(4,21)	(9,14)	(31,34)	(9,11)	(11,22)	(29,36)
+	$\hat{\rho}$	=	0.37	0.28	0.28	0.27	0.26	0.25	0.25	0.24	0.24	0.23
	(i,j)	=	(14,15)	(6, 7)	(5, 7)	(17,18)	(31,35)	(34,35)	(15,18)	(7,20)	(7,21)	(11,29)
-	$\hat{\rho}$	=	-0.45	-0.40	-0.38	-0.37	-0.37	-0.37	-0.33	-0.30	-0.28	-0.27
United Technologies	(i,j)	=	(11,13)	(15,26)	(26,35)	(10,11)	(12,28)	(23,34)	(27,32)	(3, 4)	(10,23)	(11,20)
+	$\hat{\rho}$	=	0.34	0.30	0.30	0.29	0.29	0.28	0.27	0.27	0.26	0.25
	(i,j)	=	(11,12)	(14,15)	(15,16)	(22,23)	(21,22)	(9,26)	(6,10)	(32,35)	(14,17)	(25,29)
-	$\hat{\rho}$	=	-0.72	-0.48	-0.47	-0.40	-0.40	-0.35	-0.33	-0.33	-0.33	-0.32

Table 4. Comparison of C and H Tests Using Bid Prices Rather Than Traded Prices. McDonalds Corp., 1983. Threshold is 0.1%.

	Traded Prices 252 Days	**Bid Prices 237 Days**
Significant C Windows	2(0.8%)	1(0.4%)
Significant H Windows	3(1.2%)	10(4.2%)

Table 5. Correlations of Statistics for Significant H Windows Using a Threshold of 0.10%

Name	Cor(H,C)	Cor(H,Sig)	Cor(H,KURT)	Cor(H,Range)
Alcoa	0.298	-0.106	-0.221	-0.085
American Express	*	*	*	*
Bethlehem Steel	0.268	-0.465	-0.148	0.332
Chevron	-0.061	-0.081	-0.093	-0.015
Coke	0.315	-0.232	-0.462	-0.298
GM	0.196	-0.289	-0.371	-0.302
Goodyear	0.049	0.430	-0.315	0.335
IBM	-0.239	-0.116	-0.304	-0.344
Int. Paper	0.192	-0.237	-0.485	-0.251
Kodak	*	*	*	*
McDonalds	0.096	-0.027	-0.131	-0.078
Merck	-0.510	0.493	0.094	0.389
MMM	0.375	-0.283	-0.634	-0.341
Proctor & Gamble	0.344	0.100	-0.278	-0.031
United Tech	0.693	-0.385	-0.594	-0.471

* Insufficient significant H windows to calculate a meaningful statistic.

Table 7. Summary of LaGrange Multiplier Test for Included Stocks
Test Stat = NR^2 of Regression for Daily Variance Estimate on 10 Lagged Variances. Entire Period, and Groups of 214 Days.

Name	Entire Sample	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
Alcoa	243.62**	14.69	14.06	22.95*	31.21**	13.71	30.46**	6.25
American Express	218.85**	47.22**	22.95*	30.58**	47.59**	6.61	57.51**	22.12*
Bethlehem Steel	617.94**	19.87	2.43	18.07	50.27**	32.17**	47.76**	62.76**
Chevron	70.23**	7.43	3.90	26.89**	7.87	5.90	25.85**	14.67
Coke	233.27**	23.30*	7.05	7.54	10.75	11.56	10.54	27.83**
GM	115.18**	4.87	23.35*	21.57*	28.03**	32.10**	32.55**	11.83
Goodyear	399.88**	62.22**	40.74**	17.91	1.98	19.38	16.65	15.51
IBM	321.29**	71.59**	29.36**	27.23**	70.83**	11.20	35.59**	16.12
Int. Paper	92.48**	15.77	10.98	17.48	19.42	18.30	5.08	15.10
Kodak	157.99**	7.25	43.80**	27.07**	41.76**	3.37	17.38	29.89**
McD	228.15**	11.757	58.92**	26.11**	39.35**	24.91**	15.32	6.16
Merck	299.90**	44.85**	18.33	13.14	65.42*	8.28	22.04*	22.85*
MMM	263.35**	13.06	21.32*	21.75*	49.29**	28.66**	14.00	6.69
P&G	121.50**	30.92**	1.05	26.52**	54.82**	17.56	21.77*	15.30
UTX	11.64	18.18	0.53	35.53**	10.55	17.24	44.51**	3.79

*significant at 5%
**significant at 1%

Figure -1. Realization of Pure White Noise Process
with Unit Variance

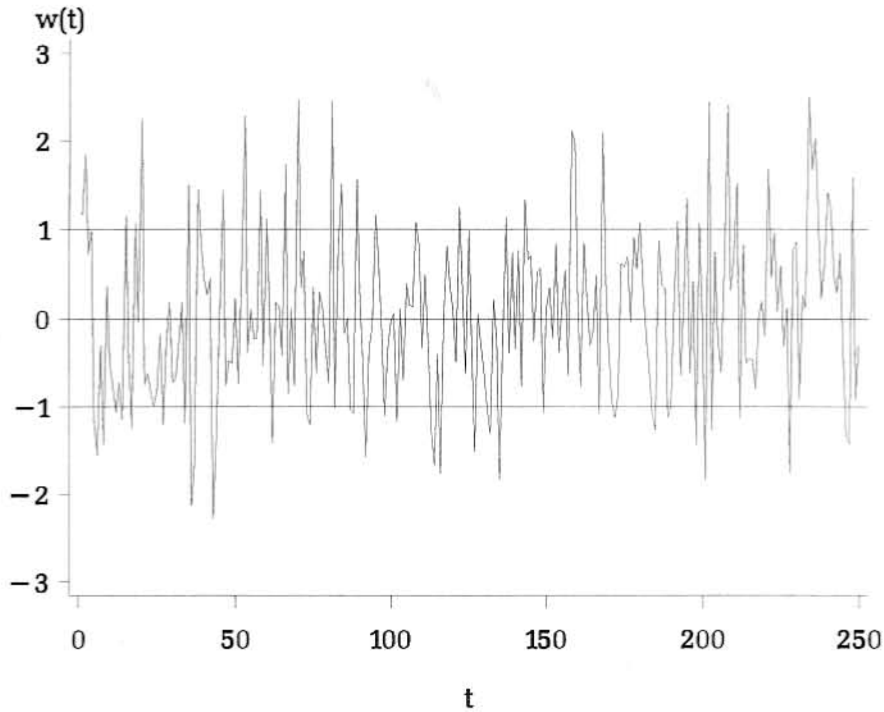

Figure 1.
Time Series Plot of the Probability Level of the Test
Statistics for Each Window for Coca–Cola During 1983.
(a) C Statistic

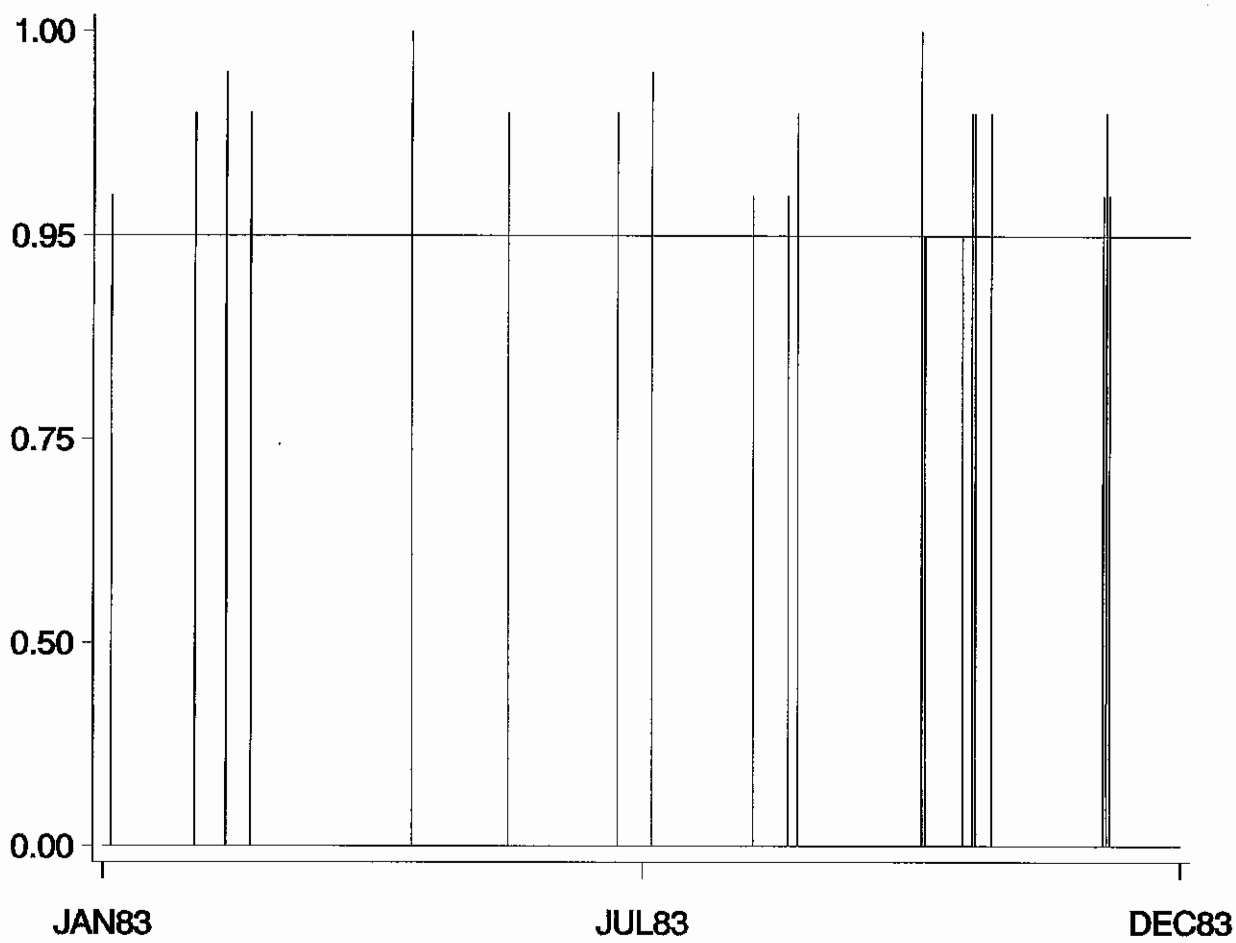

Figure 1.
Time Series Plot of the Probability Level of the Test Statistics for Each Window for Coca – Cola During 1983.
(b) H Statistic

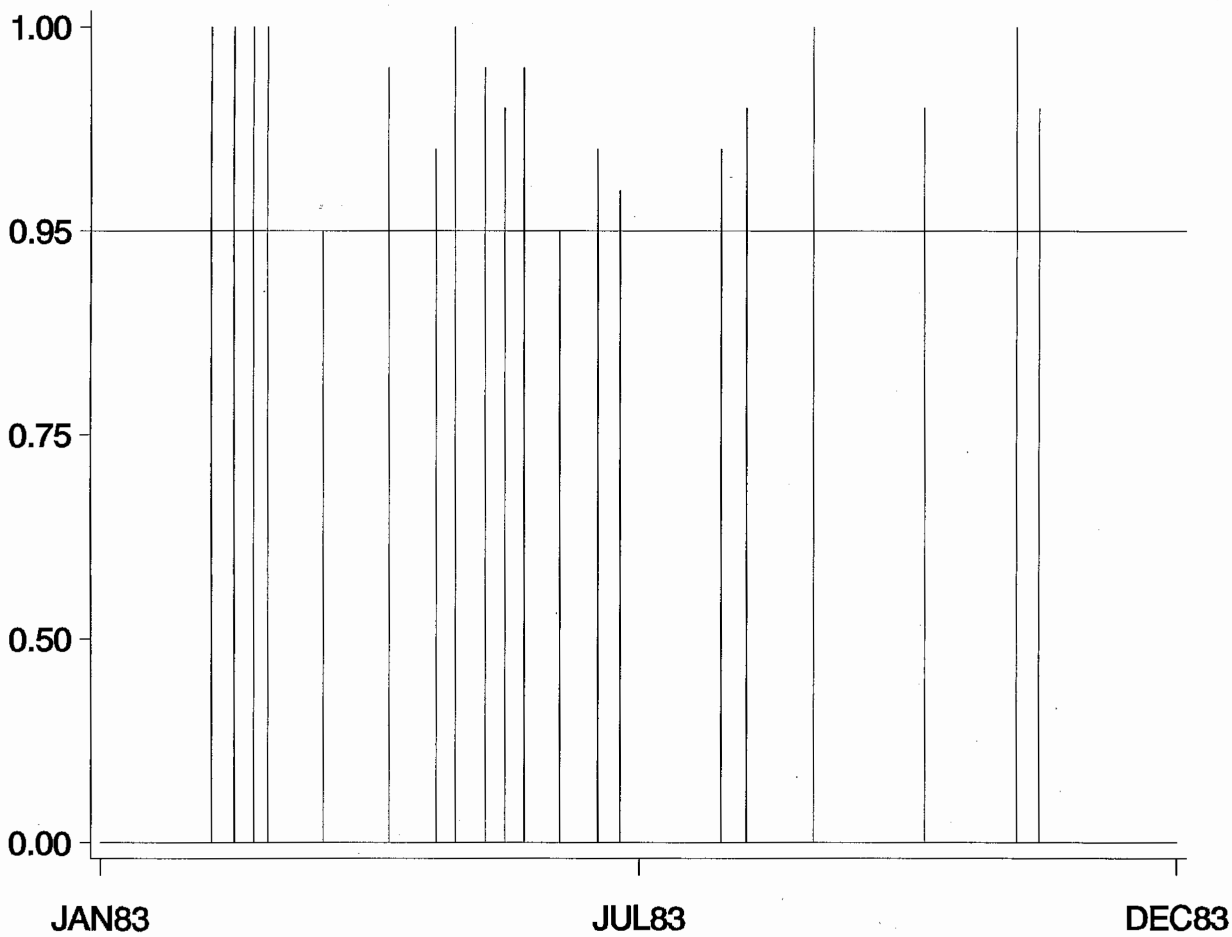

Figure 2. Plot of the sum of the absolute values of x(k−1)x(k−2), x(k−1)x(k−3), x(k−1)x(k−4), x(k−2)x(k−3), x(k−2)x(k−4), and x(k−3)x(k−4), versus x(k) over three consecutive days where the H statistic was significant. McDonalds Corp., Thursday, Friday, and Monday, March 27, 28, 31, 1980