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Randomly Modulated Periodic Signals in Alberta's Electricity Market

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Abstract

This paper uses hourly electricity prices and MW hour demand for Alberta, Canada over the deregulated period after 1996 to test for randomly modulated periodicity. In doing so, we apply the signal coherence spectral analysis to the time series of hourly spot prices and megawatt-hours (MWh) demand from 1/1/1996 to 12/7/2003 using the FORTRAN 95 program developed by Hinich (2000). We detect relatively steady weekly and daily cycles in demand but very unstable cycles in prices.

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1 Introduction

As Bunn (2004, p. 2) recently put it "the crucial feature of price formation in electricity spot markets is the instantaneous nature of the product. The physical laws that determine the delivery of power across a transmission grid require a synchronised energy balance between the injection of power at generating points and the offtake at demand points (plus some allowance for transmission losses). Across the grid, production and consumption are perfectly synchronised, without any capability for storage. If the two get out of balance, even for a moment, the frequency and voltage of the power fluctuates. Furthermore, end-users treat this product as a service at their convenience. When we go to switch on a light, we do not re-contract with a supplier for the extra energy before doing so. We just do it, and there is a tendency for millions of other people to do likewise whenever they feel like. Electricity may be produced as a commodidity, but it is consumed as a service. The task of the grid operator, therefore, is to be continously monitoring the demand process and to call on those generators who have the technical capability and the capacity to respond quickly to the fluctuations in demand."

Recent leading-edge research has applied various innovative methods for modeling spot wholesale electricity prices — see, for example, Deng and Jiang (2004), León and Rubia (2004), and Serletis and Andreadis (2004). The main objective of this paper is to use a parametric statistical model, called Randomly Modulated Periodicity (RMP), recently proposed by Hinich (2000) and Hinich and Wild (2001), to study Alberta's spot wholesale power market, defined on hourly intervals (like most spot markets for electricity are). In doing so, we use hourly electricity prices, denominated in megawatt-hours (MWh), and MWh demand over the recent deregulated period from 1/1/1996 to 12/7/2003 (a total of over 65000 observations, since there are 8760 hours in a normal year). Our principal concern is to test for periodic signals in electricity prices and electricity load — that is, signals that can be perfectly predicted far into the future since they perfectly repeat every period. In doing so, we take a univariate approach, although from an economic perspective the interest in the price of electricity is in its relationship with the electricity load and perhaps with the prices of other primary fuel commodities.

The paper is organized as follows. In Sections 2 and 3 we briefly discuss the RMP model, proposed by Hinich (2000) and Hinich and Wild (2001), for the study of varying periodic signals. In Section 4 we briefly discuss Alberta's power market and in Section 5 we test for randomly modulated periodicity in hourly electricity prices and MWh demand over the deregulated period after 1996. The final section provides a brief conclusion.

2 Randomly Modulated Periodicity

All signals that appear to be periodic have some sort of variability from period to period regardless of how stable they appear to be in a data plot. A true sinusoidal time series is a deterministic function of time that never changes and thus has zero bandwidth around the sinusoid's frequency. Bandwidth, a term from Fourier analysis, is the number of frequency components that are needed to have an accurate Fourier sum expansion of a function of time. A single sinusoid has no such expansion. A zero bandwidth is impossible in nature since all signals have some intrinsic variability over time.

Deterministic sinusoids are used to model cycles as a mathematical convenience. It is time to break away from this simplification in order to model the various periodic signals that are observed in fields ranging from biology, communications, acoustics, astronomy, and the various sciences.

Hinich (2000) introduced a parametric statistical model, called Randomly Modulated Periodicity (RMP), that allows one to capture the intrinsic variability of a cycle. A discrete-time random process $x(t_n)$ is an RMP with period $T = N\tau$ if it is of the form

$$x(t_n) = s_0 + \frac{2}{N}\sum_{k=1}^{N/2}[(s_{1k} + u_{1k}(t_n))\cos(2\pi f_k t_n) + (s_{2k} + u_{2k}(t))\sin(2\pi f_k t_n)]$$

where $t_n = n\tau$, τ is the sampling interval, $f_k = k/T$ is the k-th Fourier frequency, and where for each period the $\{u_{11}(t_1), \ldots, u_{1,N/2}(t_n), u_{21}(t_n), \ldots, u_{2,N/2}(t_n)\}$ are random variables with zero means and a joint distribution that has the following finite dependence property: $\{u_{jr}(s_1), \ldots, u_{jr}(s_m)\}$ and $\{u_{ks}(t_1), \ldots, u_{ks}(t_n)\}$ are independent if $s_m + D < t_1$ for some $D > 0$ and all $j, k = 1, 2$ and $r, s = 1, \ldots, N/2$ and all times $s_1 < \cdots < s_m$ and $t_1 < \cdots < t_n$. Finite dependence is a strong mixing condition — see Billingsley (1979).

These time series, $u_{k1}(t)$ and $u_{k2}(t)$, are called 'modulations' in the signal processing literature. If $D << N$ then the modulations are approximately stationary within each period. The process $x(t_n)$ can be written as

$$x(t_n) = s(t_n) + u(t_n),$$

where

$$s(t_n) = E[x(t_n)] = s_0 + \frac{2}{N}\sum_{k=1}^{N/2}[s_{1k}\cos(2\pi f_k t_n) + s_{2k}\sin(2\pi f_k t_n)]$$

and

$$u(t_n) = \frac{2}{N}\sum_{k=1}^{N/2}[u_{1k}\cos(2\pi f_k t_n) + u_{2k}\sin(2\pi f_k t_n)]$$

Thus $s(t_n)$, the expected value of the signal $x(t_n)$, is a periodic function. The fixed coefficients s_{1k} and s_{2k} determine the shape of $s(t_n)$. If $s_{11} \neq 0$ or $s_{21} \neq 0$ then $s(t_n)$ is periodic with period $T = N\tau$. If $s_{11} = 0$ and $s_{21} = 0$, but $s_{12} \neq 0$ or $s_{22} \neq 0$, then $s(t_n)$ is periodic with period $T/2$. If the first $k_0 - 1$ s_{1k} and s_{2k} are zero, but not the next, then $s(t_n)$ is periodic with period T/k_0.

3 Signal Coherence Spectrum

To provide a measure of the modulation relative to the underlying periodicity, Hinich (2000) introduced a concept called the signal coherence spectrum (SIGCOH). For each Fourier frequency $f_k = k/T$ the value of SIGCOH is

$$\gamma_x(k) = \sqrt{\frac{|s_k|^2}{|s_k|^2 + \sigma_u^2(k)}}$$

where $s_k = s_{1k} + is_{2k}$ is the amplitude of the kth sinusoid written in complex variable form, $i = \sqrt{-1}$, $\sigma_u^2(k) = E|U(k)|^2$ and

$$U(k) = \sum_{n=0}^{N-1} u_k(t_n)\exp(-i2\pi f_k t_n)$$

is the discrete Fourier transform (DFT) of the modulation process $u_k(t_n) = u_{1k}(t_n) + iu_{2k}(t_n)$ written in complex variable form.

Each $\gamma_x(k)$ is in the $(0,1)$ interval. If $s_k = 0$ then $\gamma_x(k) = 0$. If $U(k) = 0$ then $\gamma_x(k) = 1$. The SIGCOH measures the amount of 'wobble' in each frequency component of the signal $x(t_n)$ about its amplitude when $s_k > 0$. The amplitude-to-modulation standard deviation (AMS) is

$$\rho_x(k) = \frac{|s_k|}{\sigma_u(k)}$$

for frequency f_k. Thus,

$$\gamma_x^2(k) = \frac{\rho_x^2(k)}{\rho_x^2(k) + 1}$$

is a monotonically increasing function of this signal-to-noise ratio. Inverting this relationship, it follows that

$$\rho_x^2(k) = \frac{\gamma_x^2(k)}{1 - \gamma_x^2(k)}$$

An AMS of 1.0 equals a signal coherence of 0.71 and an AMS of 0.5 equals a signal coherence of 0.45.

To estimate the SIGCOH, $\gamma_x(k)$, suppose that we know the fundamental period and we observe the signal over M such periods. The mth period is $\{x((m-1)T + t_n),\ n = 0, \ldots, N-1\}$. The estimator of $\gamma(k)$ introduced by Hinich (2000) is

$$\hat{\gamma}(k) = \sqrt{\frac{|\bar{X}(k)|^2}{|\bar{X}(k)|^2 + \hat{\sigma}_u^2(k)}},$$

where

$$\bar{X}(k) = \frac{1}{M}\sum_{m=1}^{M} X_m(k)$$

is the sample mean of the DFT,

$$X_m(k) = \sum_{n=0}^{N-1} x((m-1)T + t_n)\exp(-i2\pi f_m t_n),$$

and

$$\hat{\sigma}_u^2(k) = \frac{1}{M}\sum_{m=1}^{M} |X_m(k) - \bar{X}(k)|^2$$

is the sample variance of the residual discrete Fourier transform, $X_m(k) - \bar{X}(k)$. This estimator is consistent as $M \to \infty$ and if the modulations have a finite dependence of span D then the distribution of

$$Z(k) = \frac{M}{N} \frac{|\bar{X}(k)|^2}{\sigma_u^2(k)}$$

is asymptotically chi-squared with two degrees-of-freedom and a noncentrality parameter $\lambda_k = (M/N)\, \rho_x^2(k)$ as $M \to \infty$ — see Hinich and Wild (2001). These $\chi_2^2(\lambda_k)$ variates are approximately independently distributed over the frequency band when $D << N$.

If the null hypothesis for frequency f_k is that $\gamma_x(k) = 0$ and thus its AMS is zero, then $Z(k)$ is approximately a central chi-squared statistic. Thus $Z(k)$ can be used to falsify the null hypothesis that $\gamma_x(k) = 0$. The tests across the frequency band are approximately independently distributed tests. The use of the transformation to the $Z(k)$'s is the only straightforward way to put statistical confidence on the signal coherence point estimates.

4 Alberta's Power Market

Electricity demand in Alberta is comprised of four primary groups: residential, farm, commercial, and industrial. Alberta has unique load requirements compared with other North American power markets. In particular, the industrial load is over 50% of all electric sales while the residential load is only 15%. This provides a very stable load curve all over the year, which helps reduce the frequency of price spikes. The main contributors to the fluctuations of demand are residential and small commercial customers.

Electricity demand is also cyclical in nature, with demand being lower in the spring and fall than in summer and winter. In fact, Alberta has higher winter consumption, due to lower temperatures that cause increased heating and shorter daylight hours. Moreover, winter hourly load in Alberta has two distinct peaks. Demand is low in the early morning hours and begins to increase through the morning hours, with a first peak around nine o'clock. During this interval, load can increase by 1500 MW. The other peak is around dinnertime, six-seven o'clock. The peaks in the day tend to float depending on the number of daylight hours. Demand also follows a weekly cycle and tends to be higher on weekdays than during the weekends.

Finally, demand for power is relatively inelastic in Alberta. There is no requirement for load to bid in the price they are willing to pay for the energy. Hence, un-bid load is treated as a price taker into the merit order of the Alberta's power market and must pay the hourly pool price for the energy consumed during that hour. Only a small percentage (around 4%) of the load is bid into the market. According to AESO Operations group, there is around 300 MW of price responsive demand: some large industrial customers agree to be curtailed at high pool prices, introducing some price sensitivity at higher price levels.

Being a deregulated market, the pool price in Alberta's power market is determined by competitive market forces; that is, the laws of supply and demand. Being components of supply and demand, imports and exports also influence electricity prices. Import and export volumes play an important role in ensuring system reliability and security in Alberta. In conditions of scarcity of supply and/or excess of demand, power must be imported via the inter tie-lines that connect the Alberta electric grid system with the neighbouring jurisdictions. In fact, the Alberta Interconnected Electric System (AIES) is connected, on the west side, to the British Columbia (BC) grid by the 800 MW Alberta-BC inter-tie, while it is linked on the east side to the Saskatchewan power system by a 150 MW DC interconnection.

Since the total available capacity of the inter-ties represents about 11% of the Alberta maximum peak load, the tie lines work as very large generating units, and thus may have a considerable impact on the pool price. This fact, in combination with Alberta's steep supply curve and inelastic demand even at high prices, has given importers and exporters significant market power, which has raised concerns among market participants. In conditions of tight supply-demand balance, the pool price is strongly impacted by the discretionary sales tactics implemented by importers. While these strategies of adjusting the volume of imports and exports in response to market outcomes are normal profit-maximizing behaviours, on the other hand, practices like abuse of market power or electricity dumping are deemed to manipulate the pool price. These undesirable practices have been the issues of a controversy among stakeholders in the Alberta's electricity market — see Bianchi and Serletis (2006) for more details.

5 RMP in Alberta's Power Market

We use the time series of Alberta hourly spot prices and megawatt-hours (MWh) demand from 1/1/1996 to 12/7/2003. Figure 1 shows a section of the demand time series (the load curve) and Figure 2 shows a section of the spot prices time series, over the same period. Electricity demand has a daily and weekly cycle but it is clear from Figure 1 that these cycles in the demand are wobbly. However, it is hard to see a daily or weekly cycle in the spot electricity prices in Figure 2. The prices time series in Figure 2 is much more spiky, shows higher volatility, and also a stronger mean-reverting pattern than the load time series in Figure 1.

We applied the signal coherence spectral analysis to the time series of spot prices and demand, using the FORTRAN 95 'Spectrum.for' program developed by Hinich and available at his web page, www.la.utexas.edu/~hinich. In doing so, we first detrended the hourly electricity demand and the hourly spot electricity price data by fitting an AR(12) model to each series — the AR(12) filter is used to make the data have a flat spectrum; it is a linear transformation and thus it does not create nor destroy coherence. The residuals of the fitted model are then analyzed for the presence of a randomly modulated periodicity with a fundamental period of one week (168 hours). An AR fit is a linear operation that cannot create signal coherence. Indeed signal coherence can only be reduced by a improperly applied detrended method.

The adjusted R square for the demand data is 0.74. The characteristic polynomial of the estimated AR(12) model has a 4th order complex root pair whose amplitudes are 0.96 and a complex root pair whose amplitudes are 0.96. The amplitude of the other root pair is 0.71. The adjusted R square for the spot price data is 0.666. The largest root magnitude is 0.81.

The SIGCOH spectrum of the demand time series is shown in Figure 3. All the long period harmonics up to the period of 8.4 hours have coherence greater than 0.5 except for the 9.99 hour harmonic. Only the fundamental and the harmonics 28 and 24 hours have coherences greater than 0.9. The shorter period components are either not very coherent or incoherent. Figure 3 also shows the conventional power spectrum (log spectrum in decibels). The harmonic peaks in the spectrum indicate that the weekly and daily cycles are not simple sinusoids but their lack of amplitude and phase stability indicated in the SIGCOH spectrum implies that the shorter period components are of little use for forecasting.

Figure 1. A Section of Alberta Electricity Demand

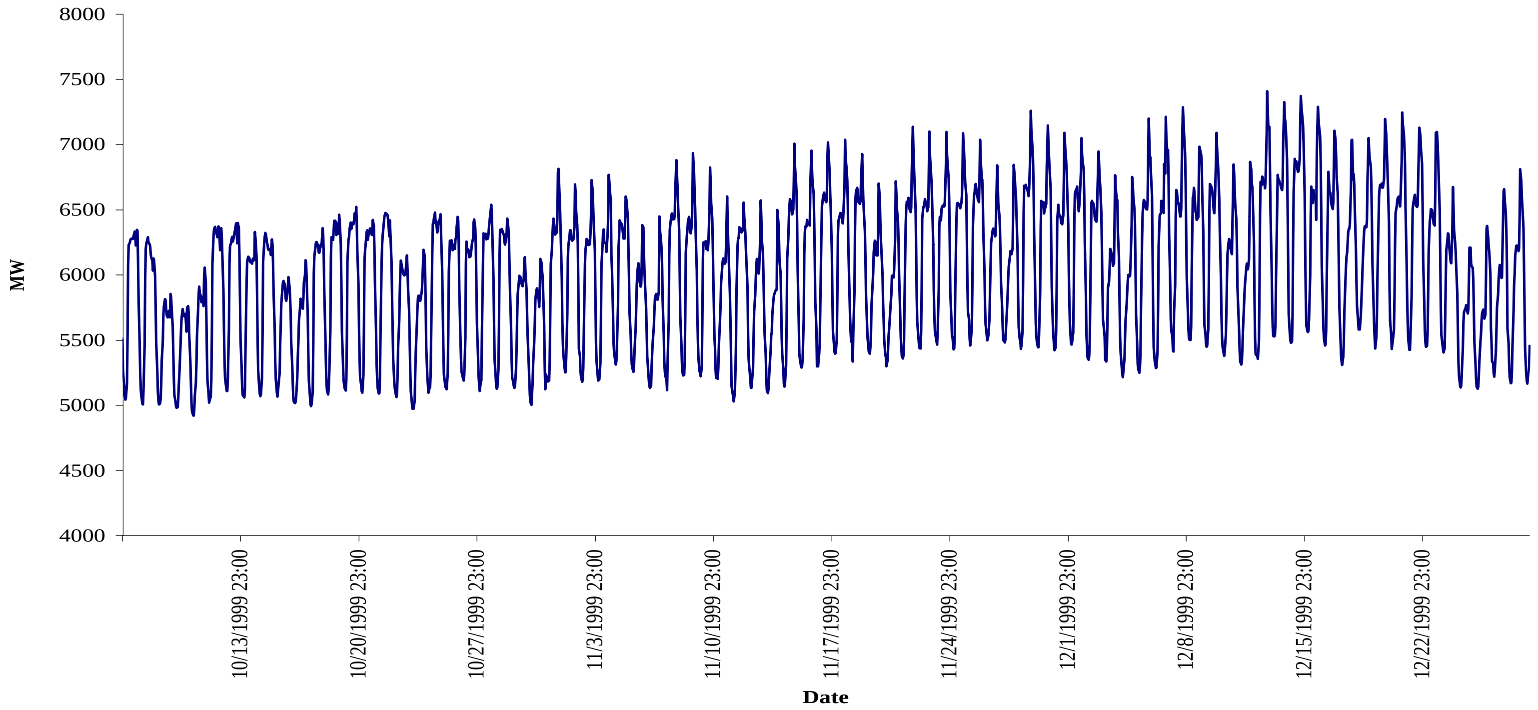


Figure 2. A Section of Alberta Electricity Spot Prices

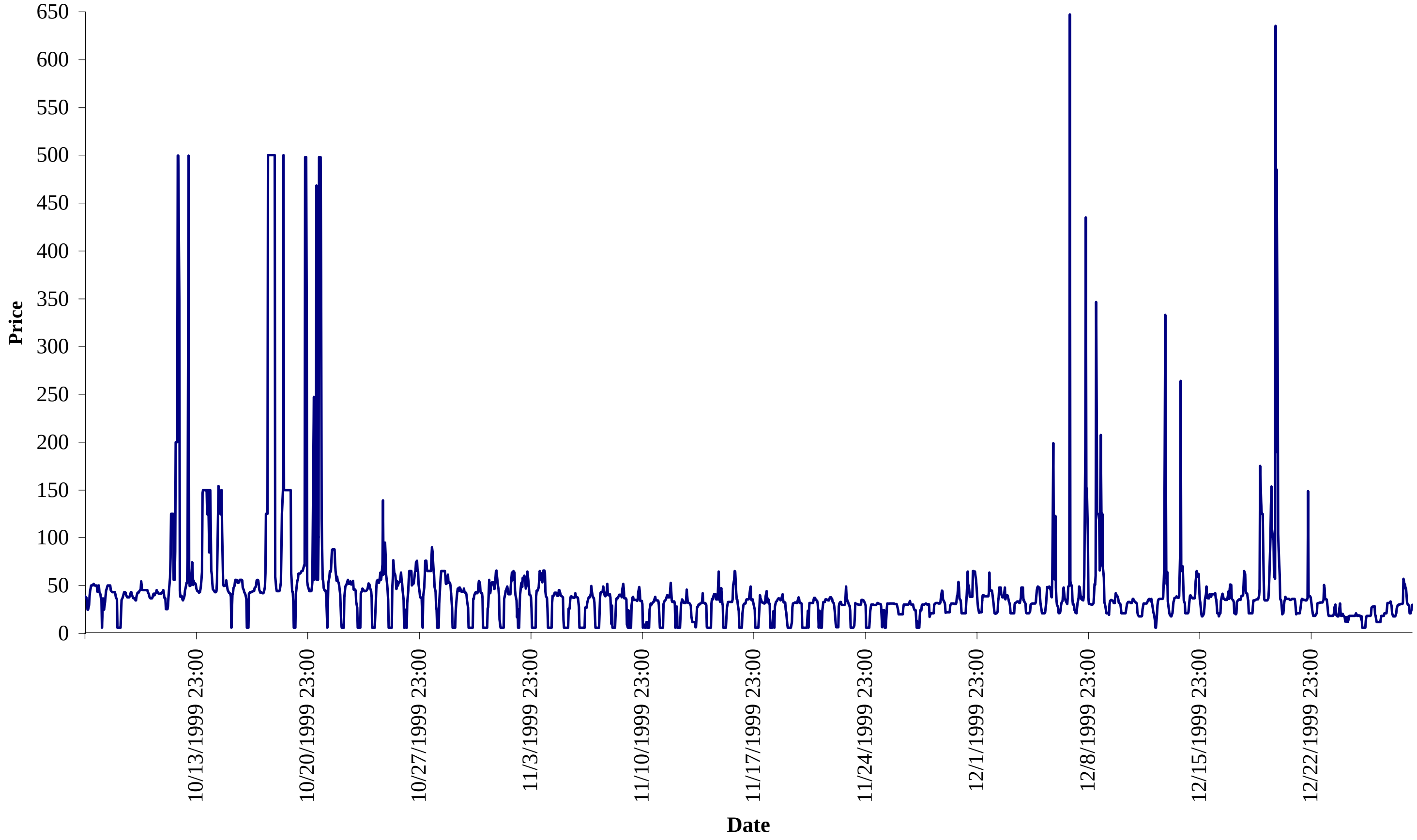
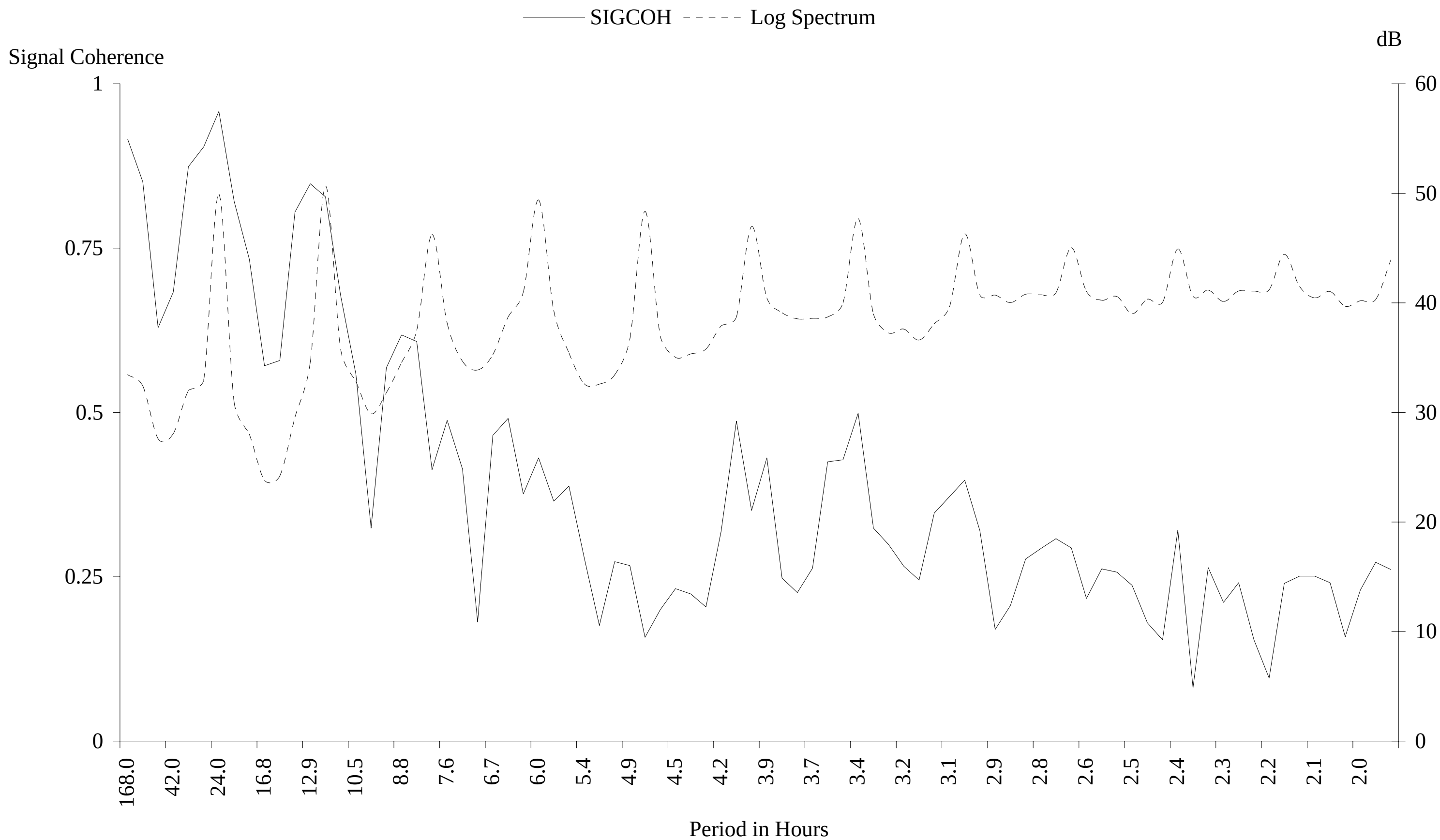


Figure 3. Power & Signal Coherence Spectra of the Residuals from an AR(12) Fit of the Alberta Electricity Hourly Spot Demand



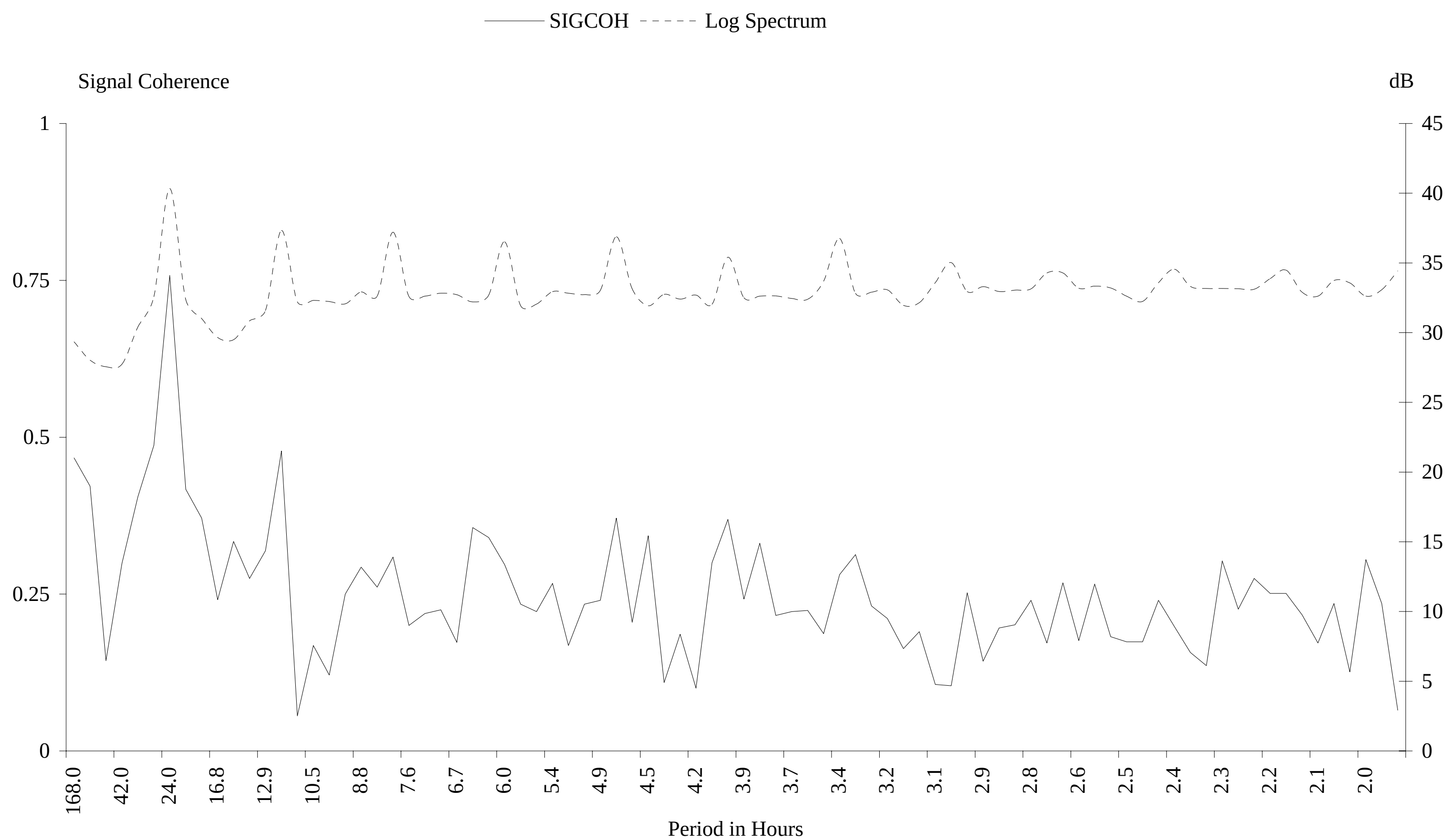
The SIGCOH spectrum of the spot prices is shown in Figure 4. Only the 24 hour harmonic has a coherence barely greater than 0.75. The rest have coherences less than 0.5, including the fundamental. The plot of the power spectrum in Figure 4 shows the standard methods for fitting a Fourier expansion of the weekly and daily cycles will not contribute much to a forecast of the spot prices.

6 Conclusion

We have applied the signal coherence spectral analysis to the time series of hourly spot prices and megawatt-hours (MWh) demand for Alberta and found that electricity prices have low coherence in the daily and weekly cycles, meaning that forecast errors will have a high error variance. However, electricity demand has lot of high coherence with the daily and weekly cycles being stable with some variation. The mean values at each half hour of the daily demand and the weekend demand should yield good forecasts for a day and the weekend for the next week after the end of the data series. Yet we expect that a statistical forecasting based on the historical demand and cofactors such as the average hourly temperature per day and patterns of industrial usage should yield better short term forecasts. Clearly, the development of a statistical technology for forecasting electricity demand is an exciting and challenging area of research — see, for example, Li and Hinich (2002).

In this paper, we have taken a univariate time series approach to the analysis of electricity prices. From an economic perspective, however, the interest in the price of electricity is in its relationship with the prices of various underlying primary fuel commodities such as, for example, natural gas, oil, or coal. As Bunn (2004, p. 2) recently put it " · · · take the case of gas, for example. This is now becoming the fuel of choice for electricity generation. The investment costs are lower than coal, or oil plant; it is cleaner and, depending upon location, the fuel costs are comparable. But with more and more of the gas resources being used for power generation, in some markets the issue of whether gas drives power prices, or *vice versa*, is not easily answered." Because the properties of univariate series need not be at all like the properties of their multivariate relationships, investigating the relationship between electricity prices and the prices of other primary fuel commodities is an area for potentially productive future research.

Figure 4. Power & Signal Coherence Spectra of the Residuals from an AR(12) Fit of the Alberta Electricity Hourly Spot Prices



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