Randomly Modulated Periodicity in the U.S. Stock Market*

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Abstract

This paper extends the work in Serletis and Shintani [6], Elder and Serletis [2], and Koustas et al. [5] by examining the empirical evidence for random walk type behavior in the U.S. stock market. In doing so, it uses the FORTRAN 95 program developed by Hinich [3] and detects a statistically significant randomly modulated periodic signal.

Keywords: Signal coherence function; Discrete Fourier transform; Dow Jones Industrial Average.
JEL classification: C13, C14, C22

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1 Introduction

This paper extends the work in Serletis and Shintani [6], Elder and Serletis [2], and Koustas et al. [5] by re-examining the empirical evidence for random walk type behavior in the U.S. stock market. In doing so, it tests the random walk hypothesis by using a parametric statistical model called Randomly Modulated Periodicity (RMP), recently proposed by Hinich [3] and Hinich and Wild [4]. In doing so it uses data on the Dow Jones Industrial Average over the period from January 3, 1928 to March 15, 2006 — a total of 19,758 observations.

The paper is organized as follows. In Sections 2 and 3 we briefly discuss the RMP model for the study of varying periodic signals. In Section 4 we test for randomly modulated periodicity in the daily Dow Jones Industrial Average and report and discuss the results. The final section provides a brief conclusion.

2 Randomly Modulated Periodicity

All signals that appear to be periodic have some sort of variability from period to period regardless of how stable they appear to be in a data plot. A true sinusoidal time series is a deterministic function of time that never changes and thus has zero bandwidth around the sinusoid's frequency. Bandwidth, a term from Fourier analysis, is the number of frequency components that are needed to have an accurate Fourier sum expansion of a function of time. A single sinusoid has no such expansion. A zero bandwidth is impossible in nature since all signals have some intrinsic variability over time.

Deterministic sinusoids are used to model cycles as a mathematical convenience. It is time to break away from this simplification in order to model the various periodic signals that are observed in fields ranging from biology, communications, acoustics, astronomy, and the various sciences.

Hinich [3] introduced a parametric statistical model, called Randomly Modulated Periodicity (RMP), that allows one to capture the intrinsic variability of a cycle. A discrete-time random process $x(t_n)$ is an RMP with period $T = N\tau$ if it is of the form

$$x(t_n) = s_0 + \frac{2}{N}\sum_{k=1}^{N/2}[(s_{1k} + u_{1k}(t_n))\cos(2\pi f_k t_n) + (s_{2k} + u_{2k}(t))\sin(2\pi f_k t_n)]$$

where $t_n = n\tau$, τ is the sampling interval, $f_k = k/T$ is the k-th Fourier frequency, and where for each period the $\{u_{11}(t_1), \ldots, u_{1,N/2}(t_n), u_{21}(t_n), \ldots, u_{2,N/2}(t_n)\}$ are random variables with zero means and a joint distribution that has the following finite dependence property: $\{u_{jr}(s_1), \ldots, u_{jr}(s_m)\}$ and $\{u_{ks}(t_1), \ldots, u_{ks}(t_n)\}$ are independent if $s_m + D < t_1$ for some $D > 0$ and all $j, k = 1, 2$ and $r, s = 1, \ldots, N/2$ and all times $s_1 < \cdots < s_m$ and $t_1 < \cdots < t_n$. Finite dependence is a strong mixing condition — see Billingsley [1].

These time series, $u_{k1}(t)$ and $u_{k2}(t)$, are called 'modulations' in the signal processing literature. If $D << N$ then the modulations are approximately stationary within each period. The process $x(t_n)$ can be written as

$$x(t_n) = s(t_n) + u(t_n),$$

where

$$s(t_n) = E[x(t_n)] = s_0 + \frac{2}{N} \sum_{k=1}^{N/2} [s_{1k} \cos(2\pi f_k t_n) + s_{2k} \sin(2\pi f_k t_n)]$$

and

$$u(t_n) = \frac{2}{N} \sum_{k=1}^{N/2} [u_{1k} \cos(2\pi f_k t_n) + u_{2k} \sin(2\pi f_k t_n)]$$

Thus $s(t_n)$, the expected value of the signal $x(t_n)$, is a periodic function. The fixed coefficients s_{1k} and s_{2k} determine the shape of $s(t_n)$. If $s_{11} \neq 0$ or $s_{21} \neq 0$ then $s(t_n)$ is periodic with period $T = N\tau$. If $s_{11} = 0$ and $s_{21} = 0$, but $s_{12} \neq 0$ or $s_{22} \neq 0$, then $s(t_n)$ is periodic with period $T/2$. If the first $k_0 - 1$ s_{1k} and s_{2k} are zero, but not the next, then $s(t_n)$ is periodic with period T/k_0.

3 Signal Coherence Spectrum

To provide a measure of the modulation relative to the underlying periodicity, Hinich [3] introduced a concept called the signal coherence spectrum (SIGCOH). For each Fourier frequency $f_k = k/T$ the value of SIGCOH is

$$\gamma_x(k) = \sqrt{\frac{|s_k|^2}{|s_k|^2 + \sigma_u^2(k)}}$$

where $s_k = s_{1k} + is_{2k}$ is the amplitude of the kth sinusoid written in complex variable form, $i = \sqrt{-1}$, $\sigma_u^2(k) = E|U(k)|^2$ and

$$U(k) = \sum_{n=0}^{N-1} u_k(t_n) \exp(-i2\pi f_k t_n)$$

is the discrete Fourier transform (DFT) of the modulation process $u_k(t_n) = u_{1k}(t_n) + iu_{2k}(t_n)$ written in complex variable form.

Each $\gamma_x(k)$ is in the $(0,1)$ interval. If $s_k = 0$ then $\gamma_x(k) = 0$. If $U(k) = 0$ then $\gamma_x(k) = 1$. The SIGCOH measures the amount of 'wobble' in each frequency component of the signal $x(t_n)$ about its amplitude when $s_k > 0$. The amplitude-to-modulation standard deviation (AMS) is

$$\rho_x(k) = \frac{|s_k|}{\sigma_u(k)}$$

for frequency f_k. Thus,

$$\gamma_x^2(k) = \frac{\rho_x^2(k)}{\rho_x^2(k) + 1}$$

is a monotonically increasing function of this signal-to-noise ratio. Inverting this relationship, it follows that

$$\rho_x^2(k) = \frac{\gamma_x^2(k)}{1 - \gamma_x^2(k)}$$

An AMS of 1.0 equals a signal coherence of 0.71 and an AMS of 0.5 equals a signal coherence of 0.45.

To estimate the SIGCOH, $\gamma_x(k)$, suppose that we know the fundamental period and we observe the signal over M such periods. The mth period is $\{x((m-1)T + t_n),\ n = 0, \ldots, N-1\}$. The estimator of $\gamma(k)$ introduced by Hinich [3] is

$$\hat{\gamma}(k) = \sqrt{\frac{|\bar{X}(k)|^2}{|\bar{X}(k)|^2 + \hat{\sigma}_u^2(k)}},$$

where

$$\bar{X}(k) = \frac{1}{M} \sum_{m=1}^{M} X_m(k)$$

is the sample mean of the DFT,

$$X_m(k) = \sum_{n=0}^{N-1} x((m-1)T + t_n) \exp(-i2\pi f_m t_n),$$

and

$$\hat{\sigma}_u^2(k) = \frac{1}{M} \sum_{m=1}^{M} |X_m(k) - \bar{X}(k)|^2$$

is the sample variance of the residual discrete Fourier transform, $X_m(k) - \bar{X}(k)$. This estimator is consistent as $M \to \infty$ and if the modulations have a finite dependence of span D then the distribution of

$$Z(k) = \frac{M}{N} \frac{|\bar{X}(k)|^2}{\sigma_u^2(k)}$$

is asymptotically chi-squared with two degrees-of-freedom and a noncentrality parameter $\lambda_k = (M/N)\,\rho_x^2(k)$ as $M \to \infty$ — see Hinich and Wild [4]. These $\chi_2^2(\lambda_k)$ variates are approximately independently distributed over the frequency band when $D << N$.

If the null hypothesis for frequency f_k is that $\gamma_x(k) = 0$ and thus its AMS is zero, then $Z(k)$ is approximately a central chi-squared statistic. Thus $Z(k)$ can be used to falsify the null hypothesis that $\gamma_x(k) = 0$. The tests across the frequency band are approximately independently distributed tests. The use of the transformation to the $Z(k)$'s is the only straightforward way to put statistical confidence on the signal coherence point estimates.

4 RMP in the U.S. Stock Market

We use daily observations on the Dow Jones Industrial Average from January 3, 1928 to March 15, 2006 — see Figure 1 for a graphical representation of the series. We applied the signal coherence spectral analysis to the differences of the natural logs of the Dow Jones Industrial Average, shown in Figure 2, using the FORTRAN 95 'Spectrum.for' program developed by Hinich[3] and available at his web page, www.la.utexas.edu/~hinich.

The spectra were computed using the nonoverlapping frame average method. The length of the frame is the longest period of the spectra. Its inverse is called the fundamental frequency of the randomly modulated periodicity.

The user must specify the frame length. We experimented with a number of frame lengths and found that a length of 390 days gave a p-value of 0.015 for the Hinich and Wild [4] test for the presence of a randomly modulated periodicity. There are 50 full frames of length 390 days in the data.

The fundamental frequency only has a signal coherence value of 0.39 with a coherence probability of 0.709. Thus the fundamental frequency has a lot of modulation. The first harmonic frequency with period 195 days has a signal coherence of 0.55 with a coherence probability of 0.994. Many of the higher harmonics have coherence probabilities less than 0.5 and thus are very unstable. The most stable harmonics are 39 days (probability = 0.967), 26 days (probability = 0.987), 7.6471 days (probability = 0.995), 6.5 days (probability = 0.980), 5.9091 days (probability = 0.985), and several short periods less than four days. The signal coherence spectrum is shown in Figure 3 and the coherence probability spectrum is shown in Figure 4.

5 Conclusion

We have applied the signal coherence spectral analysis to the daily returns series of the Dow Jones Industrial Average, over the period from January 3, 1928 to March 15, 2006. We detected sufficiently large modulations, suggesting the absence of opportunities for sufficiently large returns after transactions costs.

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Figure 1. (Logged) Dow Jones Industrial Average, 1928-2006

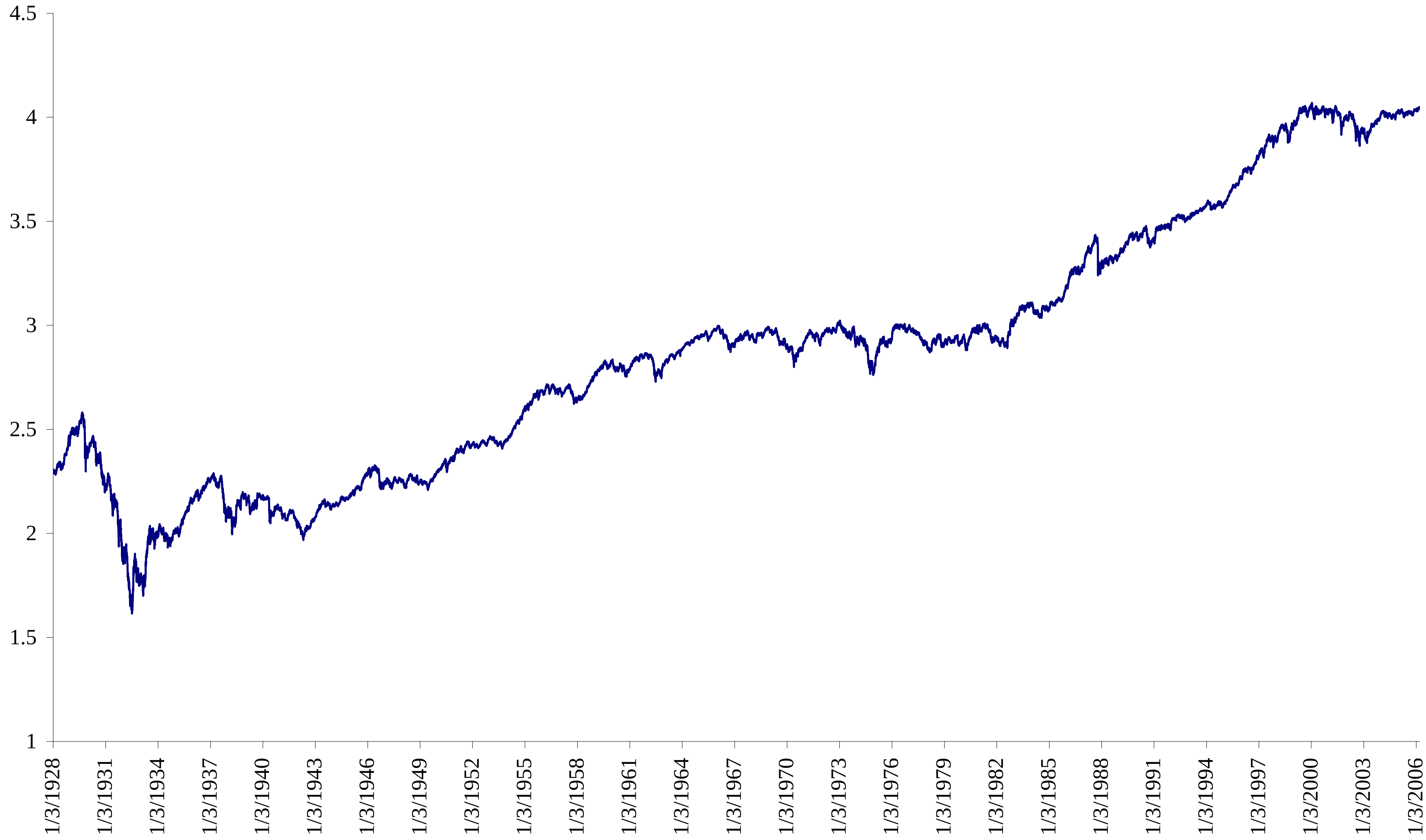

Figure 2. Logged First Differences of the Dow Jones Industrial Average

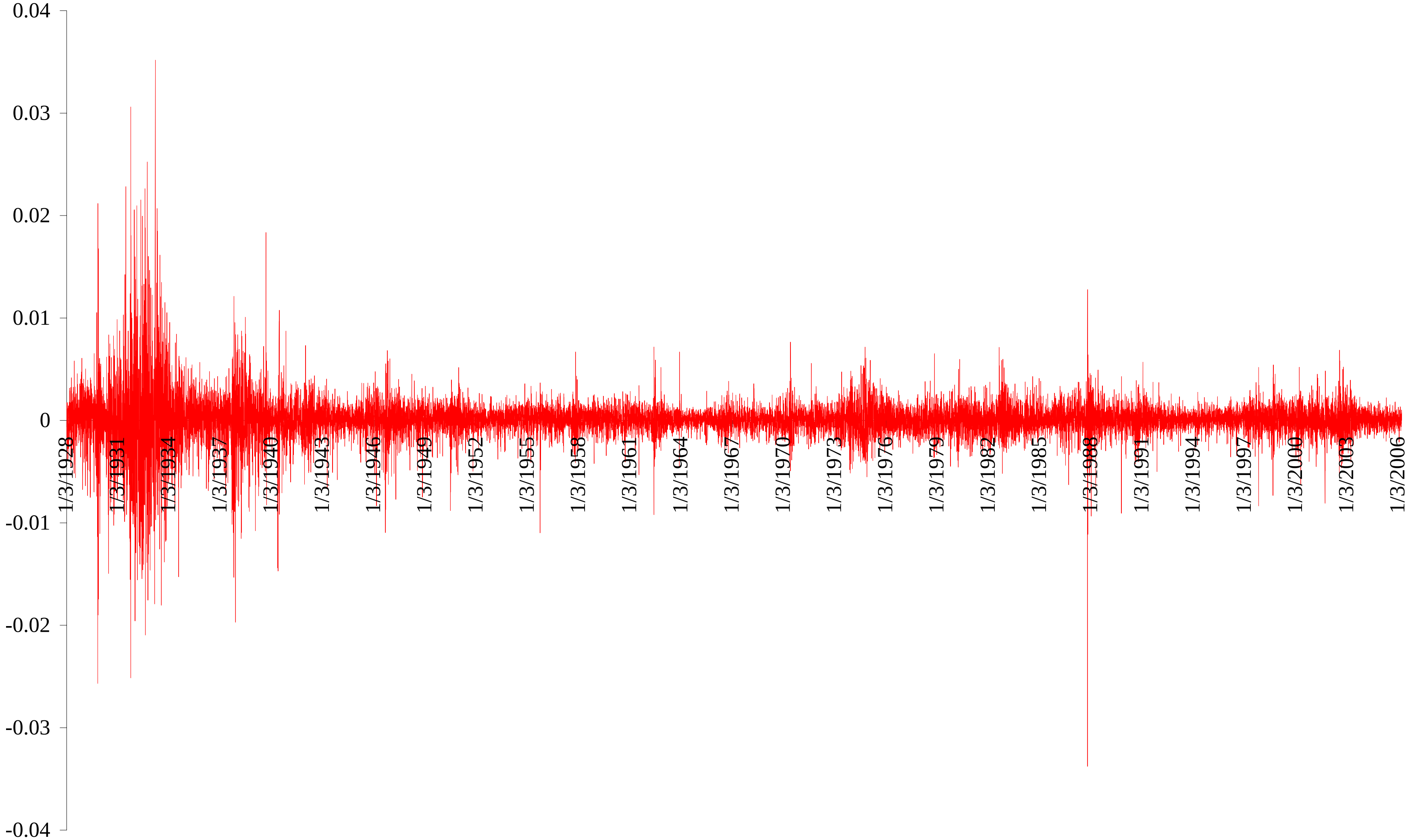

Figure 3. Signal Coherence Spectrum of the Daily Dow Jones Industrial Average

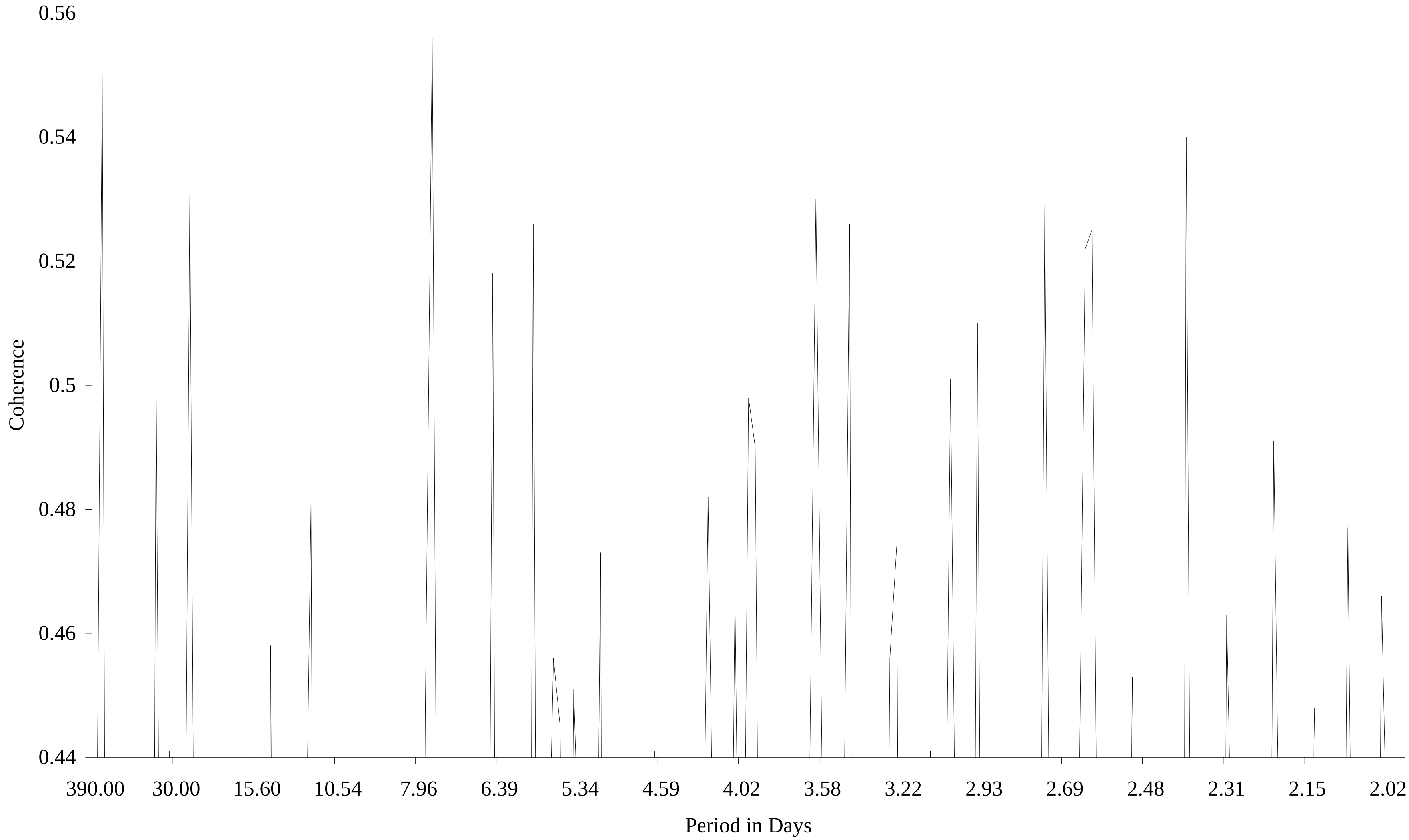

Figure 4. Signal Coherence Probability Spectrum of the Daily Dow Jones Industrial Average