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Detecting intraday periodicities with application to high frequency exchange rates

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Summary. Many recent papers have documented periodicities in returns, return volatility, bid–ask spreads and trading volume, in both equity and foreign exchange markets. We propose and employ a new test for detecting subtle periodicities in time series data based on a signal coherence function. The technique is applied to a set of seven half-hourly exchange rate series. Overall, we find the signal coherence to be maximal at the 8-h and 12-h frequencies. Retaining only the most coherent frequencies for each series, we implement a trading rule that is based on these observed periodicities. Our results demonstrate in all cases except one that, in gross terms, the rules can generate returns that are considerably greater than those of a buy-and-hold strategy, although they cannot retain their profitability net of transactions costs. We conjecture that this methodology could constitute an important tool for financial market researchers which will enable them to detect, quantify and rank the various periodic components in financial data better.

Keywords: Exchange rates; Forecasting; Periodicities; Seasonality; Spectral analysis; Trading rules

1. Introduction

Many recent papers have documented periodicities in returns, return volatility, bid–ask spreads and trading volume, in both equity and foreign exchange markets. Such systematically recurring features or regularities have sometimes been termed calendar anomalies or seasonal effects. Examples include open and close effects, lunch-time effects, day of the week effects and holiday effects. Studies of day of the week effects include French (1980), Gibbons and Hess (1981) and Keim and Stambaugh (1984), for example, all of whom have found that the average market close-to-close return in the USA is significantly negative on Monday and significantly positive on Friday. Moreover, Rogalski (1984) and Smirlock and Starks (1986) observed that this negative return between the Friday close and Monday close for the Dow–Jones industrial average occurs on Monday itself during the 1960s but moves backwards to the period between the Friday close and Monday opening in the late 1970s. By contrast, Jaffe and Westerfield (1985) found that the lowest mean returns for the Japanese and Australian stock-markets occur on Tuesdays. Harris (1986) also examined weekly and intraday patterns in stock returns and found that most of the observed day of the week effects occur immediately after the opening of the market, with a price drop on Mondays on average at this time and rises on all other weekdays.

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Recent research has also exploited the increasing availability of very high frequency and tick data together with more powerful computational abilities to analyse more closely the intraday patterns in financial markets. Wood *et al.* (1985) examined minute-by-minute returns data for a large sample of New York Stock Exchange stocks. They found that significantly positive returns are on average earned during the first 30 min of trading and at the market close, a result which was echoed by Ding and Lau (2001) using a sample of 200 stocks from the Stock Exchange of Singapore. Andersen and Bollerslev (1997a) showed that the return volatility of the German mark–dollar exchange rate exhibits the same general intraday pattern as trading volume and bid–ask spreads. Using the same set of data, Andersen and Bollerslev (1998) also studied the effects of macroeconomic announcements on the behaviour of the series. (An extensive survey of the literature on intraday and intraweek seasonalities in stock-market indices and futures market contracts is given in Yadav and Pope (1992).)

Such periodicities have typically been reconciled with the efficient markets hypothesis by appealing to market microstructure arguments (e.g. cyclical variations in market depth or liquidity, price discovery and inventory management), arrival of information, macroeconomic announcements or tax effects. Many theoretical models of investor and market behaviour have also been proposed to explain these stylized features of many financial time series, including those that account for the strategic behaviour of liquidity traders and informed traders (see, for example, Admati and Pfleiderer (1988). An alternative method for reconciling a finding of recurring seasonal patterns in financial markets is the possible existence of time-varying risk premia, implying that expected returns need not be constant over time and could vary in part systematically without implying market inefficiency.

Traditionally, studies that are concerned with the detection of periodicities in financial time series would either use a regression model with seasonal dummy variables (e.g. Chan *et al.* (1995)) or would apply spectral analysis to the sample of data (e.g. Bertoneche (1979) and Upson (1972)). Spectral analysis may be defined as a process whereby a series is decomposed into a set of mutually orthogonal cyclical components of different frequencies. Calculating the spectrum involves fitting by least squares a set of sinusoidal curves, which is equivalent to a regression with trigonometric transformations of the independent variable. The spectrum, a plot of the signal amplitude against the frequency, will be flat for a white noise process, and evidence of periodic behaviour is indicated by statistically significant amplitudes at any given frequency. By examining the spectrum, Upson (1972) observed periodicities representing cycles of duration 32, 3.8 and 2.5 weeks in US dollar–British pound data for the 1960s, whereas Bertoneche (1979) could not detect any significant departures from randomness in an application of spectral analysis to a set of weekly European stock returns. Spectral analysis was a popular tool for data analysis in economics and finance in the 1960s and 1970s (see also Granger (1966), Granger and Hatanaka (1964) and Granger and Morgenstern (1963)), although it has been largely discarded in the empirical economics and finance literature more recently. This seems to stem partly from the perceived inability of the spectral methods that were previously available to detect the relevant temporal dependences in financial time series data.

In this paper, we propose and employ a new test for detecting periodicities in financial markets based on a signal coherence function. Our approach can be applied to any fairly large, evenly spaced sample of time series data that is thought to contain periodicities. A periodic signal can be predicted infinitely far into the future since it repeats exactly in every period. In fact, in economics and finance as in nature, there are no truly deterministic signals and hence there is always some variation in the waveform over time. The notion of partial signal coherence, which is developed in this paper into a statistical model, is a measure of how much the waveform varies over time. The coherence measures that are calculated are then employed to hone in on the frequency components of the Fourier transforms of the signal that are the most stable over time. By retaining only those frequency components displaying the least variation over time, we can detect the most important seasonalities in the data, and these are then used to derive a trading rule for buying or selling a currency in a hold-out sample. The performance of the trading rule is then compared with that of buy-and-hold and randomized trading strategies. Our approach can capture a much broader range of regularities than a linear regression with time-dependent dummy variables. The model that we propose based on the signal coherence function has been shown to provide additional detectability relative to a Fisher test (see Hinich (2003)).

The remainder of this paper is organized as follows. Section 2 introduces some notation and defines the test statistics that are employed to detect the periodicities, whereas Section 3 describes the data. Section 4 presents and analyses the results whereas Section 5 concludes and offers suggestions for extensions and further research.

2. Methodology

2.1. Development of a test for signal autocoherence

This paper develops below a model for a signal with randomly modulated periodicity, and a measure known as a signal coherence function, which embodies the amount of random variation in each Fourier component of the signal. Let $\{x(t_n), n=0,1,2,\ldots\}$ be a time series of interest sampled at equally spaced times $t=n\delta$. The series would be said to exhibit randomly modulated periodicity with period T if it is of the form

$$x(t)=\frac{a_0}{K}+\frac{1}{K}\sum_{k=1}^{K}\{a_{1k}+u_{1k}(t)\}\cos(2\pi f_k t)+\frac{1}{K}\sum_{k=1}^{K}\{a_{2k}+u_{2k}(t)\}\sin(2\pi f_k t) \tag{1}$$

where $f_k=k/T$ are called Fourier frequencies and $K=T/\delta$. The $u_{ik}(i=1,2)$ are jointly dependent zero-mean random processes that are periodic block stationary and satisfy finite dependence. The modulations are in the fundamental and harmonic frequency components and so a_0 is a constant. Note that we do not require $u_{1k}(t_n)$ and $u_{2k}(t_n)$ to be Gaussian. It is apparent from equation (1) that the random variation occurs in the modulation rather than being additive noise; in statistical parlance, the specification in equation (1) would be termed a random-effects model. The modulations are produced by the data-generating mechanism and may be deterministic but in most cases the statistician analysing the data does not know the modulations process and thus they are treated as random processes. The signal $x(t_n)$ can be expressed as the sum of a deterministic (periodic) component and a stochastic process term:

$$x(t)=\frac{a_0}{K}+\frac{1}{K}\sum_{k=1}^{K}a_k\exp(2\pi \mathrm{i} f_k t)+\frac{1}{K}\sum_{k=1}^{K}u_k(t)\exp(2\pi \mathrm{i} f_k t) \tag{2}$$

where $a_k=a_{1k}-\mathrm{i}a_{2k}$ and $u_k(t_n)=u_{1k}(t_n)-\mathrm{i}u_{2k}(t_n)$. The task at hand then becomes one of quantifying the relative magnitude of the kth modulations to the magnitude of the fixed effect a_k for each k.

A common approach to processing signals with a periodic structure is to portion the observations into M frames, each of length $T=N\delta$, so that there is exactly one waveform in each sampling frame. There could alternatively be an integer multiple of T observations in each frame. The periodic component of $a(t)$ is the mean component of $x(t)$. To determine how stable the signal is at each frequency across the frames, the notion of signal coherence is employed. Signal coherence is loosely analogous to the standard R^2-measure that is used in regression analysis and quantifies the degree of association between two components for each given frequency. It is worth noting that the methodology that we propose here is based on the coherence of the signal across the frames for a single time series (which may also be termed autocoherence). This is quite different from the tests for signal coherence across markets that were used, for example, by Hilliard (1979) and Smith (1999). (Both employed the frequency domain approach to examine the extent to which stock-markets co-move across countries. Our technique is also distinct from that proposed by Durlauf (1991) and used by Fong and Ouliaris (1995) to detect departures from a random walk in five weekly US dollar exchange rate series. Both Fong and Ouliaris (1995) and Andersen and Bollerslev (1997a) detected long memory effects in the currency rates.)

The discrete Fourier transform of the mth frame, beginning at observation $\beta_m = (m-1)T + \delta$ and ending at observation mT for frequency $f_k = k/T$, is given by

$$\begin{aligned} x_m(k) &= \sum_{t=0}^{T-\delta} x(\beta_m + t)\exp(-2\pi \mathrm{i} f_k t) \\ &= a_k + U_m(k) \end{aligned} \tag{3}$$

where

$$U_m(k) = \sum_{t=0}^{T-\delta} u(\beta_m + t)\exp(-2\pi \mathrm{i} f_k t).$$

The variance of $U_m(k)$ is given by

$$\sigma_u^2(k) = \sum_{\tau=0}^{T-\delta} \exp(-2\pi \mathrm{i} f_k \tau) \sum_{t=0}^{T-\tau-1} c_u(t, t+\tau) \tag{4}$$

where $c_u(t_1, t_2) = E\{u_m^*(t_1)\, u_m(t_2)\}$, and the variance is of order $O(T)$. Provided that $u_m(t_n)$ is weakly stationary, equation (4) can be written as

$$\sigma_u^2(k) = T\,\{S_u(f_k) + O(1/T)\} \tag{5}$$

where $S_u(f)$ is the spectrum of $u(t_n)$.

Although the model for randomly modulated periodicity is not in general a cyclostationary model, when the assumption of weak stationarity is added, it becomes so (see Gladyshev (1961), Gardner and Franks (1975) and Gardner (1985, 1994) for extensive writings on cyclostationarity). The signal coherence function $\gamma_x(k)$ measures the variability of the signal across the frames and is defined as follows for each frequency f_k:

$$\gamma_x(k) = \sqrt{\left\{ \frac{|a_k|^2}{|a_k|^2 + \sigma_u^2(k)} \right\}}. \tag{6}$$

It is obvious from the construction of $\gamma_x(k)$ in equation (6) that it is bounded to lie on the interval [0,1]. The end point case $\gamma_x(k) = 1$ will occur if $a_k \neq 0$ and $\sigma_u^2(k) = 0$, which is the case where the signal component at frequency f_k has a constant amplitude and phase over time, so that there is no random variation across the frames at that frequency (perfect coherence). The other end point, $\gamma_x(k) = 0$, will occur if $a_k = 0$ and $\sigma_u^2(k) \neq 0$, when the mean value of the component at frequency f_k is 0, so that all of the variation across the frames at that frequency is pure noise (no coherence).

The signal coherence function is estimated from the actual data by taking the Fourier transform of the mean frame and for each of the M frames. The mean frame will be given by

$$\bar{x}(t_n) = \frac{1}{M} \sum_{m=1}^{M} x(\beta_m + t_n), \qquad n = 0, 1, \ldots, N-1. \tag{7}$$

Letting $\hat{a}(k)$ denote the mean frame estimate, with its Fourier transform being $\hat{A}(k)$, and letting $X_m(k)$ denote the Fourier transform for the mth frame, then $D_m(k) = X_m(k) - \hat{A}(k)$ is a measure of the difference between the Fourier transforms of the mth frame and the mean frame for each frequency. The signal coherence function can then be estimated by

$$\hat{\gamma}_x(k) = \sqrt{\left\{ \frac{|\hat{A}_k|^2}{|\hat{A}_k|^2 + (1/M) \sum_{m=1}^{M} |D_m(k)|^2} \right\}} \tag{8}$$

and $0 \leqslant \hat{\gamma}_x(k) \leqslant 1$. It can be shown (see Hinich (2000)) that the null hypothesis of zero coherence at frequency f_k can be tested by using the statistic $M\,\hat{\gamma}_x(k)^2/\{1 - \hat{\gamma}_x(k)^2\}$, which is asymptotically distributed under the null hypothesis as a non-central χ^2-distribution with 2 degrees of freedom and non-centrality parameter given by

$$\lambda_k = Ma_k^2/T\ S_u(f_k),$$

where $S_u(f_k)$ is the spectrum of $\{u(t)\}$ at the frequency f_k. We also employ a joint test of the null hypothesis that there is zero coherence across the M frames for all $K/2$ frequencies examined. This test statistic will asymptotically follow a non-central χ^2-distribution with K degrees of freedom.

2.2. *Development and testing of a trading rule*

The sample is split into ($M =$) 52 non-overlapping frames each of length 1 week, with each week of observations containing 240 half-hourly observations. (The choice of frame length is bound to be somewhat arbitrary, although in our case it represents a trade-off between having a sufficient number of frames over which to compute the tests, while having a sufficiently long frame to detect interesting periodic components.) This implies that a total of 120 periodicities will be examined: 240, 120, 80, 60, 48, …, 240/119, 2, using the whole sample of data, and the autocoherence measures are calculated across the 52 frames. Following initial exploratory analysis and estimation of the signal coherence function for the whole year's set of frames data, the sample is then split into two portions. The first 26 weeks (6240 half-hourly observations) are used for in-sample estimation of the coherence function (across the resulting 26 frames), and the remaining data are held back for out-of-sample trading rule evaluation. The out-of-sample period begins with the return of 00:30 a.m.–01:00 a.m. on July 1st, 1996.

For the out-of-sample trading rule analysis, the signal coherence function is re-estimated using the first half of the sample only, and the mean frame is 'cleaned' by removing all frequency components with signals whose random variation implies that they are not statistically coherent at the 1% level of significance. Statistically, if $\Pr\{\hat{\gamma}_x(k) = 0\} < 0.01, \hat{a}_{jk}\ (j = 1, 2)$ are kept; otherwise $\hat{a}_{jk}$ are set to 0. The coherent part of the mean frame is analogous to the fitted value of the dependent value in a standard regression model. Retaining only the most coherent frequencies for each series, we implement a trading rule that is based on these observed periodicities, 'buying' the foreign currency if the return is predicted to be positive and assuming no position if it is predicted to be negative. The trading rule is then compared with a buy-and-hold the foreign currency strategy and also in a novel way involving a simulation. To evaluate the statistical significance of the strategies, we generate random binomial 0–1 draws equal in number to the out-of-sample observations. This column of 0s and 1s is then multiplied by the actual return series and added over the hold-out sample to generate a profit from a randomized market entry and exit rule. This is repeated 10000 times to generate a distribution of artificial rules and subsequent artificial returns, which is then compared with the profit that is generated by the rules that are based on the coherent periodic signal. If the actual rule generates a profit that is larger than 95% of those generated by artificial timing, it is considered to produce statistically significant abnormal returns.

We then exploit the information that is contained in the lower half of the signal coherence function by short selling the foreign currency when it is periodically expected to exhibit negative returns, as well as taking a long-term position when it is expected to yield positive returns. The relevant simulation comparator is now one where the column of 0s and 1s is modified to a column of -1 and 1 to be mutliplied by the actual returns to give a set of returns from randomized long- and short-term positions.

3. Data

The high frequency financial data that were provided by Olsen and Associates as part of the HFDF-96 package includes 25 exchange rate series sampled half-hourly for the whole of 1996, making a total of 17 568 observations for each series. However, this series contains observations corresponding to week-end periods when all the world's exchanges simultaneously have virtually no trading. This period is the time from 23:00 h Greenwich Mean Time (GMT) on Friday when North American financial centres close until 23:00 h GMT on Sunday when Australasian markets open. The incorporation of such prices would lead to spurious zero returns and would potentially render trading strategies which recommended a buy or sell at this time to be nonsensical. Removal of these week-end observations leaves 12 575 observations for subsequent analysis and forecasting.

We do not account for differences in the dates that different countries switch to daylight saving time, since the effect of this 1-h difference is likely to be negligible as trading occurs virtually at all times at some destination around the world. Moreover, it is not clear how such an adjustment could be made. This problem would be much more serious if we were examining, say, cross-correlations between equity returns for stocks on markets that were in different time zones. Such an approach appears to be consistent with the existing literature—for example, Andersen and Bollerslev (1997b) did not correct for bank-holiday effects. Finally, it is worth noting that if asynchronous switches to daylight saving time did make a difference to our results, since it is not possible to create coherence, only to destroy it, the effect would be to render our results less strong than they otherwise would have been rather than to cause spurious findings.

The price series that are used are the average of the most recent best bid and best ask prices in that half-hour interval and are transformed into a set of continuously compounded half-hourly percentage returns in the standard fashion. The first observation in the sample corresponds to the period between 00:30 a.m. and 01:00 a.m. GMT on January 1st, 1996, whereas the last corresponds to 11:30 p.m.–midnight GMT on December 31st of the same year.

Of the 25 exchange rate series that were provided by Olsen, only seven are used in this study to avoid repetition, and for brevity. These are (using the usual mnemonic) DEM_JPY, GBP_DEM, GBP_USD, USD_CHF, USD_DEM, USD_ITL and USD_JPY. These are all quoted as units of the second (foreign) currency per unit of the first (domestic) currency. For example, xxx_yyy would be the units of yyy per unit of xxx. Hence a positive return implies that the number of units of the foreign currency yyy per unit of the domestic currency (xxx) has increased. This would imply a profit for an investor whose domestic currency is xxx, but who had switched into yyy at the start of the period and converted the terminal sum back at the end.

Some summary statistics for these seven returns series are presented in Table 1. It is clearly evident that all series are non-normal (predominantly because of fat tails rather than asymmetry), and all exhibit evidence of negative first-order autocorrelation, and conditional heteroscedas-

Table 1. Summary statistics for half-hourly exchange rate returns†

Parameter	*DEM_JPY*	*GBP_DEM*	*GBP_USD*	*USD_CHF*	*USD_DEM*	*USD_ITL*	*USD_JPY*
Mean	3.4×10^{-4}	9.7×10^{-4}	5.6×10^{-4}	8.6×10^{-4}	4.5×10^{-4}	-2.1×10^{-4}	6.5×10^{-4}
Variance	6.5×10^{-3}	4.6×10^{-3}	4.8×10^{-4}	8.5×10^{-3}	5.1×10^{-3}	9.0×10^{-3}	6.2×10^{-3}
Skewness	−0.049	−0.004	−0.167	−0.156	−0.190	−0.011	−0.019
Kurtosis	5.642	83.516	13.014	79.408	11.200	15.719	9.723
Minimum	−0.707	−1.966	−1.137	−2.431	−0.698	−0.924	−0.770
Maximum	0.659	1.992	1.203	2.403	0.777	0.966	0.758
Autocorrelation function lag 1	−0.198	−0.306	−0.205	−0.189	−0.097	−0.315	−0.150
Autocorrelation function lag 2	−0.013	−0.0053	−0.001	−0.004	0.005	−0.019	−0.002
Autocorrelation function lag 3	0.008	0.007	−0.000	0.002	0.015	−0.000	0.005
Autocorrelation function lag 4	−0.006	0.000	0.004	−0.009	−0.005	−0.005	−0.008
Autocorrelation function lag 5	0.006	0.004	−0.000	0.032	0.002	−0.017	−0.005
LB-Q(10)	500‡	3144‡	536‡	476‡	129‡	1261‡	288‡
ARCH(4)	601.1‡	23.6‡	1355‡	2559‡	693‡	1616‡	462‡
BJ norm	4×10^{5}‡	2×10^{10}‡	9×10^{4}‡	3×10^{6}‡	7×10^{4}‡	9×10^{4}‡	2×10^{4}‡
BDS	32.47‡	30.68‡	41.00‡	37.52‡	38.95‡	44.12‡	33.27‡
% 0s	7.5	6.1	5.0	5.7	5.1	5.4	4.9

†Kurtosis represents excess kurtosis; LB-Q(10) is a Ljung–Box test for autocorrelation of all orders up to 10 and is asymptotically distributed as $\chi^2(10)$ under the null hypothesis; ARCH(4) is Engle's (1982) Lagrange multiplier test for autoregressive conditional heteroscedasticity which is asymptotically distributed as $\chi^2(4)$; the BJ norm is the Bera–Jarque normality test, which is asymptotically distributed as $\chi^2(2)$ under the null hypothesis of normality; BDS is the Brock *et al.* (1987) test for independent and identically distributed data which is distributed asymptotically as standard normal under the null hypothesis (the statistic shown is for $m=5$ and $\varepsilon/\sigma=1$); % 0s gives the percentage of returns that are 0 (i.e. no price change).
‡Significant at the 1%-level.

ticity (as the Ljung–Box and Engle tests respectively show). The Brock–Dechert–Scheinkman (BDS) statistic therefore rejects the null hypothesis of independent and identical distribution at the 0.1% level of significance.

4. Results

4.1. Evidence for periodicities

Table 2 gives the p-values for tests of the joint null hypothesis that there is zero coherence at all frequencies examined for the whole 52-week sample. Clearly, there is significant evidence of coherence at one or more frequencies for the pound–mark and the dollar–yen exchange rates, whereas the result for the dollar–lira is marginal. The joint test results also suggest that there is

Table 2. p-values for the joint test of the null hypothesis that there is no signal coherence for all 120 frequencies

	DEM_JPY	*GBP_DEM*	*GBP_USD*	*USD_CHF*	*USD_DEM*	*USD_ITL*	*USD_JPY*
p-value	0.1532	0.0221	0.5342	0.2574	0.5109	0.0772	0.0000

Table 3. Periodicities with coherence statistics that are significant at the 1%-level

Period†	*Log-spectrum (dB)*	*Coherence statistic*	*Coherence statistic p-value*
(a) DEM_JPY			
120.0 (half a week = 60 h)	−2.164	0.314	0.003
4.068 (2 h 2 min)	−0.165	0.307	0.004
(b) GBP_DEM			
12.00 (6 h)	−1.357	0.312	0.003
4.286 (2 h 8 min)	−0.775	0.329	0.002
3.288 (1 h 38 min)	−1.534	0.317	0.003
(c) GBP_USD			
24.00 (12 h)	−3.688	0.302	0.005
16.00 (8 h)	−2.882	0.295	0.007
(d) USD_CHF			
30.00 (15 h)	−1.320	0.336	0.001
16.00 (8 h)	−2.110	0.345	0.001
7.059 (3 h 32 min)	−0.803	0.299	0.006
4.000 (2 h)	0.096	0.283	0.010
(e) USD_DEM			
30.00 (15 h)	0.390	0.352	0.001
16.00 (8 h)	−0.365	0.331	0.002
(f) USD_ITL			
24.00 (12 h)	−4.166	0.308	0.004
11.43 (5 h 43 min)	−3.135	0.329	0.002
2.824 (1 h 49 min)	0.139	0.353	0.001
(g) USD_JPY			
120.0 (half a week = 60 h)	−1.669	0.341	0.001
30.00 (15 h)	−0.956	0.362	0.000
16.00 (8 h)	−1.399	0.349	0.001
5.217 (2 h 37 min)	0.046	0.298	0.006
2.087 (1 h 5 min)	0.907	0.297	0.006

†In units of half-hours.

no coherence at any frequency for the other four exchange rate series. A non-rejection from the joint test does not in practice imply that there is actually no coherence at any frequency, however, since the effect of significance at one or two frequencies could be diluted by many insignificant frequencies. Hence Table 3 presents the periodicities for which the individual autocoherence estimates are statistically significant, together with the associated p-values for the χ^2-test, and the logarthm of the spectrum (in decibels). Although there is no single periodicity where all seven series show significant coherence simultaneously, there are several common features across the exchange rate returns. The mark–yen and dollar–yen rates both have statistically significant (at the 0.3%-level or higher) autocoherence at a periodicity of 120 half-hourly units (i.e. 60 h, or half a week), and all the series denominated against the dollar, except the lira, show autocoherence at the 8-h periodicity. Interestingly, the dollar–Swiss franc and the dollar–yen exchange rates also have significant coherence at the 15-h periodicity, corresponding to 8 cycles per week. In the latter case, this is the most empirically stable periodicity, with a coherence statistic of 0.362 (on a 0–1 scale), and an associated p-value of less than 0.1%. What is also evident from

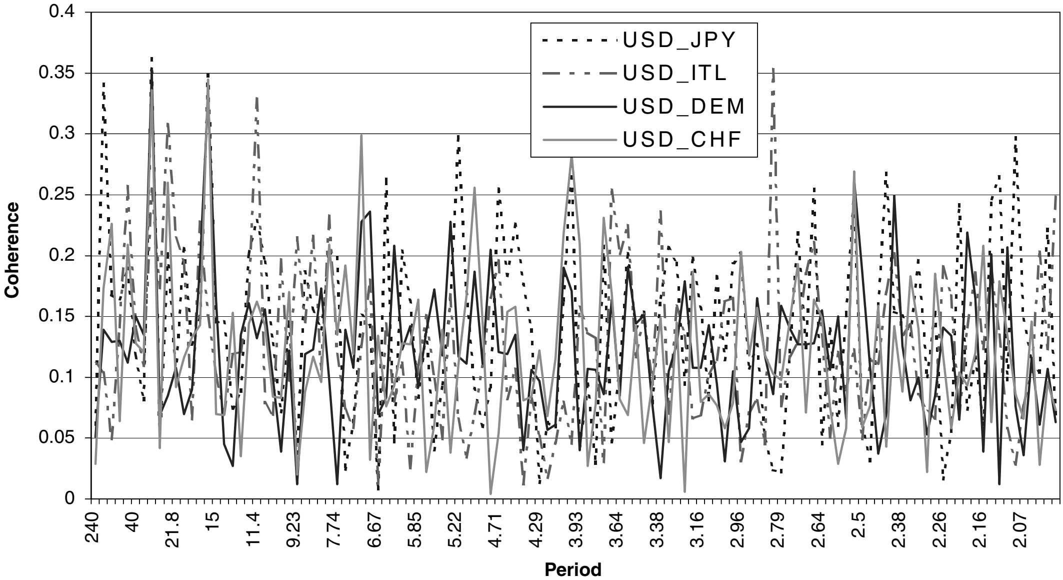

Fig. 1. Coherence against period for USD_CHF, USD_DEM, USD_ITL and USD_JPY

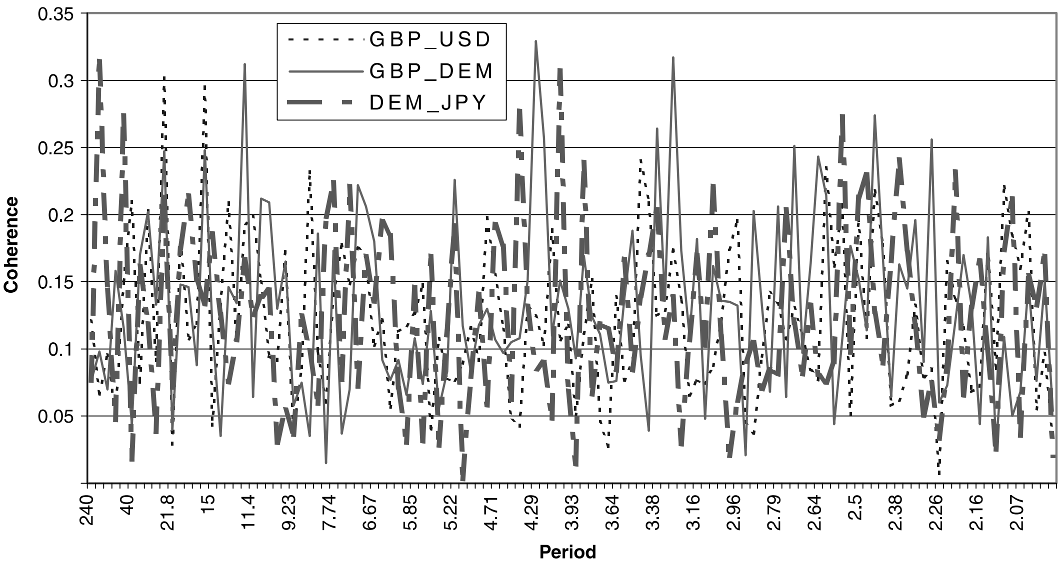

Fig. 2. Coherence against period for DEM_JPY, GBP_DEM and GBP_USD

Table 3 is that none of the coherence statistics are larger than 0.362, implying that there is still a considerable amount of variation in the waveform over the frames even for the most coherent parts. The log-spectrum in decibels is 20 times the natural logarithm of the spectral amplitude or, equivalently, it is 10 times the spectrum (in variance units). The measure in decibels, as shown in Table 3, gives on a log-scale the average sizes of the periodic movements in terms of the heights of the peaks and troughs of the coherent periodicities. Whereas autocoherence quantifies how stable these periodicities are, the amplitude measures how big the cyclical fluctuations are. It is evident from the second column of Table 3 that, in general, the higher frequency components

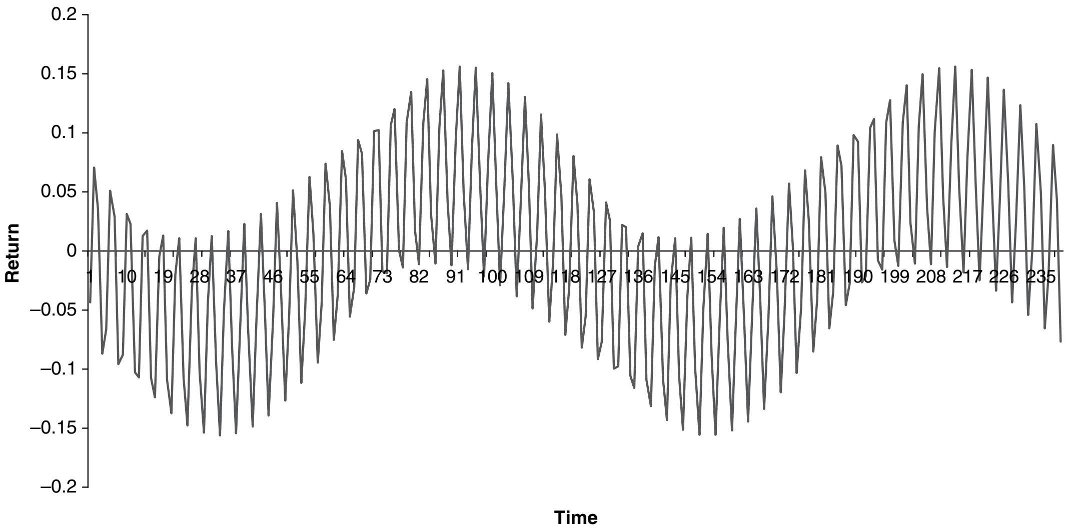

Fig. 3. Coherent part of the mean frame for a week for DEM_JPY

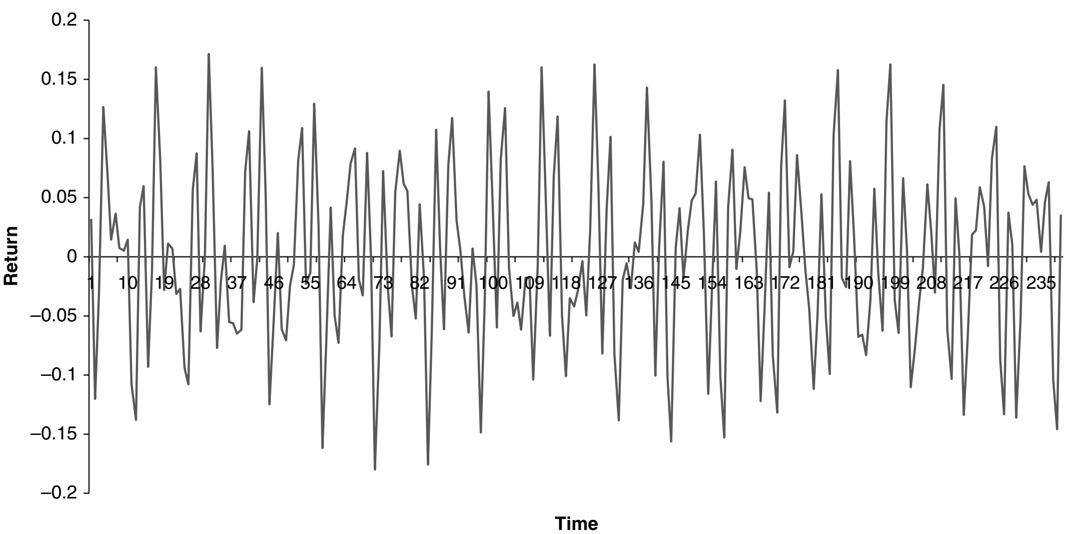

Fig. 4. Coherent part of the mean frame for a week for GBP_DEM

have the largest amplitudes, with some even being positive (on a log-scale) at frequencies of lower than 3 h for the dollar–Swiss franc, dollar–lira and dollar–yen exchange rates.

How can we explain the observed 8-, 12- and 15-h periodicities? Several factors could justify their existence, including the opening and closing times of the three major markets in different time zones (London, New York and Japan), and related changes in market volatility and liquidity through each 24-h period. For example, Andersen and Bollerslev (1998), Fig. 3, reported cycles in intraday volatility, where it is highest from 1 p.m. to 5 p.m., and lowest from 3 a.m. to 5 a.m. (GMT). It may therefore simply be that the cycles in returns are rewards for bearing time-varying intradaily risks, which are themselves cyclical. A cycle that repeats every 8 h is consistent with an effect that is driven by the opening and closing of the three markets, whereas

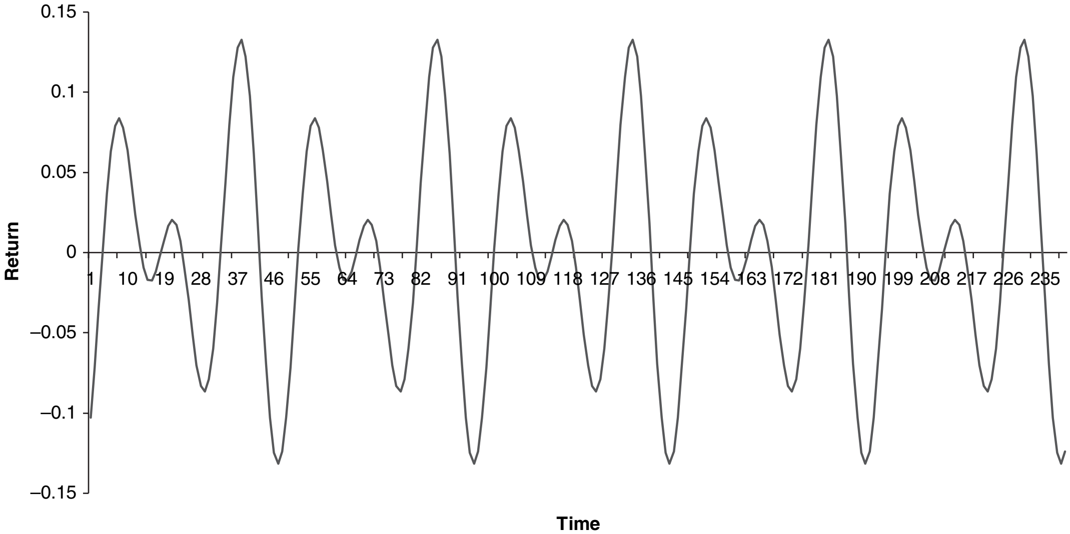

Fig. 5. Coherent part of the mean frame for a week for GBP_USD

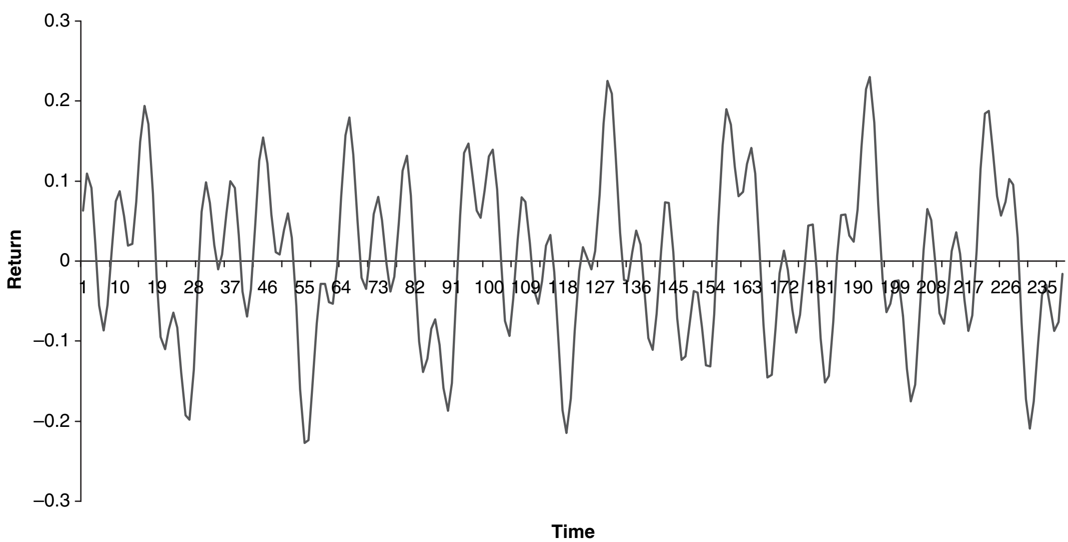

Fig. 6. Coherent part of the mean frame for a week for USD_CHF

a 15-h cycle is consistent with an effect that is attributable to the currency being heavily traded in two of the three major centres (e.g. the yen–dollar rate being heavily traded in New York and Tokyo but not London). (Periodicities of 15 h rather than 16 h would result if the time between the time when one regional market 'opens' and the regional market in the next time zone 'closes' were 15 h rather than 16 h owing to slightly shorter busy trading hours.) This is exactly the kind of result that Baillie and Bollerslev (1991) observed when hourly dummy variables were applied to intradaily exchange rate volatility. The Asian market generated much less volatility than the other two, with noticeable increases in volatility occurring around the start of the trading days for the London and New York markets.

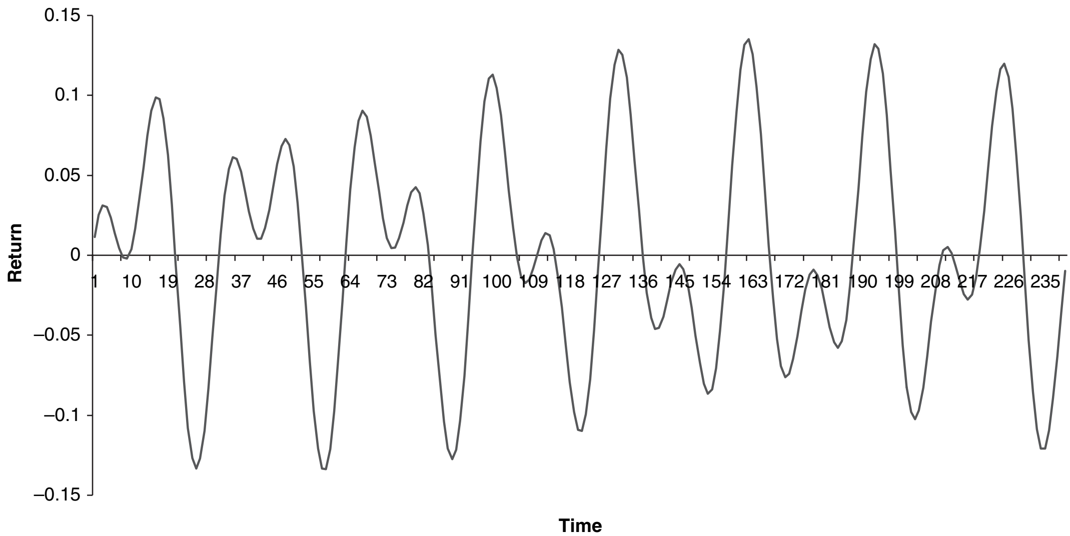

Fig. 7. Coherent part of the mean frame for a week for USD_DEM

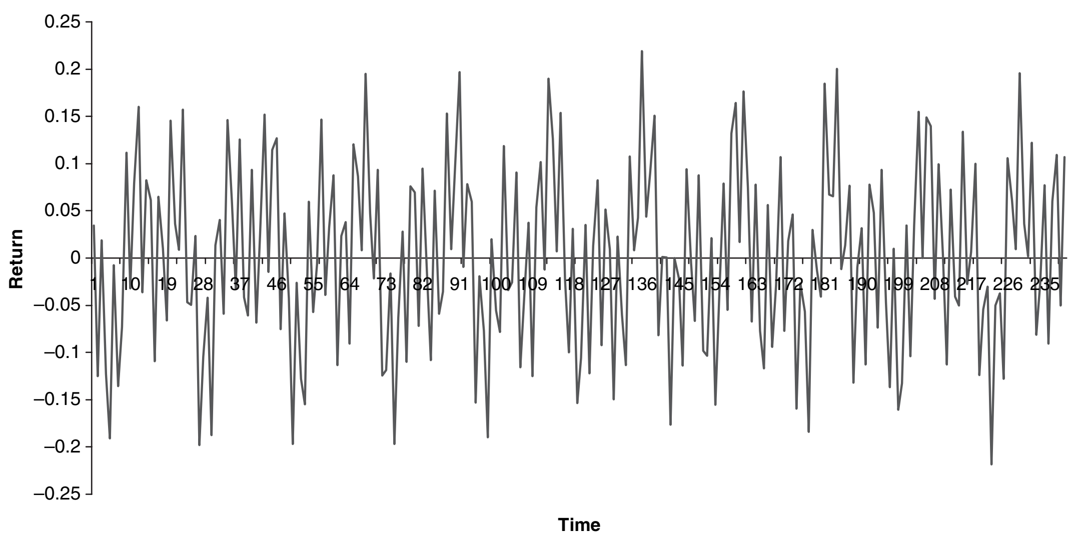

Fig. 8. Coherent part of the mean frame for a week for USD_ITL

A 12-h cycle is arguably more difficult to explain but may arise from behaviour that is caused by the full trading day of one market combined with the morning of another (e.g. if the German mark–yen exchange rate was actively traded during the full European trading day and for the morning of the Japanese trading day, but with very little trading for the Japanese afternoon and the whole of the North American trading day).

Figs 1 and 2 present plots of the coherence statistics at each frequency for the dollar-denominated and non-dollar-denominated currencies respectively. Fig. 1 seems to suggest some correlation between the coherences for the dollar–mark and dollar–Swiss franc rates, whereas in all cases the low frequency components (those with the highest periods) show less variation across frames; most coherence statistics appear to lie within the range (0.1, 0.2). Fig. 2, in contrast,

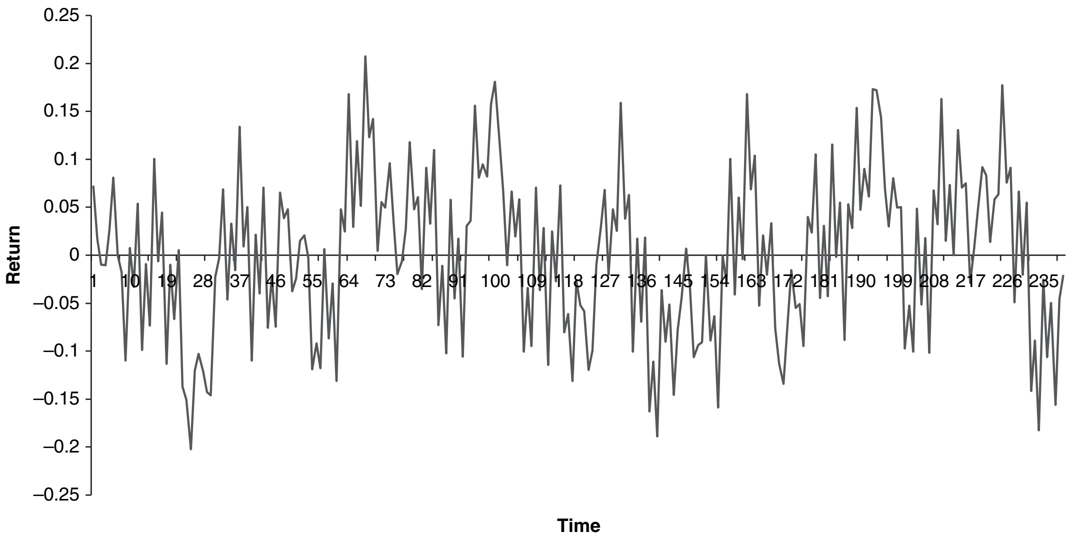

Fig. 9. Coherent part of the mean frame for a week for USD_JPY

seems to show no visible relationship across the currencies in the coherences of the waveforms at any frequency. Again, the statistics appear largest at the lowest frequencies, and smallest at the hourly frequency.

Figs 3–9 plot the coherent part of the mean frame for frames of length 1 week for each of the returns series. As we would expect, the graphs look very different from one another, at least at first glance, since different frequencies have been retained for different currencies and, even when the same frequencies are included, differences in their relative amplitudes will alter the shape of the plot. In all cases, however, the cyclical patterns are obvious in the figures now that the mean frame has been purged of frequencies with higher amounts of random variation. In Fig. 3, it is evident that the mark–yen rate has a low frequency component (with 2 full cycles per week) and a high frequency cycle around that with a period of approximately 2 h. In Fig. 4, the low frequency component of the pound–mark exchange rate is more difficult to visualize since it is smaller in amplitude than that of the mark–yen rate and there are two high frequency cycles around it. Figs 5–7, showing plots of the coherent parts of the mean frame for the pound–dollar, dollar–Swiss franc and dollar–mark exchange rates respectively, look similar in shape as all have two coherent frequencies that are close together that have exceeded the 99% threshold. The dollar–Swiss franc exchange rate also has two additional coherent high frequency components. In Fig. 8, the coherence plot for the dollar–lira exchange rate has a very jagged appearance, due to the high frequency (approximately 1 h 20 min) component with far larger amplitude than the other two. Finally, Fig. 9 showing the dollar–yen exchange rate is arguably the most complex, since the highest number of frequencies (five) have been retained.

It is also of interest to examine Figs 3–9 to determine on average at which times of the week the exchange rate moves in particular directions. The x-axis shows the half-hourly observations for a week, starting with the return for the interval midnight–00:30 a.m., and plotted against it the average continuously compounded return, in per cent, during that period. Examining the low frequency cyclical component of Fig. 3, the returns are negative (i.e. the mark weakens) for the first 60 half-hours (1 and $\frac{1}{4}$ trading days), then the mark rises until mid-week, before falling again for $1\frac{1}{4}$ days and rising on Friday. The most easily interpretable behaviour can be examined where there are time-zone differences between the foreign and domestic currencies, and coherent frequency components with periods of 8, 12 or 15 h, which there appears to be for many of the other series. For example, the pound–dollar exchange rate (Fig. 5) on average rises (the pound becomes stronger) from the early hours of the morning GMT, until midday GMT, followed by falls when the US east coast markets start trading. The dollar–German mark exchange rate on average rises (the dollar strengthens) until around 10 a.m. GMT and then falls (the mark strengthens) while the European markets are open until 4 p.m. GMT, followed by further dollar appreciation. Similarly, the mark–Swiss franc exchange rate (Fig. 6) on average rises (the dollar strengthens) until 10 a.m., with falls (the franc appreciates) until around 3 p.m., followed by franc depreciation into the European evening. The dollar–yen exchange rate (Fig. 9) appears to fall (the dollar weakens) from midnight until 4 a.m. GMT, when the US markets are closed, and then rises until around 2 p.m. GMT before falling back again. Finally, the dollar–lira exchange rate has such a large number of high frequency movements that any associations with market opening and closing times are indiscernible.

These results support those of Ito and Roley (1987), who found a systematic dollar appreciation against the yen during the US trading hours, and a systematic depreciation during Japanese trading hours. Our results are also consistent with those of Baillie and Bollerslev (1991), who observed hourly patterns in foreign exchange market volatility related to major market opening and closing times. This is, perhaps, to be expected, since the weight of trading volume will move between the different world centres through the day.

4.2. *Evaluation of trading rule profitability*

Given the results that were presented above showing evidence that there are periodically recurring patterns in all seven exchange rate return series, we now continue to investigate whether trading opportunities arise from exploiting such structure in the first half of the sample, as described above. The total gross profits, continuously compounded, and expressed as a percentage of the initial investment, for the 6-month out-of-sample period, are presented in Table 4. The row immediately following the column headers presents the gross profits for a rule that is based on a long-term position only in the foreign currency or no position (i.e. it is assumed that no short sales of the foreign currency are permitted). For comparison, part (c) of Table 4 presents the profitability of buying the foreign currency at the start of the 6-month period, holding it until the end of the year and then converting it back to the domestic currency. This is termed a 'buy and hold the foreign currency' rule. As can be seen, the trading rules that are based on the notion of signal coherence appear to generate useful entry and exit rules since the returns in all cases exceed those which were obtained by simply buying and holding the foreign currency, except for the pound–dollar exchange rate. For example, a British investor buying marks and holding them for the 6 months to December 1996 would have experienced a foreign currency appreciation of 10.8%, but switching in and out of marks using the coherent parts of the signal as a guide would have increased returns to 17%. The lira depreciated against the dollar over the period by 0.11%, so a US investor would have lost money by buying and holding lira. However, the switching rules would have generated gross profits of 7.8% over the period.

The sixth row of part (a) in Table 4 presents the percentage of times that randomly generated entry times into the foreign currency would have led to higher returns than those generated by the signal-coherence-generated rules. It is then argued that figures below, say, 5% imply that the rule generates profits that are statistically significant at the 5%-level, since this would indicate that it is less than 5% likely that the rules could generate such high profits by chance alone. The figures in the third row of Table 4 are lower than 1% for four of the cases and are only larger than 5% for the pound–dollar exchange rate. In the case of the dollar–yen rate, none of the 10000 sets of randomized rules could generate a higher return than that of the rules which

Table 4. Total profit of trading rules derived from the coherent part of the signal for 6 months: July 1st–December 31st, 1996†

Rule	*DEM_JPY*	*GBP_DEM*	*GBP_USD*	*USD_CHF*	*USD_DEM*	*USD_ITL*	*USD_JPY*
(a) Long trades only							
Coherence rule total 6-month return (long only) (%)	7.16	17.02	6.70	15.75	6.99	7.75	18.43
Standard deviation of returns (%)	5.59	5.80	4.91	6.76	5.00	7.30	5.59
Largest $\frac{1}{2}$-h gain	0.51	1.99	0.72	0.66	0.62	0.99	0.62
Largest $\frac{1}{2}$-h loss	0.56	0.64	0.85	0.72	0.59	0.92	0.45
Sharpe ratio	0.012	0.038	0.012	0.030	0.013	0.010	0.044
% of randomized rules with higher return	0.00	0.11	38.05	0.13	0.40	1.42	0.00
% of correct direction predictions	53.09	53.76	54.32	53.05	54.03	55.03	55.50
% of correct predictions that would arise if the buy–sell signals and returns were independent	50.14	50.04	50.06	50.06	49.99	50.03	50.05
Pesaran–Timmerman statistic	4.70‡	5.94‡	6.80‡	4.78‡	6.51‡	7.99‡	8.69‡
Number of round trip trades per week	52	46	15	21	10	59	52
(b) Long and short trades							
Coherence rule total 6-month return (long and short)	9.28	23.20	4.55	24.56	12.52	15.62	31.84
% of randomized rules with higher return	0.00	0.02	22.25	0.06	0.65	1.60	0.00
Standard deviation of returns (%)	7.79	8.41	7.34	10.31	6.79	9.67	7.94
Largest $\frac{1}{2}$-h gain	0.51	1.99	0.72	0.66	0.62	0.97	0.62
Largest $\frac{1}{2}$-h loss	0.56	0.64	0.85	0.72	0.59	0.92	0.45
Sharpe ratio	0.013	0.038	0.004	0.033	0.022	0.021	0.058
(c) Buy-and-hold foreign currency rule results							
Buy-and-hold foreign currency	5.02	10.84	8.85	6.94	1.45	−0.11	5.02
Bid–ask spread	0.054	0.054	0.048	0.056	0.041	0.069	0.052
Standard deviation of returns (%)	7.79	8.42	7.34	10.31	6.80	9.67	7.96
Sharpe ratio	0.004	0.015	0.013	0.006	−0.003	−0.005	0.042

†The bid–ask spread is taken as the difference between the bid and ask prices divided by the average of the bid and ask prices, and multiplied by 100 to express it as a percentage of the exchange rate, and the average is taken over the sample. The Pesaran–Timmerman statistic is asymptotically distributed standard normal under the null hypothesis of return and entry–exit timing independence.
‡Significant at the 1%-level.

are based on spectral methods. Clearly, the gross profitability of the entry and exit rules from foreign currency investment appear to show evidence of market timing ability. This is examined statistically by using the Pesaran and Timmerman (1992, 1994) nonparametric test for market timing ability, which is generalized from the Henriksson–Merton (Henriksson and Merton, 1981; Merton, 1981) test for independence between the signs of the forecast and realized values. The statistic is not derived here but is asymptotically distributed standard normal under the null hypothesis that the signs of returns and the buy–sell signals are independent. (Interested readers are referred to Pesaran and Timmerman (1992, 1994), Henriksson and Merton (1981) and Merton (1981) for a derivation of the test statistic.) The number of correct sign predictions varies between 53% and 55% across the currencies, whereas the number of correct predictions that would be expected to result if the buy–sell signals and actual returns were independent is very close to 50% in all cases. Note that the use of very high frequency data implies that a non-negligible number of actual returns per half-hour interval are 0. Although it is not clear from an examination of the formulation of the test, we define zero returns as implying correct sign predictions if the rule signal for that observation had been a 0 (so that the transactions costs that are associated with buying the foreign currency are correctly avoided). However, incorrect sign predictions are argued to result if the rule generated no buy signal, but the return turned out to be positive. The relatively high proportion of correct sign predictions combined with the large sample size leads the Pesaran–Timmerman statistic to reject the null hypothesis at the 1%-level convincingly in all cases.

The trading profitability of rules exploiting the sell signals that were generated for those observations when the coherence plots are negative are also examined in part (b) of Table 4. In this case, a positive expected return for a given observation is taken to imply a buy order, whereas a negative expected return is taken to imply short selling, with all results being expressed as a percentage of the initial position. In all cases except one, enabling short selling leads to considerable increases in gross profitability—in fact, gross profitability is typically doubled, as we may have expected. For example, the 18.4% and 7% gross profits from trading the yen against the dollar and the mark against the dollar respectively become 31.8% and 12.5%. However, the profit that is made by taking long-term positions only in the dollar against the pound actually fell when short-term positions are permitted. This result does not seem to arise from anomalous behaviour in the series, such as a larger trend component or larger outliers than were present in the other series. Rather, it is possible simply that the behaviour of the pound–dollar exchange rate was more different from the others between the in-sample and out-of-sample periods; an examination of the coherence statistics revealed that this was indeed so.

Comparing the standard deviations of the long-only, long and short, and buy-and-hold rules, the last two have very similar risk profiles, whereas the long-only rule implies, by definition, less risk since there will be a flat position in the foreign currency for part of the period. Also presented are the largest half-hourly gains and largest half-hourly losses for each of the three strategies. In all cases except that of the dollar–lira exchange rate, the largest gains and largest losses are identical for the long-only and long and short rules, suggesting that the extreme returns in both directions arise from being long rather than short. The Sharpe ratios for each rule and currency are also calculated by taking the average half-hourly risk-free rate of return (proxied by the domestic country 3-month Treasury bill rate) from the average half-hourly rule rate of return, and dividing by its standard deviation. Again, across the three panels of Table 4, in all cases except the pound–dollar exchange rate, the Sharpe ratios are larger for the coherence-based rules than for the buy-and-hold approach. This suggests that, after accounting for any differences in the riskiness that are inherent in the rules, they can still outperform a buy and hold the foreign currency rule. Also, the long and short rule Sharpe ratio is typically only very slightly higher than that of the long-only approach. Thus, even though gross profitability generally increases considerably when short positions are permitted, the increased risk counters this to leave the risk-adjusted performance only slightly altered.

Fig. 10 shows the cumulative returns for the long and short trading rules based on the signal coherence concept. Buying and selling German marks is the only activity that is seen to lose an American investor money for a protracted period. Trading in German marks using the methods described above would have been a loss-making activity from July until mid-September when

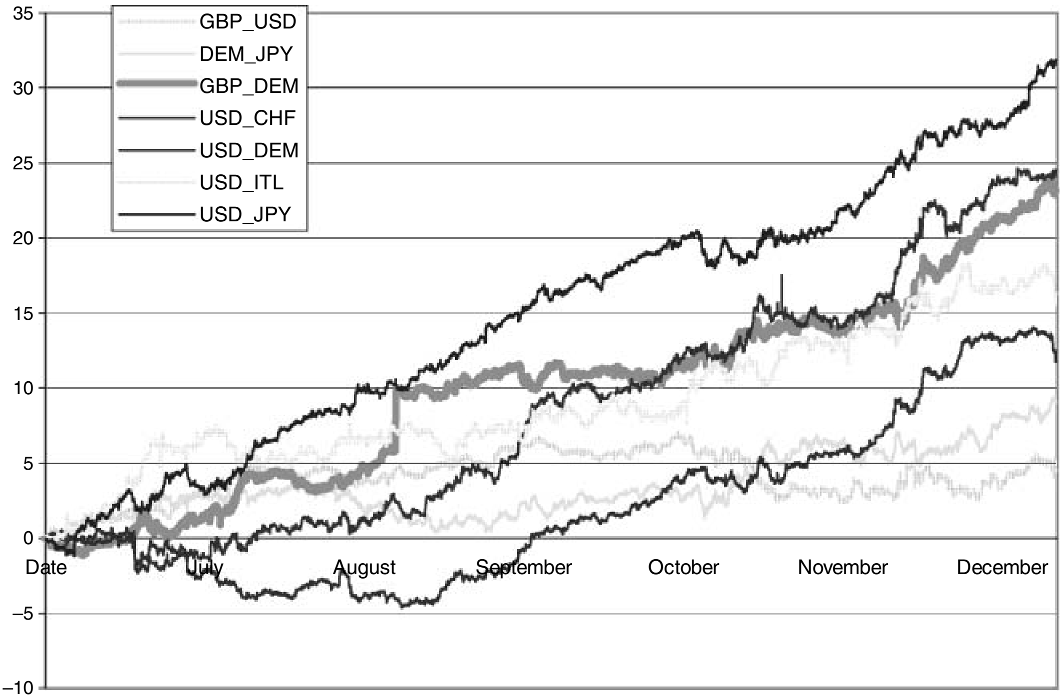

Fig. 10. Cumulative profit for July–December 2001 using long and short positions

there was a substantial reversal of fortune. UK investors trading marks or US investors trading yen would have earned steady profit streams, whereas the rules corresponding to German investors would have given a fairly flat return profile, indicating a relatively lacklustre performance.

Finally, the last row of part (a) in Table 4 displays the number of 'round-trips' that are required to apply the coherence-based rules, i.e. the number of purchases and sales of the foreign currency per week. For a large interbank trade, the bid–ask spreads on foreign exchange trades are very small—of the order of 0.05%—and there are no other fees involved. The average bid–ask spreads for each currency over the sample period, expressed as a percentage of the mid-point quotes, are given in the last row of Table 4. It is clear that, in spite of such small transactions costs, the rules cannot generate any positive net returns, since such a large number of trades are made. This is true for all the exchange rates that were tested, including both the dollar–yen, where gross profits were quite phenomenal, and for the dollar–mark, where the smallest number of trades was suggested.

The lack of net trading profits as a result of excessive transactions costs suggests the use of a filter rule, so that the number of round-trips is reduced. We experimented with various filter rules, including those that imply buying or selling the foreign currency when the predicted return fell outside the threshold-limited equivalent to the bid–ask spread. Although the filter rules were successful in reducing the number of trades, they also considerably reduced the gross profitability, so these could not generate net positive returns either.

5. Conclusions

This paper has proposed and employed a new methodology for evaluating and quantifying the autocoherence of financial time series, which was then tested on a set of seven half-hourly exchange rate returns. Significant coherence for at least one frequency across frames was revealed for all series. Overall we find the signal coherence to be maximal at the 15-h, 12-h and especially 8-h frequencies. In the last case, this may be attributable to the opening and closing of the world's three financial market time zones.

The mean frame estimate was cleaned by removing all incoherent frequency components, and the remaining estimates were used to generate trading rules based on these most stable cyclical features. The strategies were employed to construct several performance evaluation measures that are commonly used by financial market practitioners. These trading rules could generate in most cases phenomenal gross profits that were statistically significant and showed evidence of significant market timing ability, but the very large number of trades required meant that these were more than wiped out by transaction costs.

Our analysis could be extended and enhanced in various ways. First, in the out-of-sample evaluation, improved trading performance is likely to result from the use of a rolling window of data, where a one-frame-ahead forecast is produced and the trading rule implemented, and then the sample rolled forward by one frame. Second, the use of a 1% significance level cut-off for determining the coherent parts of the signal is somewhat arbitrary, and sensitivity analysis could be conducted to determine the effect on profitability of increasing or reducing this threshold. If this threshold is set too high, then important cycles in the data will not be employed, whereas if it is set too low it is comparable with the effect of including irrelevant regressors in a standard time series regression model used for forecasting.

A third possibility would be an examination of a much longer run of lower frequency data, so that the frequency of transacting would be reduced and potential returns per trade increased to mitigate the effect of the bid–ask spread on profitability. These factors suggest that the profitability that is demonstrated in this paper is probably an understatement of that which is possible if the approach is further refined and optimized. In any case, there are many financial agents who do not require immediacy when making foreign exchange trades. US multinational companies, for example, may desire to reduce their holdings in marks and to increase their holding in pounds, and they may be willing to make the trades at any time in the near future, although the exact time is not of concern to them. Such agents would be forced to incur the transaction costs whenever they trade, and our research suggests that there are better methods for selecting the time to trade than doing so at random. Some large financial institutions (e.g. hedge funds) may be able to trade at much lower transaction costs than the figures that we quote, thus significantly improving net returns. We conjecture that the methodology that is employed in this paper could be a widely applicable tool for market microstructure researchers and statistical arbitrageurs, which will enable them to detect, quantify and rank the various periodic components in financial data better.

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