Episodic Nonlinear Event Detection in the Canadian Exchange Rate

Melvin J. HINICH and Apostolos SERLETIS

This article uses daily observations for the Canadian dollar–U.S. dollar nominal exchange rate over the recent flexible exchange rate period and a new statistical technique, recently developed by Hinich, to detect major political and economic events that have affected the exchange rate.

KEY WORDS: Autoregressive conditional heteroscedasticity; Bicorrelations; Bicovariance; Nonlinearity.

1. INTRODUCTION

As the Bank of Canada's former governor, Gordon Thiessen (2000–2001, p. 47), put it,

> One of the issues that has often surfaced over the years is the exchange rate for the Canadian dollar. Indeed, over the past couple of years, it has been a topic of considerable public discussion. That discussion has revolved around such questions as: Should we continue floating, or should we peg our currency to the U.S. dollar? In fact, should we even keep our own currency, or should we adopt the U.S. currency?

The attention to the exchange rate regime stems mostly from the decline of the Canadian dollar against the U.S. dollar through the 1990s, but also from the recent creation of a single European currency, the euro, to replace the national currencies of 12 member countries of the European Monetary Union. The debate in Canada has revolved around exchange rate alternatives, particularly around the issue of whether a floating currency is the proper exchange rate regime or whether we should fix the exchange rate between the Canadian and U.S. currencies, as we did from 1962 to 1970 (see, e.g., Murray and Powell 2002; Murray, Powell, and Lafleur 2003).

Exchange rate fluctuations are difficult to reconcile with linear models that focus on commodity prices, productivity, interest rate differentials, and demand and supply shocks (see Schembri 2001). In this article we use a method, recently proposed by Hinich (1996) for detecting episodic nonlinearity in time series data, to detect major political and economic events that have affected the Canadian dollar–U.S. dollar nominal exchange rate. In particular, we pursue a reverse form of event study in which we let data analysis determine the events rather than hypothesize about an event and then use statistics to "prove our hypothesis." We show that there are periods of nonlinearity that cannot be captured by standard volatility models. We argue that the development of a new statistical model for third-order and higher dependent processes that appear to be stationary white noise will increase our understanding of exchange rate movements.

The article is organized as follows. The next section outlines the testing methodology used. Section 3 describes the data and presents the evidence, and Section 4 demonstrates that the statistical structures present in the data cannot be captured by ARCH-type models. The final section provides a brief summary and conclusion.

Melvin J. Hinich is ???, Applied Research Laboratories, The University of Texas, Austin, TX 78713. Apostolos Serletis is ???, Department of Economics, University of Calgary, Calgary, Alberta T2N 1N4, Canada (E-mail: *serletis@ucalgary.ca*). The authors thank Mark Kaiser, an associate editor, and a referee for comments that greatly improved the article. Serletis also gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada.

2. THE HINICH PORTMANTEAU BICORRELATION TEST

Hinich (1996) suggested a modified version of the Box–Pierce (Box and Pierce 1970) portmanteau Q-statistic for autocorrelation and a third-order portmanteau statistic, which can in a sense be viewed as a time-domain analog of the bispectrum test. A full theoretical derivation of the test statistics and a number of Monte Carlo simulations to assess their size and power have been given by Hinich (1996) and Hinich and Patterson (1995).

Let $\{x(t)\}$ denote a time series sampled at a fixed rate. As is the custom in the nonengineering time series literature, the time unit is suppressed and t is an integer. In this article the time series is daily Canadian exchange rate relative to the U.S. dollar. The method is to break the observed series into frames of equal length and apply a number of statistics to each frame, generating a multivariate time series of frame statistics, which are then used to detect events depending on the algorithm used. The method is a simplification and generalization of the methods used to process radar and sonar signals. The method also can be applied to the whole series.

Let t_p denote the time of the first observation in the pth frame will length T. Thus the $(p+1)$th frame begins at $t_p + T$. The data in each frame are standardized by subtracting the sample mean of that frame and dividing by the frame's standard deviation. Let $\{z_p(t)\}$ denote the standardized data in the pth frame. The bicorrelation test statistic introduced by Hinich (1996) for detecting third-order correlation in a time series is

$$H_p = \sum_{r=2}^{L} \sum_{s=1}^{r-1} (T-s)^{-1} B_p^2(r,s),$$

where

$$B_p(r,s) = \sum_{t=1}^{T-r} z_p(t) z_p(t+r) z_p(t+s).$$

The distribution of H_p is approximately chi-squared with $L(L-1)/2$ degrees of freedom for large T if $L = T^c (0 < c < .5)$ under the null hypothesis that the observed process is pure white noise (iid). The parameter c is chosen by the user. Thus, under the pure white noise null hypothesis, $U = F(H_p)$ has a uniform (0, 1) distribution, where F is the cumulative distribution function of a chi-squared distribution with $L(L-1)/2$

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degrees of freedom. The program operates on the transformed variate $U = F(H_p)$.

Based on the results of Monte Carlo simulations, using $c = .4$ is recommended to maximize the power of the test while ensuring a valid approximation to the asymptotic theory even when T is small. Simulations for the size of this test statistic presented by Hinich and Patterson (1995) have shown that the test is conservative for small sample sizes.

The test is of a null of pure white noise against an alternative that the process has m nonzero correlations or bicorrelations in the set $0 < s < r \leq L$ (i.e., there exists second or third-order dependence in the data-generating process), and relies on the zero bicovariance property of pure noise. The test is particularly useful in detecting nonlinear dependencies, because it has much better small-sample properties and does not have such stiff data requirements as many of its competitors, such as the BDS test (Brock, Dechert, and Scheinkman 1987; see Brock Hsieh, and LeBarron 2001 for a useful survey).

Rather than reporting the H statistic as a chi-squared variate, the T23 program written by Hinich reports the statistics as p values using the appropriate chi-squared cumulative distribution value to transform the computed statistic to a p value.

3. THE DATA

The analysis presented here is based on daily mid-price spot exchange rates of the Canadian dollar relative to the U.S. dollar for the period January 2, 1973–June 16, 2006 (a total of 8,401 observations). As shown in Figure 1, the Canadian nominal exchange rate experienced four long swings in this period: (1) a 30.76% depreciation from January 2, 1973 to February 4, 1986; (2) a 26.04% appreciation from February 4, 1986 to January 7, 1992; (3) a 28.99% depreciation form January 7, 1992 to February 27, 2002; and (4) a 43.31% appreciation from February 27, 2002 to June 16, 2006 (the last day in our sample). In what follows, the raw exchange rates are transformed into a series of 8,400 log-returns (see Fig. 2), which can be interpreted as a series of continuously compounded daily returns (see Brock et al. 1991).

There is no significant correlation in the data, but there is third-order nonlinearity in the data. The H_p statistic for the pure white noise hypothesis for the whole dataset is $<.001$ using a bootstrapped p value of $p = .901\text{E}{-}02$ set to have a false alarm (size) of .5%. This threshold is the $1 - 99.5\%$ quantile of the $U = F(H_p)$ statistics computed for 250 random draws with replacement of the whole dataset. If we were to use the standard approach in statistics and econometrics when the null is rejected, then we would attribute the nonlinearity to the whole series assuming that the process is stationary. There is no obvious nonstationarity in the data, but we show that the nonlinearity is due to bursts of third-order dependence in the data that are not correlated in any way with the variances and ranges of the data in the frames.

The data are split into a set of 240 nonoverlapping frames of length 35 observations (i.e., approximately 7 trading weeks); the program does not use the last frame if it is not full. This frame length is long enough to validly apply the tests and yet short enough for the data-generating process to have remained roughly constant. The results are basically the same if we double or triple the frame length, but then we would have greater uncertainty about when the event occurred. Figure 3 shows the

Figure 1. Time Series of the Canadian Dollar–U.S. Dollar Nominal Exchange Rate.

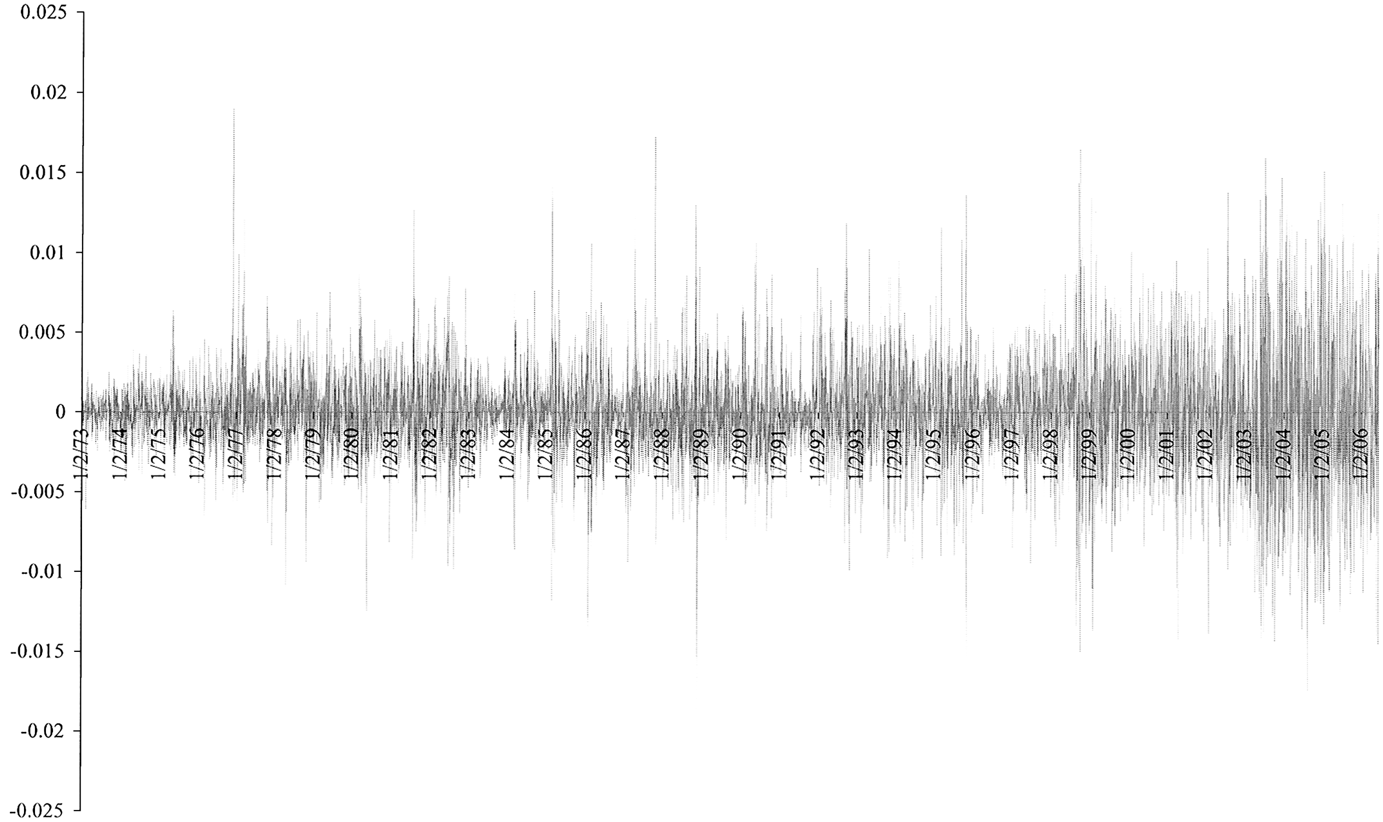

Figure 2. Exchange Rate Returns.

standard deviations of the log-returns for the 240 nonoverlapping frames. As can be seen, the volatility slowly increases over the period, which, however, does not affect our statistical analysis, because we are using frames with 35 observations.

We can apply the tests to either the raw returns for each frame or the residuals of an autoregressive (AR) fit of each frame using the same order for the AR process. The AR fit is made to remove serial correlation in the frame, which would invalidate the null hypothesis of independence of the H test. The justification for considering the residuals is to demonstrate that the nonstationarity must be a consequence of the nonlinearity that is episodically present in the data rather than a form of linear dependence (which has been removed); thus only significant H statistics will cause a rejection of the null of pure noise. We used a second-order AR process, AR(2), to whiten each frame.

A sampling with replacement bootstrap was used to determine a threshold for the H statistic with test size set to .5%. A total of 200 sets of 35 residuals from the AR(2) fit of each frame were selected at random with replacement. The $1 - .5\%$ quantile was computed for the 200 $U = F(H_p)$ statistics for these resample frames. The bootstrapped threshold was 1.97%. A similar bootstrap method was used to find the .5% threshold for the Hinich (1996) modification of the Box–Pierce test for serial correlation. The bootstrapped threshold was 1.19%. No frame had a significant bootstrapped portmanteau test for correlation using a four-lagged sum of squared correlations.

A total of 10 significant frames from the 240 frames of the residuals were found using a .5% threshold for the p values of the H statistics. (These 10 significant frames represent 4.16% of the total number of frames.) As can be seen in Figure 4, the R^2 values for the frames are small, generally < 0.4. The patterns of the significant frames are shown for the $1 - p$ values in Figure 5. The first significant frame started on May 12, 1976, and the final one started on July 6, 1991. Thus episodic nonlinearity kept occurring in the Canadian dollar–U.S. dollar exchange rates after the period studied by Brooks, Hinich, and Molyneux (2000).

It is interesting to note the absence of episodic nonlinearity in the Canadian exchange rate after 1991, when Canada adopted inflation targeting as its monetary policy regime. In particular, in February 1991 the Bank of Canada's Governor and the Minister of Finance jointly announced a series of declining inflation targets, with a band of $+$ and -1 percentage point around them. The targets were 3% by the end of 1992, falling to 2% by the end of 1995, to remain within a range of 1–3% percent thereafter. The 1–3% percent target range for inflation was renewed in December 1995, in early 1998, and again in May 2001, to apply until the end of 2006.

4. AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY MODELS

The class of autoregressive conditional heteroscedasticity (ARCH) models, a nonlinear modeling strategy originally suggested by Engle (1982), has been widely used to model financial time series. This class of models relaxes the assumption of the classical linear regression model that the variance of the disturbance term is both conditionally and unconditionally constant. These models have been found to accurately describe a number of the important characteristics of data from diverse financial disciplines.

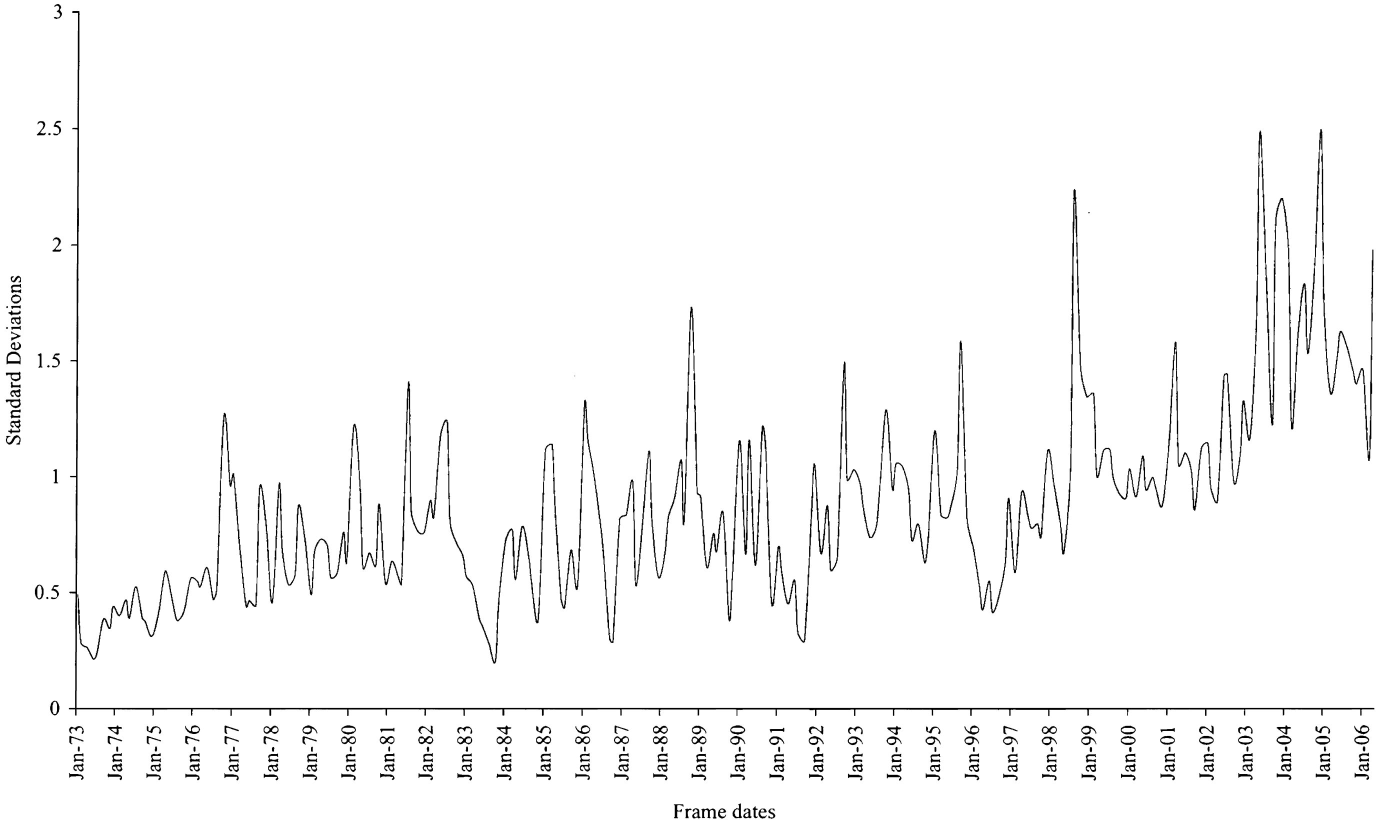

Figure 3. Standard Deviations of the 35-Day Frames.

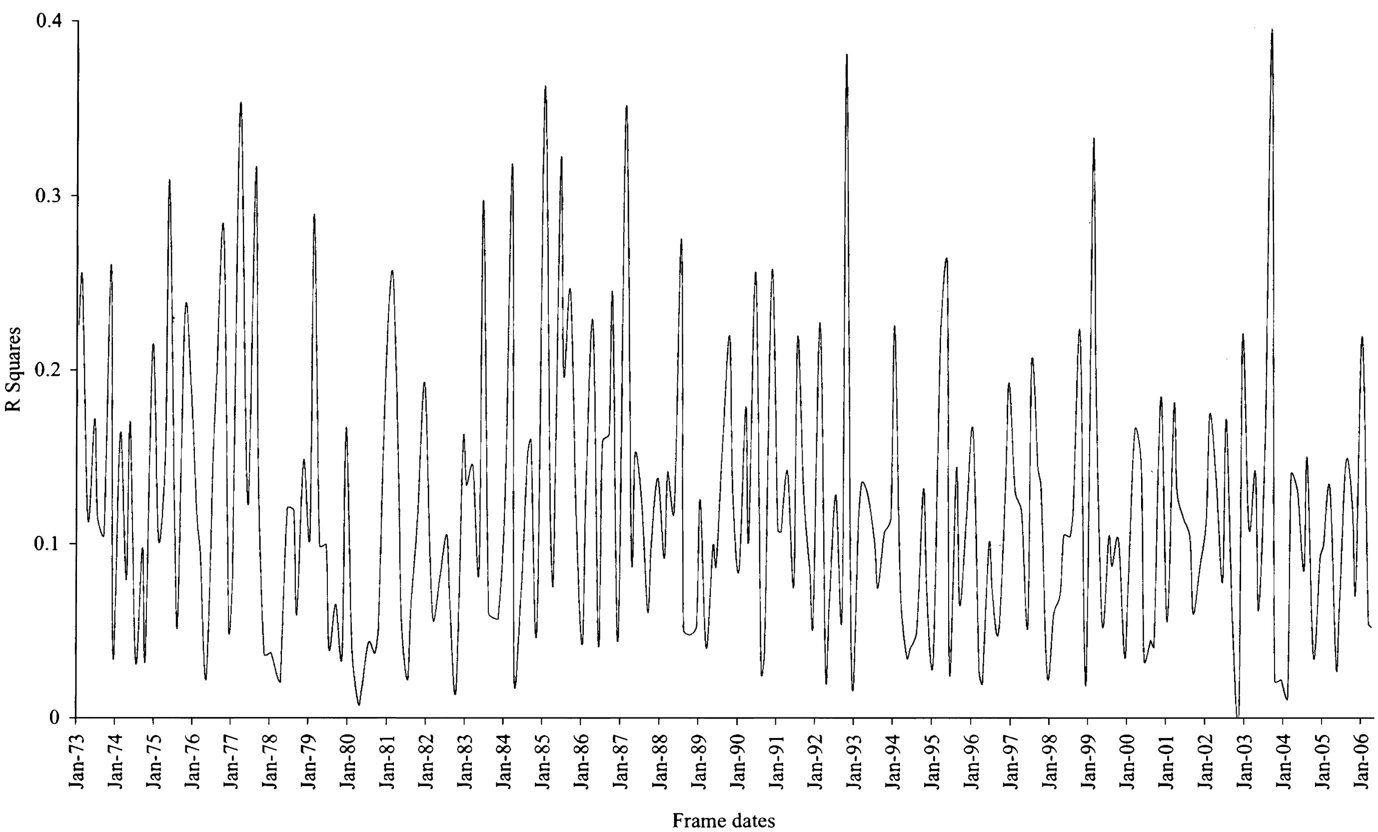

Figure 4. R^2's of the 35-Day Frames.

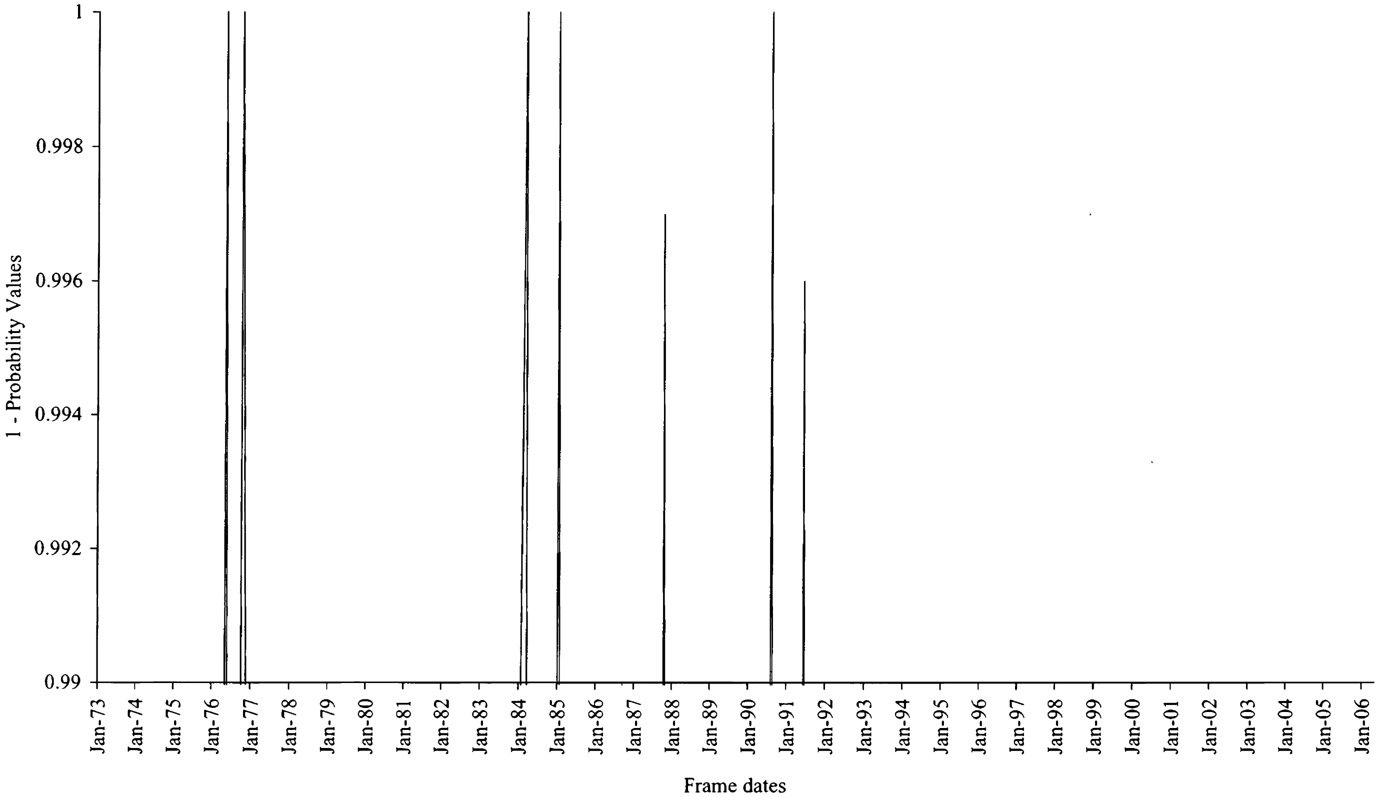

Figure 5. The $1 - p$ Values for the Significant H Statistics.

A model that allows the conditional variance to depend on the past realization of the series is the ARCH model, introduced by Engle (1982), according to which the conditional variance is assumed to depend on lagged values of squared residuals, as follows:

$$\sigma_t^2 = w_0 + \sum_{i=1}^{p} \alpha_i e_{t-i}^2,$$

with $p \geq 0$. Note that the disturbances in the ARCH(p) model are serially uncorrelated but not independent, as they are related through second moments. An extension of the ARCH model is the generalized ARCH (GARCH) model proposed by Bollerslev (1986). In the GARCH(p, q) model, we have

$$\sigma_t^2 = w_0 + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2,$$

where $w_0 > 0$, $\alpha_i \geq 0$, $i = 1, \ldots, p$, and $\beta_j \geq 0$, $j = 1, \ldots, q$. Here, the conditional variance is assumed to depend on lagged values of squared residuals and also on lagged values of itself; an AR component is introduced. Recently, Hansen and Lunde (2005) compared 330 ARCH-type models in terms of their ability to describe the conditional variance in the deutschemark–dollar exchange rate and found no evidence that the GARCH(1, 1) was outperformed by more sophisticated models.

Assuming, consistent with our test results so far that the AR(2) model renders the residuals uncorrelated, we now check whether the finding of nonlinearity might be a false rejection due to GARCH(1, 1) effects. Toward that end, we transform the returns into a set of binary data denoted by $\{y_p(t)\}$, where $y_p(t) = 1$ if $z_p(t) \geq 0$ and $y_p(t) = -1$ if $z_p(t) < 0$. If the variation in the data is due solely to an ARCH or GARCH process, then the 0 and 1 series will be a Bernoulli process, because the e_t's are independently distributed.

Besides the H statistic, the T23 program computes a number of statistics for each window, including the mean, standard deviation, skewness, kurtosis, and, if an AR(p) fit is applied to each frame, the R^2 for the fit. In addition, a portmanteau correlation test statistic, called the C statistic, which is a modified version of the Box–Pierce Q-statistic, is reported. Unlike the Box–Pierce Q-statistic that is usually applied to the residuals of a fitted ARMA model, the C statistic is a function of the standardized observations and the number of lags used and depends on the sample size. Despite these differences, both statistics perform the same function—to detect the presence of linear dependence in the form of significant autocorrelation (known as second-order correlation in statistical terms).

The tests were applied to the same 240 frames for the binary-transformed data for each series using a .5% threshold for the p values of the C statistic. The bootstrapped threshold for the C statistics was 1.97%. Figure 6 displays the significant C statistics for the binary-transformed residuals. There are three significant frames for the C statistic, which is 1.25%. The results show more significant frames than would be expected purely by chance given the very strict .5% nominal threshold level bootstrapped for the C statistic. Hence the previous rejection of linearity is unlikely to be a result simply of GARCH effects in an otherwise linear model.

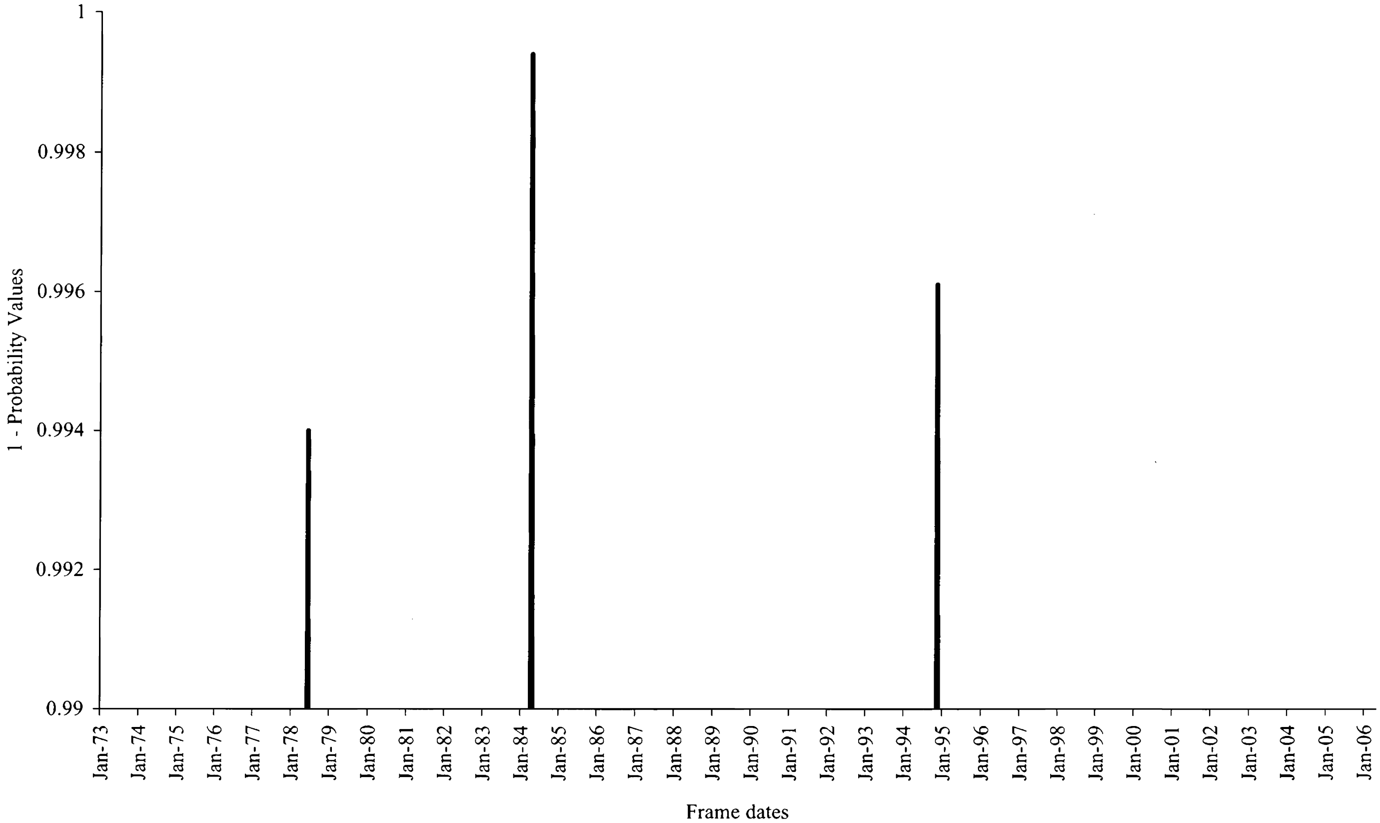

Figure 6. The $1 - p$ Values for the Significant C Statistics.

5. CONCLUSION

Detecting nonlinearity in time series data has become an important area of statistical and econometric research. Various new methodologies have been developed to test for the presence of nonlinearity as a consequence of the increasingly widely held view that the economic and political systems are nonlinear. For example, evidence of nonlinearity has been reported by Hsieh (1989), Brooks (1996), Brooks et al. (2000), Brooks and Hinich (2001), Serletis and Shahmoradi (2004), and Pinno and Serletis (2005). In fact, Brooks et al. (2000) and Brooks and Hinich (2001) have presented evidence that the nonlinearity is episodic.

In this article we extended the work of Brooks et al. (2000) and Brooks and Hinich (2001) on the Canadian dollar–U.S. dollar nominal exchange rate to a longer period using daily data over the recent floating exchange rate period, of January 2, 1973 to June 16, 2006. We show that there are periods of nonlinearity that cannot be captured by standard volatility models. We believe that when surprises hit the market, they generate a pattern of nonlinear exchange rate movements relative to previous movements, because the traders are unsure of how to react and hence they respond slowly, whereas normal news generates much quicker responses. Our results indicate that a different but as-yet unknown market response mechanism is generating foreign exchange rates, suggesting that there is now a need to develop a statistical model for third-order and higher dependent processes that appear to be stationary white noise.

Moreover, our results have implications for what stylized facts theoretical models of exchange rate determination should be attempting to explain. Of course, a better understanding of exchange rate movements is relevant to the recent debate in Canada of whether a floating currency is the proper exchange rate regime or whether Canada should consider alternative monetary arrangements. In this regard, and to the extend that we detected no episodic nonlinearity in the Canadian exchange rate after 1991, when Canada adopted inflation targeting as its monetary policy regime, we conclude that Canada should continue the current exchange rate regime (allowing the exchange rate to float freely with no intervention in the foreign exchange market by the Bank of Canada), as well as the current monetary policy regime (of inflation targeting).

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