Model Identification Of ARCH/GARCH Using Non-Linearity Tests

An Application On Asean-5 Foreign Exchange Markets

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ABSTRACT

This study provides an alternative framework for non-linear model identification / diagnostics by using a battery of non-linearity tests, with an application on the ASEAN-5 foreign exchange markets. To achieve that end, the differing power of the Brock-Dechert-Scheinkman (BDS) test and Hinich bispectrum test are utilized for they provide valuable identification information on the adequacy of the current framework of ARCH/GARCH models in capturing the non-linear dynamics in ASEAN-5 currencies. The results from the BDS test indicate strong evidences of non-linearity. However, this conveys little information on the nature of the detected non-linearity since the BDS test has high power against vast class of alternatives. Further application of the Hinich bispectrum test provides valuable non-linear identification information, in which the results provide strong evidences against the adequacy of the ARCH/GARCH models in explaining the non-linear dynamics in ASEAN-5 exchange rate returns series.

Key words: Non-linearity; BDS test; Hinich bispectrum test; ARCH, GARCH, Asean foreign exchange markets.

1. Introduction

This paper goes beyond the aim of only providing evidences of non-linearity in the ASEAN-5 exchange rate returns series. Through the application of a battery of non-linearity tests, namely the BDS and Hinich bispectrum test, this study provides an alternative framework for non-linear model identification on ARCH/GARCH.

The study of the foreign exchange market has attracted the interest of many and it has become even more important in the post-Bretton Woods era. Since the inception of the floating exchange rate regime in 1973, most foreign exchange markets have experienced continuous and sometimes dramatic fluctuations and volatility. There is no exception for the ASEAN-5 currencies, which is the focus of this paper, especially in 1997 when currency crisis swept the economy of South East Asia.

Many different empirical exchange rate models have been developed over the years in an attempt to explain this volatile phenomenon in the foreign exchange market. One of the features of these models of exchange rates is that they are assumed in linear form, or can be made linear by use of a simple transformation. However, this class of models does not perform well in explaining movements in exchange rates (Meese and Rogoff, 1983; Boothe and Glassman, 1987; Alexander and Thomas, 1987; Wolff, 1987). Recently, interest has been shown in the possibility that non-linearity accounts for the apparent unpredictability of exchange rates. Developments in the mathematical and statistical analysis of dynamic systems have been the main driving force behind the shift to non-linear studies. The richness of these new non-linear tests lies in their ability to uncover a more complex form of dependencies in a time series that appear to be random. This line of research has gain popularity over the years with substantial literature supporting the presence of non-linearity in the exchange rate return series (see example, Hsieh, 1989; De Grauwe *et al*., 1993; Steurer, 1995; Brooks, 1996). In the words of Campbell *et al.* (1997), "A natural frontier for financial econometrics is the modelling of non-linear phenomenon".

The evidences of non-linearity in exchange rate return series have profound implications for it covers a vast range of areas as follows:

Market Efficiency: The existence of statistically significant non-linear components in a return series will be inconsistent with market efficiency, at least if accompanied by risk-neutrality and negligible transaction costs[1] (Abhyankar *et al.*, 1995; Brooks, 1996; McMillan and Speight, 2001).

Model Adequacy: According to Hinich and Patterson (1989), the test for non-linearity can be viewed as general tests of model adequacy for linear models. The rationale behind this is that if there is still dependence in the residuals of linear model, the original linear model can no longer be viewed as an accurate representation of the data.

Derivatives Markets: The evidence that securities follow non-linear dynamics have strong implications on the pricing of derivative securities and the development of dynamic hedging strategies. Specifically, if the assumed stochastic processes do not adequately depict the full complexity of the true generating processes, then any derivative securities in question may be mis-priced and thus expose investors to unwanted risks due to imperfect hedges.

Most of the studies in the area of non-linearity have been focusing on currencies of developed countries, especially the US dollar, Japanese yen, British pound and German deutschmark, while currencies of ASEAN-5 have received little attention. Though most of the studies acknowledged the prevalence of non-linearity in exchange rate returns data, no conclusion can be inferred for ASEAN-5 currencies, which are very likely to exhibit characteristics different from those observed in developed foreign exchange markets. Besides market thinness, investors in these ASEAN-5 markets tend to react slowly and gradually to new information, which is in contrast with the highly efficient developed markets.

Motivated by the above consideration, this paper would like to investigate the universal applicability of those findings from developed countries. To the knowledge of the writers, this is the first attempt using non-linearity tests to examine the presence of non-linear structure in ASEAN-5 exchange rate return series. Another contribution of this study to the literature is the application of a battery of non-linearity tests as an alternative framework for non-linear model identification, which has not been done in previous studies, with the exception of a recent paper by Ashley and Patterson (2001). Specifically, this study employs the Brock-Dechert-Scheinkman (BDS) test and Hinich bispectrum test in a sequential way so as to provide valuable identification information on the adequacy of the current framework of ARCH/GARCH models in capturing the non-linear dynamics in ASEAN-5 exchange rate returns series.

This paper is organized as follows: Section 2 provides a review of the literature on non-linear studies, while Section 3 provides a brief description of the data and the methodology used in this study. Section 4 presents the empirical results as well as the analysis of the findings. Finally, concluding remarks are given at the end of the paper.

2. Literature Review

There are wide variety of tests employed in the literature to detect non-linearity and chaos, just as there is no commonly agreed definition for non-linear system and chaos. Barnett and Serletis (2000) highlight that none of the tests for non-linearity completely dominates the others. This is supported by the Monte Carlo studies conducted by Ashley *et al.* (1986), Ashley and Patterson (1989), Hsieh (1991), Liu *et al.* (1992), Lee *et al.* (1993), Brock *et al.* (1996), Barnett *et al.* (1997) and Ashley and Patterson (2001). If this is the case, the available non-linearity tests can be utilized in a complementary way, rather than competing. The application of a battery of non-linearity tests in a sequential way can provide deeper insight into the nature of non-linear generating mechanism of a time series (Barnett *et al*., 1995, 1997; Barnett and Serletis, 2000; Ashley and Patterson, 2001).

Most of the empirical studies in the literature have been extensively applying the BDS test (see example, Scheinkman and LeBaron, 1989; Hsieh, 1989, 1991; De Grauwe *et al.*, 1993; Steurer, 1995; Abhyankar *et al*., 1995, 1997; Brooks, 1996; Barkoulas and Travlos, 1998; Opong *et al.*, 1999; Mahajan and Wagner, 1999). The sampling distribution of the BDS test statistic is not known, either in finite samples or asymptotically, under the null of non-linearity. However, it is possible to use the BDS test to produce a test of linearity against the broad alternative of non-linearity. In particular, after the linear structure has been removed by fitting the best possible linear model, the BDS test can then be used to test the residuals for remaining non-linear dependence. However, the issue that needs to be addressed is whether such method of linear filtering will change either the asymptotic or the finite sample distribution of the BDS test statistic. Brock (1987) proves that using residuals in linear models instead of raw data does not alter the asymptotic distribution of the BDS test statistic. The simulations results in Hsieh (1991) provide further support.

Though applying the BDS test to the residuals of a filtered data will give a strong support for the conclusion of non-linearity[2], it conveys very little information as to what kind of non-linear process that generated the data. This is because the BDS test has great power against vast class of non-linear processes (Hsieh, 1991; Barnett *et al.* 1997; Ashley and Patterson, 2001). With high power against such a vast class of alternatives, the BDS test can only be used as a “non-linearity screening test”. In fact, this is the limitation of previous studies that only provide evidences of non-linearity, assuming at the outset that the non-linearity takes a particular form.

Another popular non-linearity test is the Hinich bispectrum test (Hinich, 1982), which produces a test statistic having known asymptotic sampling distribution under the respective null hypotheses of linearity and Gaussianity. However, the alternative hypothesis is not as broad as that for the BDS test. With the bispectrum test, the alternative hypothesis is all non-linear processes having non-flat bispectrum. In other words, the bispectrum test has no power against those forms of non-linearity that display flat bispectrum and non-flat higher order polyspectra (Barnett *et al*., 1997). Thus, this approach appears to have limitation when the data fails to reject the null of linearity. Failure of rejection does not imply the acceptance of linearity for it might be due to some non-linear processes in which the bispectrum test has low power against. Thus, further test is needed in this case to determine the presence of non-linearity.

To overcome the above-mentioned limitations, both the BDS and Hinich bispectrum test can be used in a complementary, rather than competing way. Moreover, the application of both the BDS and Hinich bispectrum test in a sequential way can provide a deeper insight into the types of non-linear process (Barnett *et al*., 1995, 1997; Barnett and Serletis, 2000; Ashley and Patterson, 2001). In this study, the differing power of the BDS and Hinich bispectrum test in detecting ARCH and GARCH is utilized as an alternative framework for non-model identification. Specifically, the low power of the Hinich bispectrum test relative to the BDS test for the ARCH/GARCH models suggests that the bispectrum test is useful as a marker for these ARCH/GARCH models. Barnett *et al.* (1997) demonstrate in their Monte Carlo studies that the Hinich bispectrum test has low power in detecting non-linearity in the forms of ARCH and GARCH types. The results wrongly accept linearity for data simulated from the ARCH and GARCH models. The fact that the Hinich bipsectrum test has low power against ARCH and GARCH is well acknowledged in the literature (see example, Hsieh, 1989; Brooks, 1996).

The results from the application of both the BDS and Hinich bispectrum test provide important implications for the interpretation of the recent voluminous literature that attempts to model exchange rate return series using this ARCH family of models (see example, Hsieh, 1989; Bera and Higgins, 1997; Brooks, 1997; Franses and Homelen, 1998). Though, there are some recent empirical evidences against the ARCH/GARCH models (see example, Krager and Kugler, 1993; Brooks and Hinich, 1998), they are at present the widely accepted framework to account for the most apparent type of non-linear dependence in exchange rate return series. Thus, this study has strong implications for the adequacy of this current framework of ARCH/GARCH[3] in explaining the non-linear dynamics in ASEAN-5 exchange rate returns series.

3. Research Design and Methodology

An Overview

The purpose of this study is to show that there is in fact a non-linear mechanism that drives the observed ASEAN-5 exchange rate return series. However, it is not sufficient to only provide evidences of non-linearity. This study takes a further step to apply a battery of non-linearity tests in a sequential way so as to provide an alternative framework for non-linear model identification, specifically the ARCH/GARCH models. Both the BDS and Hinich bispectrum test are used in view of their differing power in detecting ARCH/GARCH models.

In this paper, the BDS test, as the first run test, is applied to the residuals of a pre-filtered linear model. If the null of i.i.d. is accepted, there is little point to continue further, since the BDS test provides strong evidence against the presence of non-linearity. In the case where the null is rejected, the Hinich bispectrum test can then be used to permit the class of relevant non-linearity to be narrowed. In particular, the Hinich bispectrum test is useful as a marker for the ARCH/GARCH models. Since linearity has been ruled out by the BDS test, the acceptance of the null by the Hinich bispectrum test might be due to the presence of non-linear processes which the Hinich test has low power against, specifically the ARCH and GARCH (Hsieh, 1989; Brooks, 1996; Barnett *et al.*, 1997). On the other hand, rejection of the null hypothesis by the Hinich bispectrum test provides evidences against the presence of ARCH and GARCH.

The Data

The daily spot exchange rates for ASEAN-5 currencies (expressed as the price of a country's currency in terms of the U.S. dollar) are obtained over the period January 2,1990 to March 30, 2001. However, the daily data of Indonesia rupiah (IDR/USD) and the Philippines peso (PHP/USD) for the first half of 1990s are difficult to obtain. Thus, for these two currencies, the sample period covers November 16, 1995 to March 30,2001. Though the sample sizes for these two currencies are smaller, they are far exceeded the limit of acceptability for both the BDS and Hinich bispectrum test.

The daily data of Singapore dollar (SGD/USD), Malaysia ringgit (MYR/USD) and Thai baht (THB/USD) used in this study are drawn from the Federal Reverse Statistical

Release[4]. The Federal Reserve is the central bank of the United States, which was founded by Congress in 1913. The source for the Indonesia rupiah (IDR/USD) and the Philippines peso (PHP/USD) daily rates comes from the Pacific Exchange Rate Service[5].

The raw exchange rate data are transformed into the differenced-log return series (r_t). All subsequent analysis is being performed on these transformed ASEAN-5 exchange rate return series, which can be interpreted as a series of continuously compounded percentage daily returns (Brock *et al.*, 1991). Formally, it can be written as:

$$r_t = 100\,[\ln(S_t) - \ln(S_{t-1})] \qquad (1)$$

where S_t is the exchange rate at time t, and S_{t-1} the rate on the previous trading day.

This transformation has become standard in the finance literature (see example, Hsieh, 1989; De Grauwe *et al.*, 1993; Steurer, 1995; Brooks, 1996; Mahajan and Wagner, 1999). Thus, this transformation is done to conform to the literature and to allow comparison with other studies in this domain. Another possible justification for using returns rather than raw data is that the raw data is likely to be non-stationary.

Stationarity is a pre-requisite for both the BDS and Hinich bispectrum test. Hsieh (1991) points out that non-stationarity in the data series can cause a rejection of the null hypothesis of i.i.d. on the basis of the BDS test. On the other hand, non-stationarity may cause a spurious rejection of the null of linearity in the bispectrum test (Hinich and Patterson, 1985).

It is important to note that the choice of level or of return will not affect the results of the BDS test and Hinich bispectrum test. De Grauwe *et al.* (1993) highlight that if the system is non-linear, then this structure will show up in the levels as well as in the returns.

Brock-Dechert-Scheinkman Test (BDS Test)

Brock, Dechert, and Scheinkman (Brock *et al.*, 1987) developed a statistical test and the BDS statistic. The original BDS paper takes the concept of the correlation integral[6] and transforms it into a formal test statistic which is asymptotically distributed as a normal variable under the null hypothesis of i.i.d. against an unspecified alternatives. A revision of this original paper has been done in Brock *et al*. (1996).

The BDS test is based on the correlation integral as the test statistic. Given a sample of independent and identically distributed observations, $\{x_t$: t = 1, 2, ….., $n\}$, Brock *et al*. (1987, 1996) show that:

$$W_{m,n}(\varepsilon) = \sqrt{n}\,\frac{T_{m,n}(\varepsilon)}{V_{m,n}(\varepsilon)} \tag{2}$$

has a limiting standard normal distribution, where $W_{m,n}(\varepsilon)$ is the BDS statistic. n is the sample size, m is the embedding dimension, and the metric bound, ε, is the maximum difference between pairs of observations counted in computing the correlation integral. $T_{m,n}(\varepsilon)$ measures the difference between the dispersion of the observed data series in a number of spaces with the dispersion that an i.i.d. process would generate in these same spaces, that is $C_{m,n}(\varepsilon) - C_{1,n}(\varepsilon)^m$. $T_{m,n}(\varepsilon)$ has an asymptotic normal distribution with zero mean and variance $V^2_m(\varepsilon)$.

This BDS test has an intuitive explanation. The correlation integral $C_{m,n}(\varepsilon)$ is an estimate of the probability that the distance between any two m-histories, $x_t^m = (x_t, x_{t+1}, \ldots\ldots, x_{t+m-1})$ and $x_s^m = (x_s, x_{s+1}, \ldots\ldots, x_{s+m-1})$ of the series $\{x_t\}$ is less than ε, that is,

$C_{m,n}(\varepsilon) \rightarrow$ prob$\{|x_{t+i} - x_{s+i}| < \varepsilon$, for all i = 0, 1, ……… m-1$\}$, as $n \rightarrow \infty$

If the series $\{x_t\}$ are independent, then, for | t-s | > m, $C_{m,n}(\varepsilon) \rightarrow \prod_{i=0}^{m-1} \text{prob}\{| x_{t+i} - x_{s+i} | < \varepsilon\}$, as $n \rightarrow \infty$. Furthermore, if the series $\{x_t\}$ are also identically distributed, then $C_{m,n}(\varepsilon) \rightarrow C_1(\varepsilon)^m$, as $n \rightarrow \infty$.

The BDS statistic therefore tests the null hypothesis that $C_{m,n}(\varepsilon) = C_{1,n}(\varepsilon)^m$, which is the null hypothesis of i.i.d.[7].

The need to choose the values of ε and m can be a complication in using the BDS test. For a given m, ε cannot be too small because $C_{m,n}(\varepsilon)$ will capture too few points. On the other hands, ε cannot be too large because $C_{m,n}(\varepsilon)$ will capture too many points. For this purpose, we adopt the approach used by advocates of this test. In particular, we set ε as a proportion of standard deviation of the data, σ. Hsieh and LeBaron (1988a, b) have performed a number of Monte Carlo simulation tests regarding the size of the BDS statistic under the null of i.i.d. and the alternative hypotheses. The Monte Carlo evidences show that the 'best' choice of ε is between 0.50 and 1.50 times the standard deviation.

On the other hand, at our chosen setting of ε, we produced the BDS test statistic, $W_{m,n}(\varepsilon)$ for all settings of embedding dimension from 2 to 10, in line with the common practice of most researchers (Hsieh, 1989; De Grauwe *et al.*, 1993; Brooks, 1996; Mahajan and Wagner, 1999; Opong *et al.*, 1999). However, it is important to take note that the small samples properties of BDS degrade as one increases the embedding dimension. Thus, in this study, the results with embedding dimensions of 2 to 5 are given the most serious consideration[8].

Asymptotically, the computed BDS statistics, $W_{m,n}(\varepsilon) \sim N(0,1)$ under the null of i.i.d. against an unspecified alternative. Thus, this would suggest a two-sided test. However, this is a very tricky issue. Brooks (1996) and Opong *et al.* (1999) clearly state that the BDS test is a two-sided test so that the rejection of the null of i.i.d. occurs when the estimated value of the $W_{m,n}(\varepsilon)$ is more extreme (in either tail) than the corresponding statistic from the normal tables. However, Barnett *et al.* (1995, 1997) run it as a one-tailed test. In this study, the BDS test is taken as a one-tailed test[9].

Hinich Bispectrum Test

Hinich (1982) has laid out a statistical test for determining whether an observed stationary time series $\{x(t)\}$ is linear. It is possible that $\{x(t)\}$ is linear without being Gaussian, but all of the stationary Gaussian time series are linear. The Hinich bispectrum test involves estimating the bispectrum of the observed time series, which is the double Fourier transform of the third-order cumulant function.

In this section, we present a brief description of the testing procedures presented by Hinich (1982). Let $\{x(\text{t})\}$ denote a third order stationary time series, where the time unit t is an integer. The third-order cumulant function of $\{x(\text{t})\}$ is defined to be $C_{xxx}(m,n) = E[x(t+n)x(t+m)x(t)]$ for each (m,n) when $E[x(\text{t})] = 0$, in which $n \leq m$ and $m = 0, 1, 2, \ldots\ldots\ldots$

Since third-order cumulants are hard to interpret, and their estimates are even harder to fathom, the double Fourier transform of the third-order cumulant function (called the bispectrum) is calculated.

The bispectrum at frequency pair (f_1, f_2) is the double Fourier transform of $C_{xxx}(m,n)$:

$$B_x(f_1, f_2) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C_{xxx}(m,n) \exp[-i2\pi(f_1 m + f_2 n)] \tag{3}$$

assuming that $|C_{xxx}(m,n)|$ is summable. The symmetries of $C_{xxx}(m,n)$ translate into symmetries of $B_x(f_1,f_2)$ that yield a principal domain for the bispectrum, which is the triangular set $\Omega = \{0 < f_1 < 1/2,\ f_2 < f_1,\ 2f_1 + f_2 < 1\}$.

Since the spectrum of {x(t)}is $S_x(f) = \sigma_u^2 |A(f)|^2$, it follows that:

$$\psi^2(f_1, f_2) \equiv \frac{|B_x(f_1, f_2)|^2}{S_x(f_1)S_x(f_2)S_x(f_1 + f_2)} = \frac{\mu_3^2}{\sigma_u^6} \qquad (4)$$

for all f_1 and f_2 in Ω, where $A(f) = \sum_{n=0}^{\infty} a(n)\exp(-i2\pi fn)$. The left hand side of equation (4) defines the square of the skewness function of {x(t)}, $\psi(f_1,f_2)$. Linearity and Gaussianity of {x(t)} are tested through the null hypotheses that $\psi(f_1,f_2)$ is constant over all frequencies and that $\psi(f_1,f_2)$ is zero over all frequencies respectively using the estimated bispectrum.

The test statistics for both hypotheses are reduced to

$$\hat{S} = 2\sum_m \sum_n \left|\hat{X}_{m,n}\right|^2 \qquad (5)$$

at the frequency pair (m,n) where $\hat{X}_{m,n} = \dfrac{\hat{B}_x(m,n)}{[N/M^2]^{1/2}[\hat{S}_x(g_m)\hat{S}_x(g_n)\hat{S}_x(g_{m+n})]^{1/2}}$

Under the null hypothesis of Gaussianity, $\hat{S}$ is distributed chi-squared with 2P degree of freedom, with P being the number of squares whose centres are in the principal domain. Hinich (1982) showed that, asymptotically, the transformation of $\hat{S}$ is well approximated by a normal distribution with zero mean and unit variance. Thus, the significance of the test statistics is readily determined from standard normal tables.

On the other hand, if {x(t)} is linear but not Gaussian, the sample dispersion of $2\left|\hat{X}_{m,n}\right|^2$ should not differ significantly from the population dispersion of $\chi^2(2,\hat{\lambda})$, where $\hat{\lambda} = \{\hat{S}/P\} - 2$. Linearity test statistics, examine whether the sample dispersion is significantly different from that of $\chi^2(2,\hat{\lambda})$. The distribution of the standard normal is used to produce a one-sided test, in which the null is rejected if the test statistic is greater than the critical value at the chosen level of significance.

It is important to note that this dispersion can be measured in many ways. Hinich (1982) and Ashley *et al.* (1986) recommend the use of the 80 percent quantile of the empirical distribution. This statistic is robust with respect to outliers and its asymptotic sampling distribution is easily calculated. In this study, we use 90 percent quantile to get a more plausible result instead of the 80 percent.

Another important consideration in the implementation of the bispectrum test is the parameter M, the frame size. The choice of M governs the trade-off between the bias and variance of the estimator. In this respect, the larger (smaller) the M, the smaller (larger) the finite sample variance and the larger (smaller) the sample bias. Due to this trade-off, there is no unique M that is appropriate to use in performing non-linearity test. In this study, we set M equal to 30 for our sample sizes to increase the power of the test.

4. Results and Analysis

Descriptive Statistics

Before proceed to the formal non-linearity tests, we perform the preliminary analysis on the ASEAN-5 exchange rate return series in order to get a better view of some of the important statistical features of these series of returns.

Table 1 provides summary statistics for all the ASEAN-5 exchange rate return series. The means are quite small. The range of daily changes, however, is relative high. Moving beyond the basic mean and standard deviation measurements to higher-order moments, all the returns series have longer tail than the normal distribution, which are evidenced from the skewness statistics. On the other hand, the distributions of returns for all the series are highly leptokurtic, in which the tails of its distribution taper down to zero more gradually than do the tails of a normal distribution. Not surprisingly, given the non-zero skewness levels and excess kurtosis demonstrated within these series of returns, the Jarque-Bera (JB) test strongly rejects the null of normality for all ASEAN-5 exchange rate return series. These results conform to the consensus in the literature that the distributions of exchange rate return series are non-normal (see example, Hsieh, 1988; Steurer, 1995; Brooks, 1996).

<<Table 1: Summary Statistics for ASEAN-5 Differenced-log Returns Series (r_t)>>

One area that deserves our attention is the stationarity of the exchange rate return series, which is a pre-requisite for both the BDS and Hinich bispectrum test. The results from the

Augmented Dickey Fuller (ADF) test in Table 2 show that the null hypothesis of a unit root can be rejected for all the exchange rate return series even at the 1% level of significance. Similar conclusions are made based on the results of Phillips-Perron (PP) test summarized in the same table. Thus, the results indicate that all the transformed return series of ASEAN-5 exchange rate do not contain a unit root and thus are stationary. Those statistics confirm the appropriateness of the differenced logarithmic transformation in rendering the exchange rate return series stationary.

<<Table 2: Unit Root Test Results for ASEAN-5 Differenced-log Returns Series (r_t)>

Finally, it is important to note that a rejection of the null hypothesis of i.i.d.for the BDS test could be due to the presence of linear serial dependencies in the data. Table 3 gives the autocorrelation coefficient and the Ljung-Box Q-statistics for all the ASEAN-5 exchange rate returns series at selected lags[10]. The result show that linear serial dependencies play a significant role in the dynamics of ASEAN-5 daily return series.

<< Table 3: Autocorrelation Coefficients and Ljung-Box Q-Statistics for ASEAN-5 Differenced-log Returns Series (r_t)>>

Linear Filtering

For further testing of the presence of non-linear dependencies in each of the ASEAN-5 exchange rate return series using the BDS test, the effects of linear serial dependencies have to be filtered out. This is because the rejection of the i.i.d. null in the BDS test can be due to non-white linear and non-white non-linear dependence. After fitting the best possible linear model, the BDS test can then be used to test the residuals for remaining non-linear dependence.

In this section, we fit an autoregressive AR(p) model[11] to all the ASEAN-5 exchange rate return series. It should be emphasized that the objective is not to build a statistically adequate empirical model of exchange rate returns, but rather to choose an acceptable specification, which will remove autocorrelation from the returns series. Specifically, our aim is to ensure that the estimated residual series did not exhibit any serial dependencies for at least 30 lags, corresponding to a one-month period. The identified models are p = 29, 16, 28, 19, and 13 respectively, for the MYR, SGD, THB, IDR, and PHP.

BDS Test

Table 4 reports the results of the BDS test on the residuals of the fitted AR(p) model. The first column of Table 4 indicates the metric bound, ε, which is the maximum difference between pairs of observations counted in computing the correlation integral. ε is set as a proportion of σ, which is the standard deviation of the time series being analysed. The second column is the embedding dimension, *m*. The BDS statistics, $W_{m,n}(\varepsilon)$, are calculated for all combinations of *m* and ε where *m* = 2, 3,…….10 and ε = 0.50σ, 0.75σ, 1.00σ, 1.25σ and 1.50σ, with a total of 45 combinations. These statistics of $W_{m,n}$ (ε) for each of the ASEAN-5 exchange rate return series are reported in columns 3 to 7 respectively.

The results from Table 4 reveal substantial non-linear dependence in all of the ASEAN-5 exchange rate return series. Although we report the results with embedding dimensions varying from 2 to 10, the results with embedding dimensions of 2 to 5 should be given the most serious consideration. This is because the small samples properties of BDS degrade as one increase the dimension. Specifically, as one gets beyond m=5, the small sample properties are not very good (in terms of normal approximations) at sample sizes comparable to ours.

All of the values are positive and significant at least at the 5% level of significance, even at the suggested dimensions of 2 to 5. For example, for the MYR return series, the null of i.i.d. can be rejected at the 1% level of significance for all the 45 combinations. Similar results are obtained for SGD, THB, IDR and PHP return series. These results indicate that each of the ASEAN-5 exchange rate return series is not truly random since some patterns show up more frequently than would be expected in a truly random series. Since the possibility of linear serial dependencies in the data has been ruled out by the AR-filtering, the rejection of i.i.d. null reveal that the BDS test is in fact detecting strong non-linear dependence in the data.

< Table 4: BDS Test Results on Residuals from AR(p) Fit for ASEAN-5 Currencies>

The results from the BDS test clearly show that there is in fact a non-linear mechanism that drives each of the ASEAN-5 exchange rate return series. However, it is not sufficient to only provide evidences of non-linearity. Using the Hinich bispectrum test in the subsequent section can provide valuable identification information, specifically on the adequacy of ARCH/GARCH in capturing the detected non-linearity in all the ASEAN-5 exchange rate returns series.

Hinich Bispectrum Test

Since pre-whitening has been done in the previous section, the Hinich bispectrum test is then applied to the residuals of the pre-whitening AR(p) model. The Hinich bispectrum test has the nice property that it is invariant to linear filtering of the data, even if the filter is estimated (Ashley *et al*., 1986). Thus, one need not worry about the possibility that the linear pre-whitening AR(p) model has failed to remove all linear serial dependencies in the data.

The results for the bispectrum Gaussianity test for each of the ASEAN-5 exchange rate return series are shown in Table 5. The results reveal that the null is strongly rejected for all the ASEAN-5 exchange rate return series. These results confirm the non-normality of the exchange rate return series suggested by Jarque-Bera (1987) normality test results in the earlier section.

Although Gaussianity and linearity tests are linked, a rejection of Gaussianity does not necessarily rule out linearity. If exchange rates are linear, but not Gaussian, then the sample dispersion of $2\left|\hat{X}_{m,n}\right|^2$ should not differ significantly from the population dispersion of $\chi^2(2,\hat{\lambda})$, where $\hat{\lambda} = \left\{\hat{S}/P\right\} - 2$. The linearity test statistics examine whether the sample dispersion is significantly different from that of $\chi_2{}^2(2,\hat{\lambda})$. Table 5 reports the p-value for the 90 percent quantile bispectrum linearity test for all the ASEAN-5 exchange rate return series. The results reject the null hypothesis of a linear generating mechanism at the 1% level of significance for all the ASEAN-5 exchange rate return series. These indicate the existence of non-linear dependencies within the daily returns, at least in the form that can be detected by the bispectrum test.

<<Table 5: Gaussianity and Linearity Test Results on Residuals of AR(p) Fit>>

Figure 1 to Figure 5 illustrate the standardized bispectrum estimates for all the ASEAN-5 exchange rate return series, which offer an intuitive account of the Gaussianity and linearity testing procedures. The contour plots display the estimated bispectrum over the two-dimensional principal domain, viewed from above the surface. Recall that the bispectrum is independent of frequency and is constant if the series conforms to a linear model, and is zero if the series is Gaussian. Clearly, the standardized bispectrum estimates are non-zero over its triangular principal domain (observations outside the principal domain are set equal to zero). The findings in Table 5 reflect the rejection of Gaussianity. Although Gaussianity and linearity tests are linked, a rejection of Gaussianity does not necessarily rule out linearity. Thus, linearity tests then examine whether the standardized bispectrum estimates are constant over their principal domain. Since the nulls have been rejected, as reported in Table 5, we would expect to see a number of peaks in the bispectrum, as indeed we do in Figure 1 to 5.

The horizontal and vertical axes of the plots show the frequencies of points in the principal domain measured in cycles per day. If the process were linear, we would not find interaction between various frequencies in the sample returns (Hinich and Patterson, 1985). The contour plots in Figure 1 to 5 show the combinations of frequencies where inter-frequency interaction occurs when the input to a non-linear filter is Gaussian white noise. Thus, the plots provide a convenient way to observe that the ASEAN-5 exchange rate return series are indeed generated by a non-linear mechanism.

<Figure 1: Contour Plot of the Estimated Bispectrum for MYR Return Series>

<Figure 2: Contour Plot of the Estimated Bispectrum for SGD Return Series>

<Figure 3: Contour Plot of the Estimated Bispectrum for THB Return Series>

<Figure 4: Contour Plot of the Estimated Bispectrum for IDR Return Series>

<Figure 5: Contour Plot of the Estimated Bispectrum for PHP Return Series>

5. Conclusions

The outcomes of our econometric investigation support the presence of non-linearity on all the ASEAN-5 exchange rate return series. It is interesting to note that all the ASEAN-5 exchange rate return series share similar statistical properties based on the results of our preliminary analysis. Even the results of the BDS test reveal that there is in fact a non-linear mechanism that drives each of the ASEAN-5 exchange rate return series.

Moreover, the bispectrum test results support our findings in the earlier BDS test, which indicates strong evidences of non-linearity. The rejection of the null of linearity in the bispectrum test is a strong support for the presence of non-linearity (Barnett *et al.*, 1997). The beauty of the Hinich bispectrum test lies in its ability to provide a direct test for a non-linear generating mechanism, irrespective of any linear serial dependencies that might be present. Consequently, when this test rejects the null, one need not worry about the possibility that the linear pre-whitening model has failed to remove all linear serial dependence in the data (Ashley and Patterson, 2001). This has helped us to cast away our worries that the rejection of the null in the BDS test could be due to the possibility of imperfect pre-whitening[12].

These bispectrum test results, however, do yield additional information on model identification beyond merely confirming the results of the earlier BDS test. Since the bispectrum test has relatively low power against the ARCH/GARCH models, these results provide strong evidences against the adequacy of the current framework of ARCH/GARCH in modelling the ASEAN-5 exchange rate return series. Thus, all the ASEAN-5 exchange rate return series are more likely being generated by a process that is of a form in addition to, or instead of ARCH/GARCH.

ENDNOTES

1. It is important to take note that the market could still be weak-form efficient with evidences of non-linearity if transaction costs are so high that any profitable opportunities are not arbitrageable since it would be too costly to take advantage of them.

2. Rejection of the null of 'independent and identical distribution' (i.i.d) indicates the presence of non-linearity (since linear dependence has been filtered out), while the acceptance implies no evidence of non-linearity.

3. Of course, rejection of the null hypothesis by the Hinich bispectrum test does not specify what non-linear model will provide an adequate representation of the actual generating mechanism.

4. These daily data are obtained from the Federal Reserve Board's official website at http://www.federalreserve.gov/releases/H10/hist on 18/4/2001. The H.10 release contains daily rates of exchange of major currencies against the U.S. dollar.

5. These daily data are obtained from the web location of Pacific Exchange Rate Service at http://pacific.commerce.ubc.ca/xr/data.html on 18/4/2001.

6. In Grassberger and Procaccia (1983), the correlation integral was introduced as a measure of the frequency with which temporal patterns are repeated in the data. For example, the correlation integral $C(\varepsilon)$ measures the fraction of pairs of points of a time series $\{x_t\}$ that are within a distance of ε from each other.

7. The null of i.i.d. implies that $C_{m,n}(\varepsilon) = C_{1,n}(\varepsilon)^m$ but the converse is not true.

8. In a personal communication, LeBaron recommends the use of embedding dimension from 2 to 5 at sample sizes comparable to ours.

9. LeBaron recommends running the BDS test as a one-tailed test. According to him, the argument is that for any sensible alternative to i.i.d., BDS should be large and positive.

10. We have computed the autocorrelation coefficients of the return series for all ASEAN-5 exchange rates up to 30 lags. The choice of 30 lags is based on the reason that this covers the residuals for a full month period.

11. According to Barnett *et al.* (1995, 1997), filtering out all possible linear possibilities with certainty is difficult at best, but nevertheless pre-filtering by ARIMA fit is often viewed as a reputable means for pre-whitening. However, for simplicity, most researchers (see example, Hsieh, 1989, 1991; Steurer, 1995; Brooks 1996; Barkoulas and Travlos, 1998; Opong *et al.*, 1999; Mahajan and Wagner, 1999) fit the autoregressive AR (p) model to the data for linear filtering. According to Brooks (1996), the process of log differencing has already removed the unit root in the series, and since any moving average model can also be represented by an infinite order autoregression, the class of possible linear specifications is restricted to those of an autoregressive form.

12. This concern is well directed since much of the Monte Carlo research that has been published on the BDS test (see example, Brock *et al.*, 1991) has emphasized the pre-testing issue and the potential dependence of the properties of the test on the prior linear filter. Some of the test's sensitivity to non-linearity could be as a result of remaining linear dynamics in the data. However, the Hinich bispectrum test has a nice property that it is not confounded by linear serial dependence remaining in the data due to imperfect pre-whitening.

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